A Game-theoretic and Algorithmic Study of the Toll Rates of Hong Kong Road Tunnels

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Outline

• Introduction
• Methodology
• Experimental Result
• Conclusion & Limitations
Introduction

Congestion

Figure 1: Cross-Harbour Tunnel
History of Congestion Pricing

- Singapore – “Area License Scheme”
  - Priced zone, time-based charges
- Cambridge, England (Not implemented)
  - Priced zone, congestion-based charges
- California, US – “91 Express Lanes”
  - Extra priced lanes
- Netherlands (Not implemented)
  - Road pricing system
Limitations of Past Schemes

Cases of -

• ALS
  – “Crude”, high cost imposed to commuters

• England; Netherland
  – Complicated, unpredictable
  – Little support from government/public

• 91 Express Lane
  – Extra investment
Congestion in Highway/Tunnel

• Cases comparison

Figure 1: Cross-Harbour Tunnel

Figure 2: Evergreen Point Floating Bridge (SR 520 Bridge)
Methodology

• Congestion Game
  – Resources - *tunnels*
  – Players - *drivers*
  – Payoff functions - *toll rates*
  – Equilibrium

Figure 4: a illustration of congestion game with a Pigou-like network
Methodology

• (Potential) Optimal Solution

Assuming: flow := $x_4$ payoff := $f_4(x)$

Total Payoff = $x_4 \times f_4(x_4)$

Gradient: $f_4(x_4) + x_4 \times f_4'(x_4)$

Payoff’ = Payoff + Extra cost (toll) = Gradient

Toll = $x_4 \times f_4'(x_4)$
Methodology

• Modeling
  – Cannot start everything from scratch

• Self-proposed scenario
  In a 50-day duration:
  – Resources – 2 tunnels
  – Players – 100 drivers
  – Payoff functions – $f(\text{congestion}, \text{toll})$
  – Other settings
Methodology

• Self-proposed scenario
  – ……
  – Payoff function – $f(\text{congestion}, \text{toll})$

Payoff = Achievement ($= 1$) – Congestion – Toll
Methodology

• Self-proposed scenario
  – Payoff function
    Payoff = Achievement (= 1) – Congestion – Toll
  – Following question:
    What is the value of “congestion”?  
  – Two proposals:
    Congestion = \[
    \frac{\text{Number of players}}{\text{Total Number of players}} - C
    \]
    \[
    = \frac{C}{\text{Maximum Utility} - \text{Number of Players}}
    \]
Methodology

• Self-proposed scenario
  – Other settings
  1. Initial flow distribution
  2. The probability of switching path choice
Methodology

- (Based on the scenario)

- Propose a congestion-driven toll rates
  - Rate with fixed increasing ratio
  - **Bounded** Rate with fixed increasing ratio
  - (Potential) Enhancement
Methodology - summary

• Modeling

• Self-proposed scenario

• Proposing a congestion-driven toll rates
Experimental Result

Structure:

1. Base case – paths without toll
2. Paths with fixed tolls
   - Initial flow distribution
   - Switching probability
3. Paths with congestion-driven toll rates
4. Paths with bounded toll rates
5. (Additional testing)
Experimental Result

- Base case – paths without toll
Experimental Result

Paths with fixed tolls

- Initial flows: (100, 0), (50, 50), (0, 100) (irrelevant)
- Switching probability: 0.05, 0.10, 0.15 (irrelevant)
- Toll rates: (20, 30), (20, 40), (20, 50) respectively
Experimental Result

Paths with fixed tolls

- Initial flow - (100, 0), (50, 50), (0, 100)
- Average during last 40 days: 0.59
Experimental Result

Paths with fixed tolls

– Switching probability - 0.05, 0.10, 0.15
Experimental Result

Paths with fixed tolls

– Switching probability - 0.05, 0.10, 0.15

– Average during last 40 days:
  • 0.60, 0.59, 0.59

– Standard deviation during last 40 days:
  • 0.022, 0.033, and 0.049
Experimental Result

Paths with fixed tolls

- Toll rates: (20, 30), (20, 40), (20, 50) respectively

- Average: 0.53, 0.59 and 0.64
Experimental Result

Paths with congestion-driven toll rates

– Proposed algorithm:
  While flow $i > flow \ j$:
    increase the rate of path $i$ by fixed ratio $R$
    decrease the rate of path $j$ by $R$

– $R = 0.02, 0.05, 0.10$
Experimental Result

Paths with congestion-driven toll rates

– Proposed algorithm:

– Average: 0.50, 0.49, 0.50
Experimental Result

Paths with congestion-driven toll rates

– Toll rates’ trend

– Average toll rates on (50, 99): 27.9, 26.3 and 19.97
Experimental Result

Paths with congestion-driven toll rates

• Toll rates’ trend
  – As the daily increment ratio $R$ increases, the toll rates decreases with time.

• Reasoning of the phenomena
  – Program being over-sensitive to the small congestion
  
  – $(1 - 0.1) \times (1 + 0.1) = 0.99 < 1,$
Experimental Result

Paths with congestion-driven toll rates

- Modified algorithm:
- New mechanism: 3 consecutive congestions trigger the change

![Path 1's Toll rates - Modified version](chart.png)
Experimental Result

Paths with bounded toll rates

- Bounds for two paths’ rates: [10, 20], [40, 50]
- Predictable result – same as “fixed toll rates”

RATIO OF PATH 1 PLAYERS

Round 1 | Round 2 | Round 3
Experimental Result

Additional testing:

• Formula of congestion being applied

\[
\frac{\text{Number of player}}{\text{Total Number of players}}
\]

• Substitute formula

\[
\frac{C}{\text{Maximum Utility} - \text{Number of Players}}
\]
Experimental Result

Additional testing:

Ratio of Path 1's Players

Ratio converges to 0.63 and 0.68

“Unsurprising” result
Discussion & Conclusion

1. Base case – paths without toll
2. Paths with fixed tolls
   – Initial flow distribution
   – Switching probability
3. Paths with congestion-driven toll rates
   – Issue occurred: decreasing trend of both toll rates
   – Algorithm being over-sensitive
   – Fixed with new mechanism (consecutive congestion)
4. Paths with bounded toll rates
   – Equivalent to case of “fixed tolls”
5. (Additional testing)
Limitations

• Model being hypothetical
• Feasibility
  – Real-life tolls are always bounded
  – Even flow distribution (no relative congestion) does not exist
• Complication of the real-life congestion
  – Irrational decision by players
  – Unpredictable emergencies
Reference

Documents


Reference

Documents

• “Toll Rates of Road Tunnels and Lantau Link.” Transport Department of HKSAR. 27 September 2017.


• “Nash equilibrium”. Wikimedia Foundation, Inc. 27 November 2017.
Reference

Figures

Figure 1. Cross-Harbour Tunnel, Retrieved from:

Figure 2. SR 520 Bridge, Retrieved from:
https://c1.staticflickr.com/8/7796/17033643714_ca5ec0bb39_b.jpg

Figure 3. SR 520 Bridge Toll rates, Retrieved from:
https://www.wsdot.wa.gov/Tolling/520/520tollrates.htm
Thank you.