A Game-theoretic and Algorithmic Study of the Toll Rates of Hong Kong Road Tunnels

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Outline

- Introduction
- Methodology
- Experimental Result
- Conclusion & Limitations

Introduction

Congestion

Figure 1: Cross-Harbour Tunnel

History of Congestion Pricing

- Singapore – “Area License Scheme”
  - Priced zone, time-based charges
- Cambridge, England (Not implemented)
  - Priced zone, congestion-based charges
- California, US – “91 Express Lanes”
  - Extra priced lanes
- Netherlands (Not implemented)
  - Road pricing system

Limitations of Past Schemes

Cases of -

- ALS
  - "Crude", high cost imposed to commuters
- England; Netherland
  - Complicated, unpredictable
  - Little support from government/public
- 91 Express Lane
  - Extra investment

Congestion in Highway/Tunnel

- Cases comparison

Figure 1: Cross-Harbour Tunnel

Figure 2: Evergreen Point Floating Bridge (SR 520 Bridge)

Methodology

- Congestion Game
  - Resources - *tunnels*
  - Players - *drivers*
  - Payoff functions - *toll rates*
  - Equilibrium

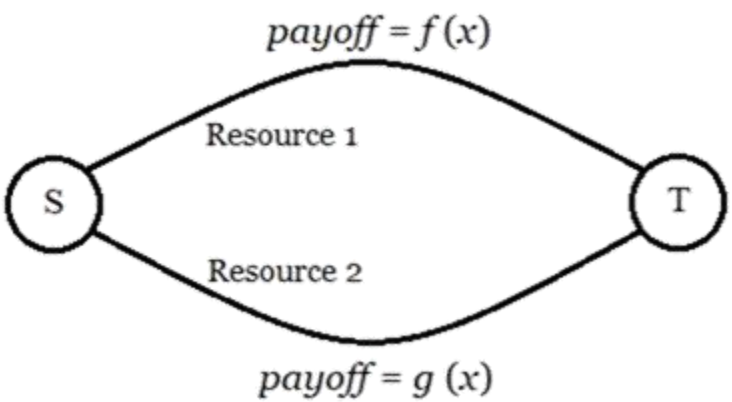
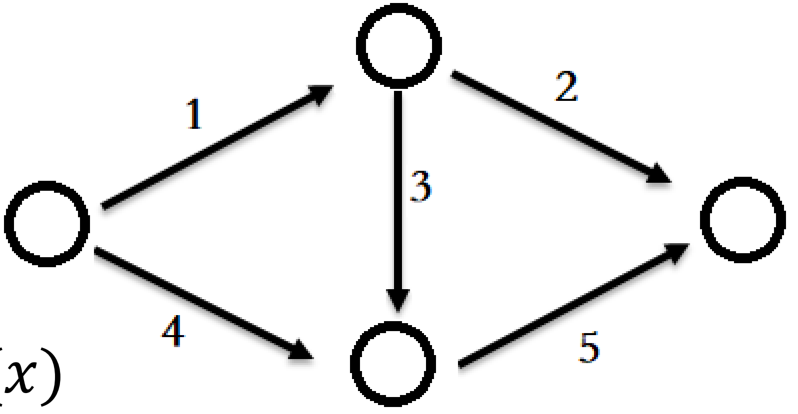


Figure 4: a illustration of congestion game with a Pigou-like network

Methodology

- (Potential) Optimal Solution



Assuming: flow $:= x_4$ payoff $:= f_4(x)$

Total Payoff $= x_4 \ * f_4(x_4)$

Gradient: $f_4(x_4) + x_4 \ * \ f_4'(x_4)$

Payoff' = Payoff + Extra cost (toll) = Gradient

Toll $= x_4 \ * \ f_4'(x_4)$

Methodology

- Modeling
  - Cannot start everything from scratch
- Self-proposed scenario

  In a 50-day duration:

  - Resources *– 2 tunnels*
  - Players *– 100 drivers*
  - Payoff functions *– f(congestion, toll)*
  - Other settings

Methodology

- Self-proposed scenario
  - ……
  - Payoff function *– f(congestion, toll)*

$$Payoff = Achievement\ (= 1) - Congestion - Toll$$

Methodology

- Self-proposed scenario
  - Payoff function

$$\text{Payoff} = \text{Achievement } (= 1) - \text{Congestion} - \text{Toll}$$

  - *Following question:*

What is the value of "congestion"?

