A Game-theoretic and Algorithmic Study of the Toll Rates of Hong Kong Road Tunnels

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Outline

• Introduction
• Methodology
• Progress
• Difficulties & Future Plan
Introduction

Congestion

Figure 1: Cross-Harbour Tunnel
Methodology

• Toll Rates

Figure 2: Evergreen Point Floating Bridge (SR 520 Bridge)
Methodology

- Toll Rates

<table>
<thead>
<tr>
<th>Monday - Friday</th>
<th>Good To Go! Pass</th>
<th>Pay By Mail</th>
</tr>
</thead>
<tbody>
<tr>
<td>Midnight to 4:59 a.m.</td>
<td>$1.25</td>
<td>$3.25</td>
</tr>
<tr>
<td>5 a.m. to 5:59 a.m.</td>
<td>$2.00</td>
<td>$4.00</td>
</tr>
<tr>
<td>6 a.m. to 6:59 a.m.</td>
<td>$3.40</td>
<td>$5.40</td>
</tr>
<tr>
<td>7 a.m. to 8:59 a.m.</td>
<td>$4.30</td>
<td>$6.30</td>
</tr>
<tr>
<td>9 a.m. to 9:59 a.m.</td>
<td>$3.40</td>
<td>$5.40</td>
</tr>
<tr>
<td>10 a.m. to 1:59 p.m.</td>
<td>$2.70</td>
<td>$4.70</td>
</tr>
<tr>
<td>2 p.m. to 2:59 p.m.</td>
<td>$3.40</td>
<td>$5.40</td>
</tr>
<tr>
<td>3 p.m. to 5:59 p.m.</td>
<td>$4.30</td>
<td>$6.30</td>
</tr>
<tr>
<td>6 p.m. to 6:59 p.m.</td>
<td>$3.40</td>
<td>$5.40</td>
</tr>
<tr>
<td>7 p.m. to 8:59 p.m.</td>
<td>$2.70</td>
<td>$4.70</td>
</tr>
<tr>
<td>9 p.m. to 10:59 p.m.</td>
<td>$2.00</td>
<td>$4.00</td>
</tr>
<tr>
<td>11 p.m. to 11:59 p.m.</td>
<td>$1.25</td>
<td>$3.25</td>
</tr>
</tbody>
</table>

Figure 3: Toll rates for passing SR 520
Methodology

- Congestion Game
  - Resources - *tunnels*
  - Players - *drivers*
  - Payoff functions - *toll rates*

Figure 4: a illustration of congestion game with a Pigou-like network
Methodology

• Congestion Game
  – Equilibrium: an approximate result of players’ choices
  – Optimal: the case with minimum total cost (max payoff)
  – “Price of Anarchy” (POA): an index of efficiency

\[
POA = \frac{\text{total cost in equilibrium case}}{\text{total cost in optimal case}}
\]
Progress

• Model development
  • Resource
  • Player
Progress

- Model development
  - Resource

- Current Progress
Progress

• Model development
  • Resource
  • Player

Pure strategy

Mixed strategy
Difficulties & Future Plan

- Math
- Evaluation of our model

• What to do next
  - Evaluation
  - Continue with current model
  - Offering mixed strategy to players
Reference

Documents

Figures
Figure 1. Cross-Harbour Tunnel, Retrieved from: https://upload.wikimedia.org/wikipedia/commons/thumb/a/aa/HK_Cross_Harbour_Tunnel.jpg/1024px-HK_Cross_Harbour_Tunnel.jpg

Figure 2. SR 520 Bridge, Retrieved from: https://c1.staticflickr.com/8/7796/17033643714_ca5ec0bb39_b.jpg

Figure 3. SR 520 Bridge Toll rates, Retrieved from: https://www.wsdot.wa.gov/Tolling/520/520tollrates.htm
Thank you.