  - *Two proposals:*

$$\text{Congestion} = \frac{\text{Number of player}}{Total\ Number\ of\ players}$$

$$\frac{C}{Maximum\ Utility - Number\ of\ Players}$$

Methodology

- Self-proposed scenario
  - Other settings
  1. Initial flow distribution
  2. The probability of switching path choice

Methodology

- (Based on the scenario)
- Propose a congestion-driven toll rates
  - Rate with fixed increasing ratio
  - ***Bounded*** Rate with fixed increasing ratio
  - (Potential) Enhancement

Methodology - summary

- Modeling
- Self-proposed scenario
- Proposing a congestion-driven toll rates

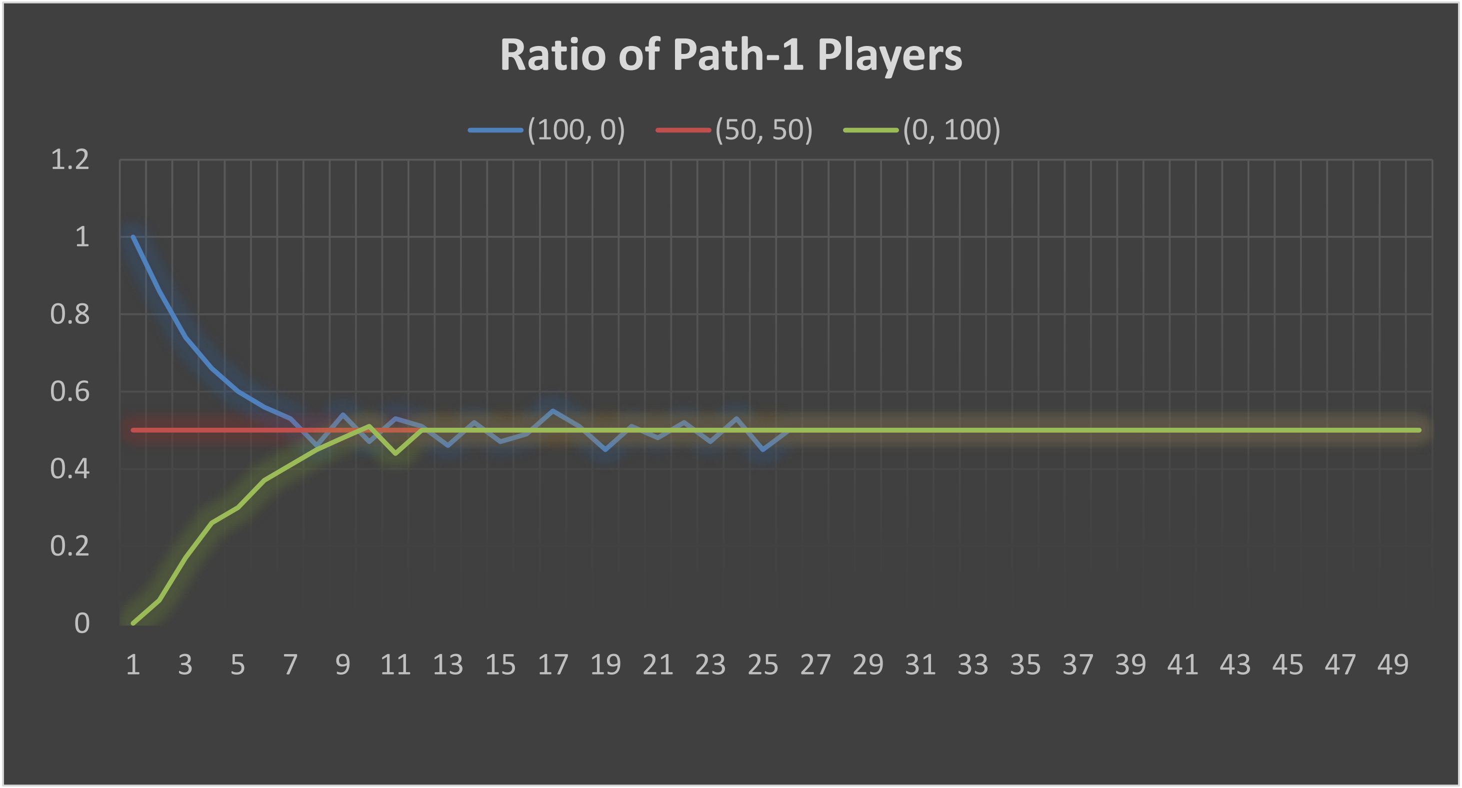
Experimental Result

Structure:

1. Base case – paths without toll
2. Paths with fixed tolls
   - Initial flow distribution
   - Switching probability
3. Paths with congestion-driven toll rates
4. Paths with bounded toll rates
5. (Additional testing)

Experimental Result

- Base case – paths without toll



Experimental Result

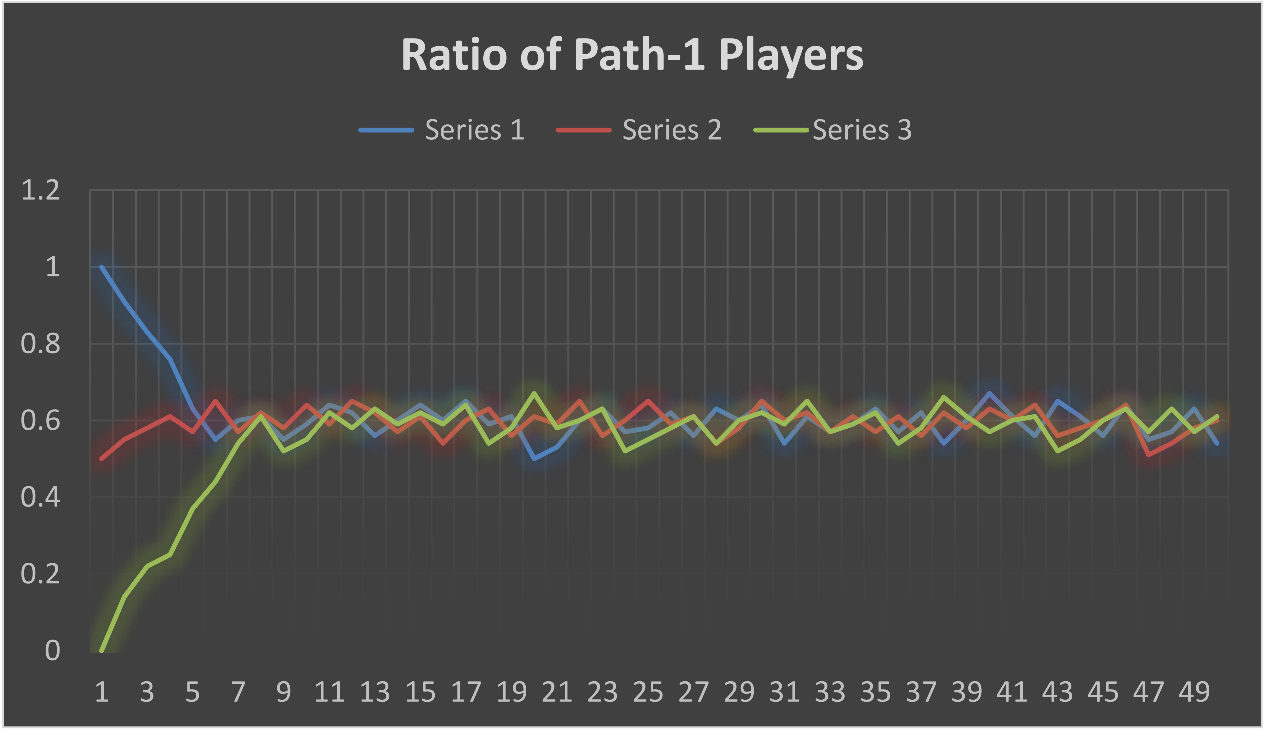
Paths with fixed tolls

- Initial flows: (100, 0), (50, 50), (0, 100) (irrelevant)
- Switching probability: 0.05, 0.10, 0.15 (irrelevant)
- Toll rates: (20, 30), (20, 40), (20, 50) respectively

Experimental Result

Paths with fixed tolls

- Initial flow - (100, 0), (50, 50), (0, 100)

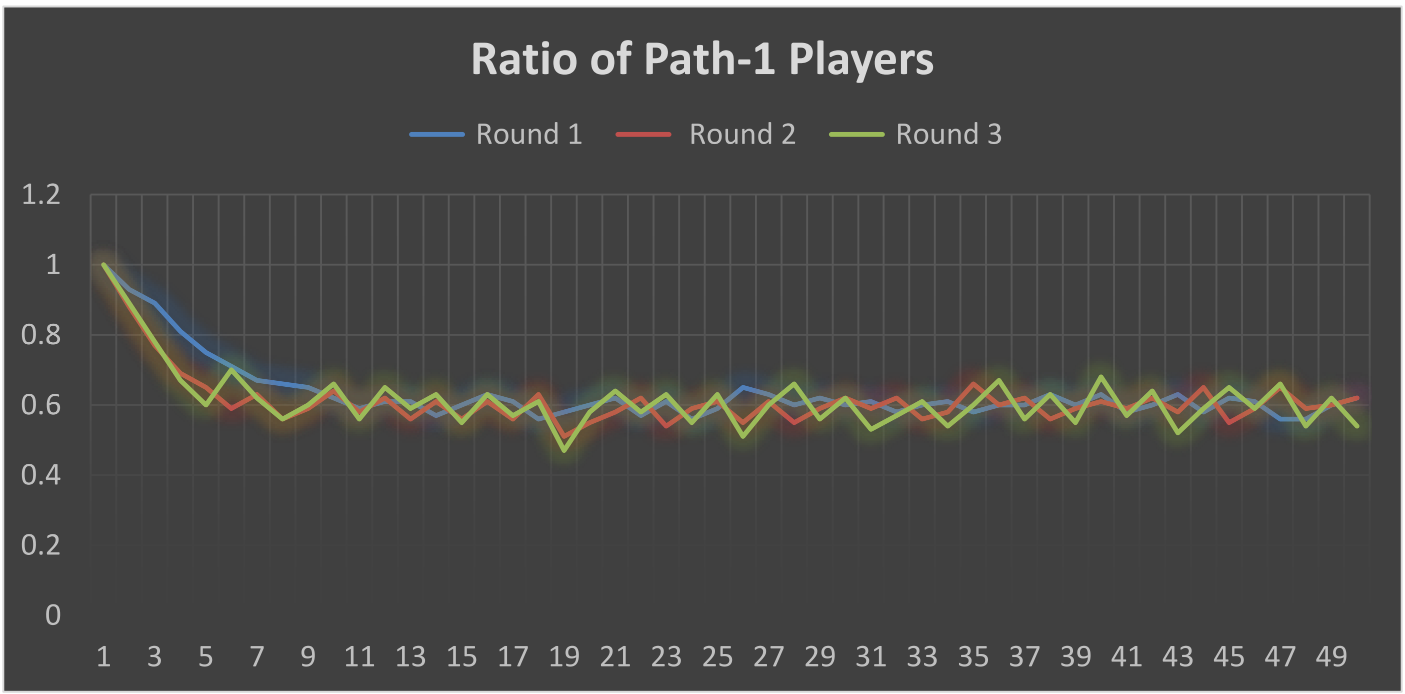


- Average during last 40 days: 0.59

Experimental Result

Paths with fixed tolls

- Switching probability - 0.05, 0.10, 0.15



Experimental Result

Paths with fixed tolls

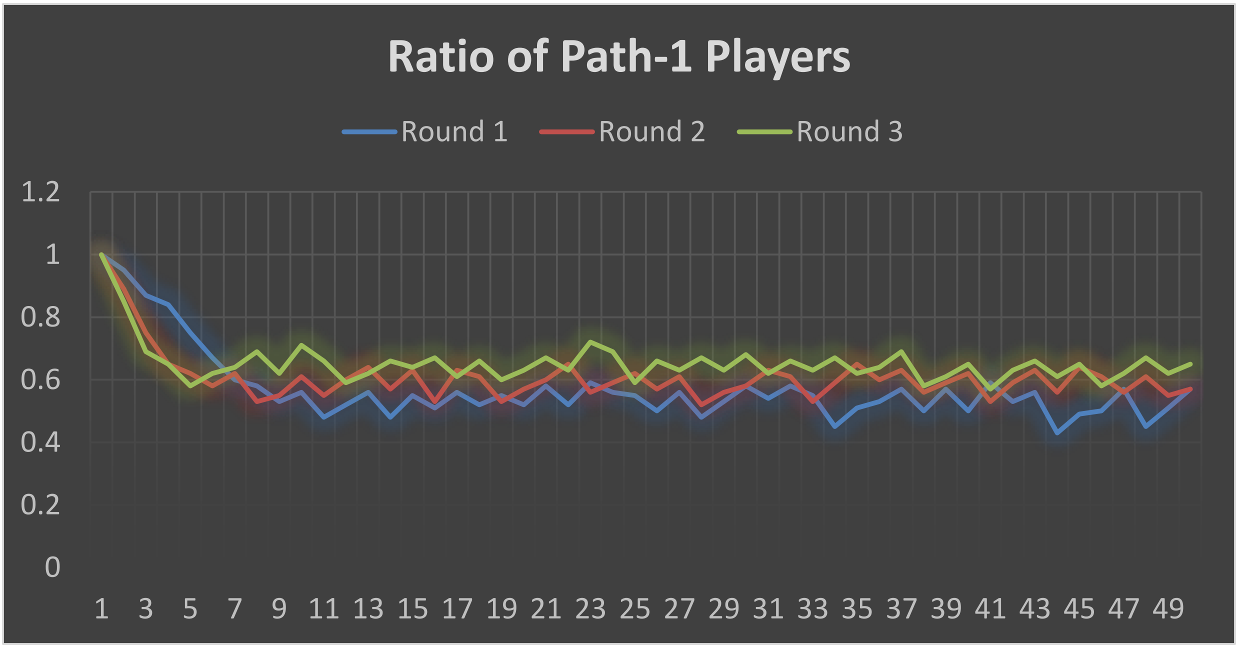
- Switching probability - 0.05, 0.10, 0.15

- Average during last 40 days:
  - 0.60, 0,59, 0.59
- Standard deviation during last 40 days:
  - 0.022, 0.033, and 0.049

Experimental Result

Paths with fixed tolls

- Toll rates: (20, 30), (20, 40), (20, 50) respectively



- Average: 0.53, 0.59 and 0.64

Experimental Result

Paths with congestion-driven toll rates

- Proposed algorithm:

  While flow i > flow j:

  increase the rate of path i by fixed ratio R

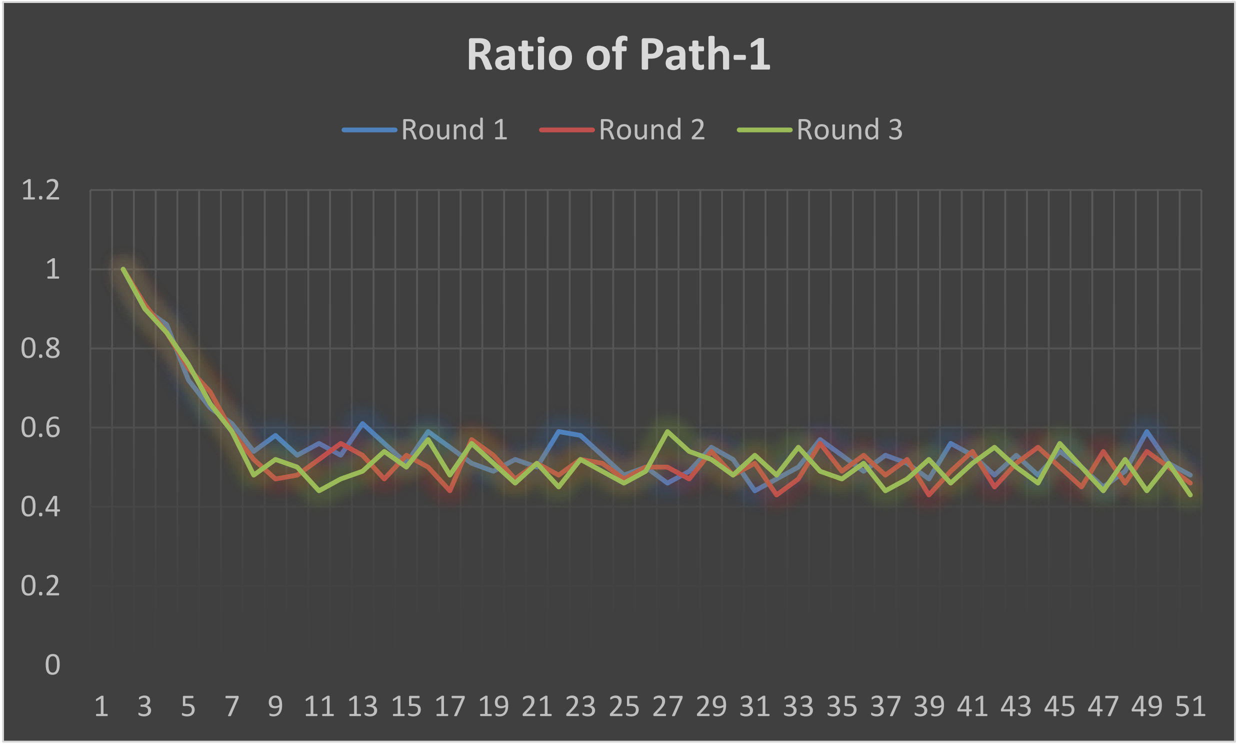
  decrease the rate of path j by R

- R = 0.02, 0.05, 0.10

Experimental Result

Paths with congestion-driven toll rates

– Proposed algorithm:

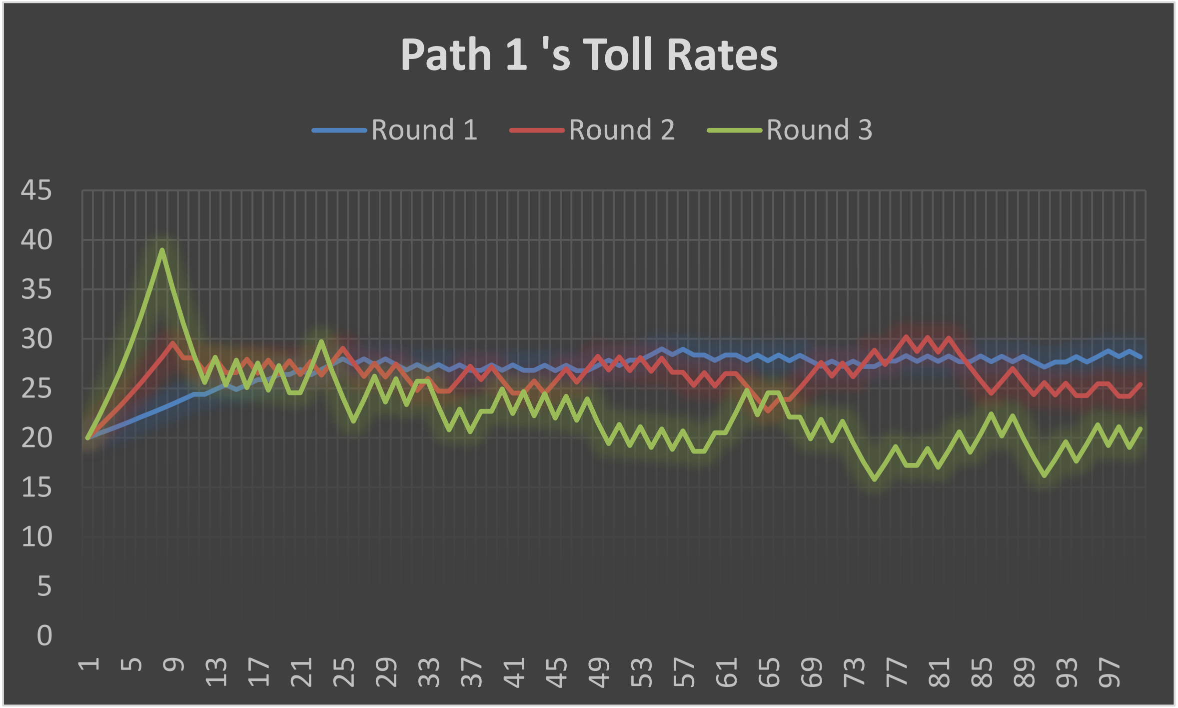


– Average: 0.50, 0.49, 0.50

Experimental Result

Paths with congestion-driven toll rates

– Toll rates' trend



– Average toll rates on (50, 99): 27.9, 26.3 and 19.97

Experimental Result

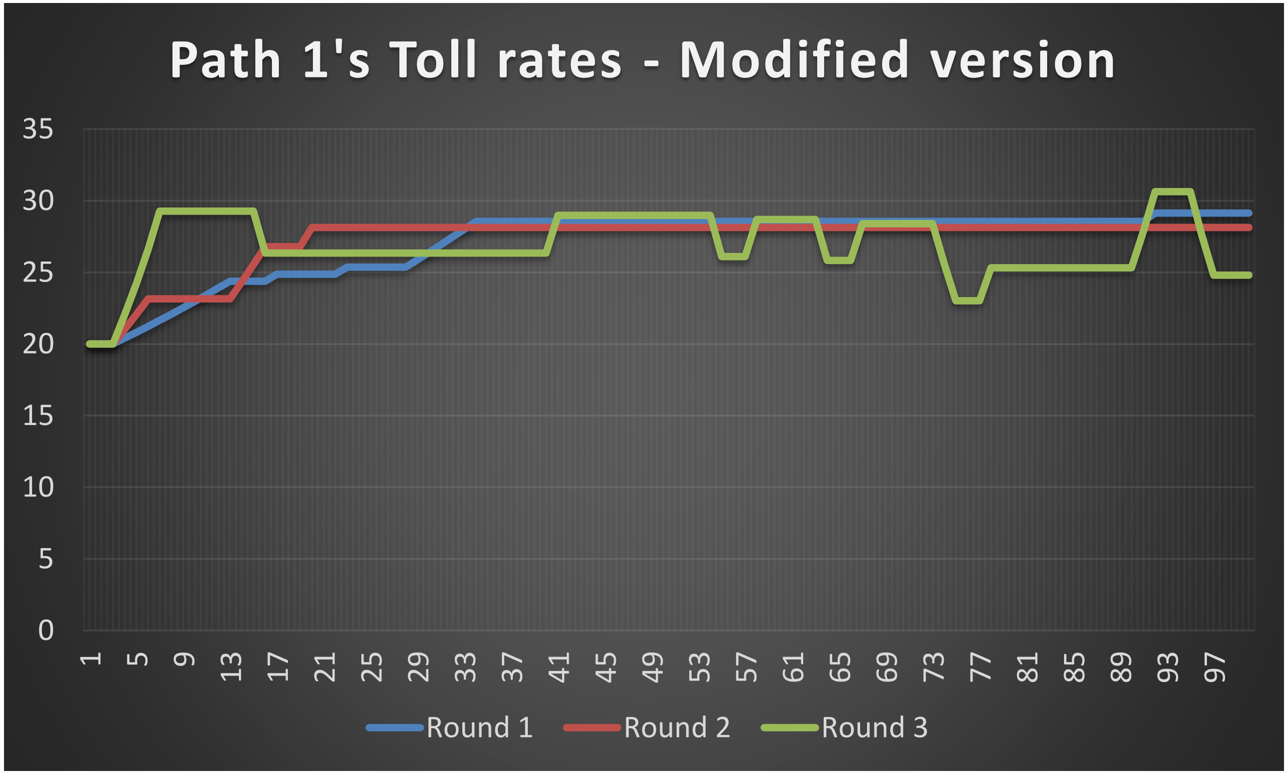
Paths with congestion-driven toll rates

- Toll rates' trend
  - As the daily increment ratio R increases, the toll rates decreases with time.
- Reasoning of the phenomena
  - Program being over-sensitive to the small congestion
  - $(1 - 0.1) * (1 + 0.1) = 0.99 < 1,$

Experimental Result

Paths with congestion-driven toll rates

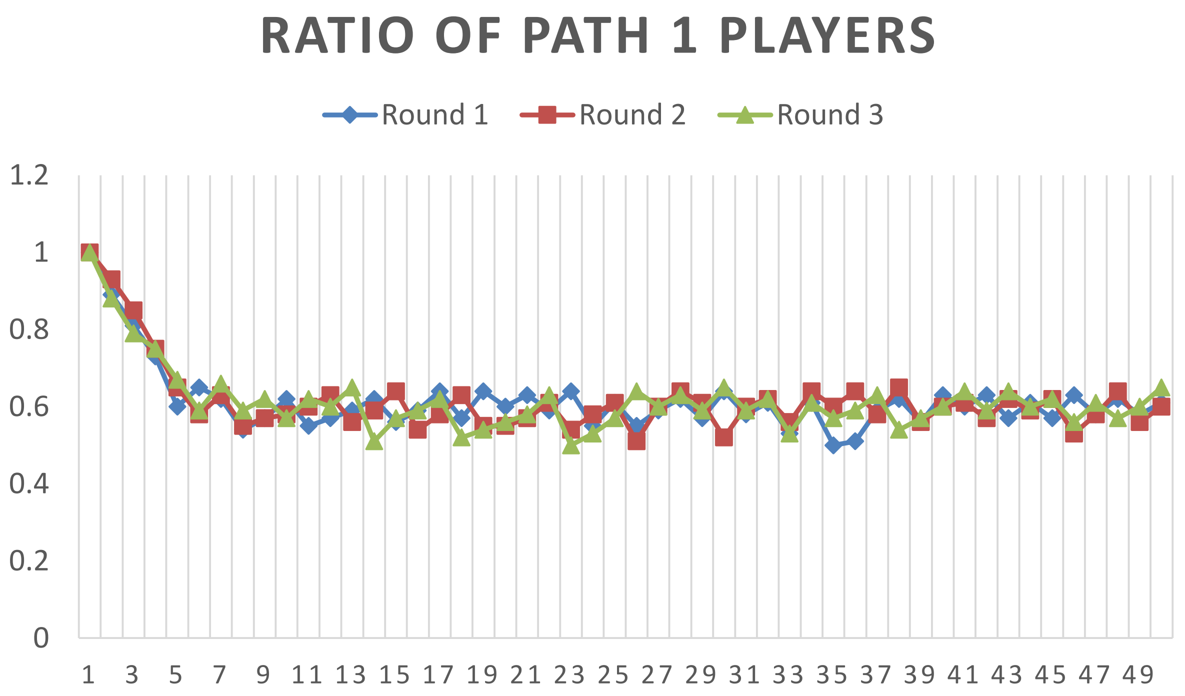
– Modified algorithm:
– New mechanism: 3 consecutive congestions trigger the change



Experimental Result

Paths with bounded toll rates

- Bounds for two paths' rates: [10, 20], [40, 50]
- Predictable result – same as "fixed toll rates"



Experimental Result

Additional testing:

- Formula of congestion being applied

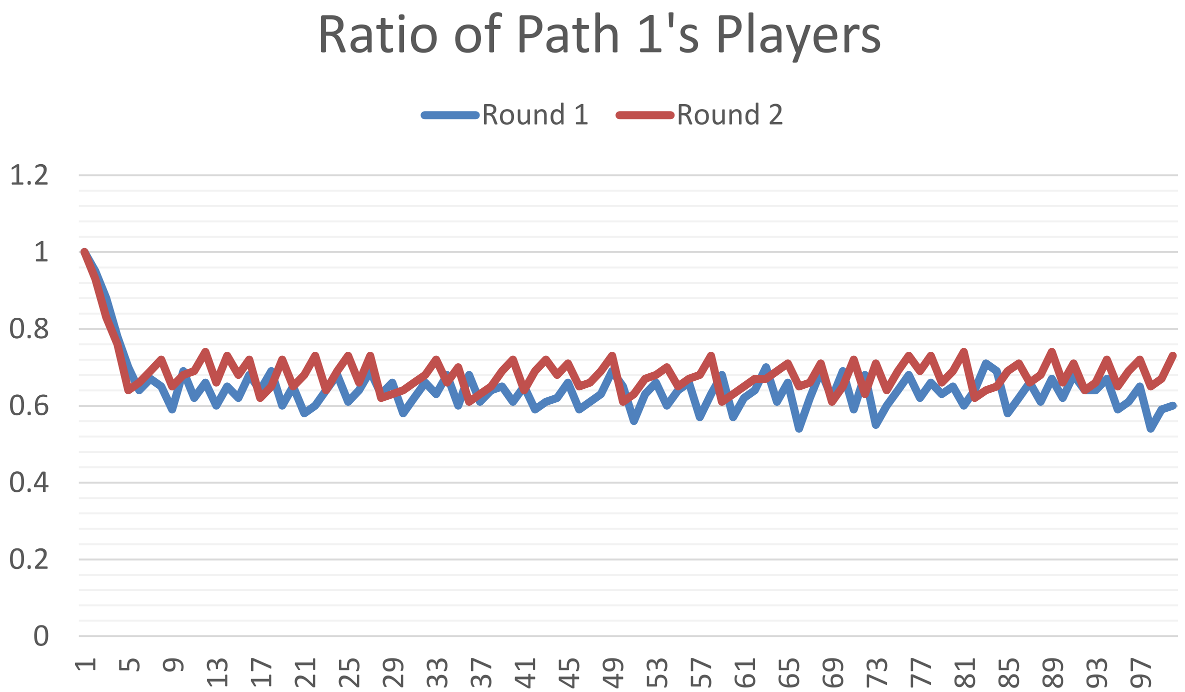
$$\frac{\text{Number of player}}{Total\ Number\ of\ players}$$

- Substitute formula

$$\frac{C}{Maximum\ Utility - Number\ of\ Players}$$

Experimental Result

Additional testing:



Ratio converges to 0.63 and 0.68

"Unsurprising" result

Discussion & Conclusion

1. Base case – paths without toll
2. Paths with fixed tolls
   - Initial flow distribution
   - Switching probability
3. Paths with congestion-driven toll rates
   - Issue occurred: decreasing trend of both toll rates
   - Algorithm being over-sensitive
   - Fixed with new mechanism (consecutive congestion)
4. Paths with bounded toll rates
   - Equivalent to case of “fixed tolls”
5. (Additional testing)

Limitations

- Model being hypothetical
- Feasibility
  - Real-life tolls are always bounded
  - Even flow distribution (no relative congestion) does not exist
- Complication of the real-life congestion
  - Irrational decision by players
  - Unpredictable emergencies

Reference

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Figures

Figure 1. Cross-Harbour Tunnel, Retrieved from:
https://upload.wikimedia.org/wikipedia/commons/thumb/a/aa/HK_Cross_Harbour_Tunnel.jpg/1024px-HK_Cross_Harbour_Tunnel.jpg

Figure 2. SR 520 Bridge, Retrieved from:
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Figure 3. SR 520 Bridge Toll rates, Retrieved from:
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Thank you.