

# A Game-theoretic and Algorithmic Study of the Toll Rates of Hong Kong Road Tunnels

ZHANG Zhili

# Outline

- Introduction
- Methodology
- Progress
- Difficulties & Future Plan

# Introduction



## Congestion

Figure 1: Cross-Harbour Tunnel

# Methodology

- Toll Rates



Figure 2: Evergreen Point Floating Bridge (SR 520 Bridge)

# Methodology

- Toll Rates

Monday - Friday	Good To Go! Pass	Pay By Mail
Midnight to 4:59 a.m.	\$1.25	\$3.25
5 a.m. to 5:59 a.m.	\$2.00	\$4.00
6 a.m. to 6:59 a.m.	\$3.40	\$5.40
7 a.m. to 8:59 a.m.	\$4.30	\$6.30
9 a.m. to 9:59 a.m.	\$3.40	\$5.40
10 a.m. to 1:59 p.m.	\$2.70	\$4.70
2 p.m. to 2:59 p.m.	\$3.40	\$5.40
3 p.m. to 5:59 p.m.	\$4.30	\$6.30
6 p.m. to 6:59 p.m.	\$3.40	\$5.40
7 p.m. to 8:59 p.m.	\$2.70	\$4.70
9 p.m. to 10:59 p.m.	\$2.00	\$4.00
11 p.m. to 11:59 p.m.	\$1.25	\$3.25

Figure 3: Toll rates for passing SR 520

# Methodology

- Congestion Game
  - Resources - *tunnels*
  - Players - *drivers*
  - Payoff functions - *toll rates*

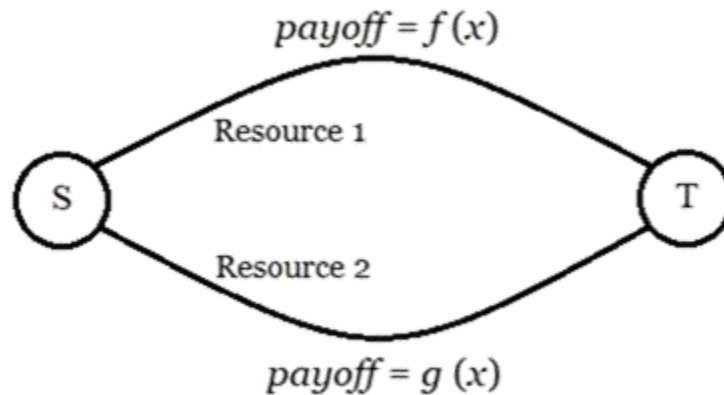


Figure 4: a illustration of congestion game with a Pigou-like network

# Methodology

- Congestion Game

- Equilibrium: an approximate result of players' choices
- Optimal: the case with minimum total cost (max payoff)
- “Price of Anarchy” (POA): an index of efficiency

$$\text{POA} = \frac{\text{total cost in equilibrium case}}{\text{total cost in optimal case}}$$

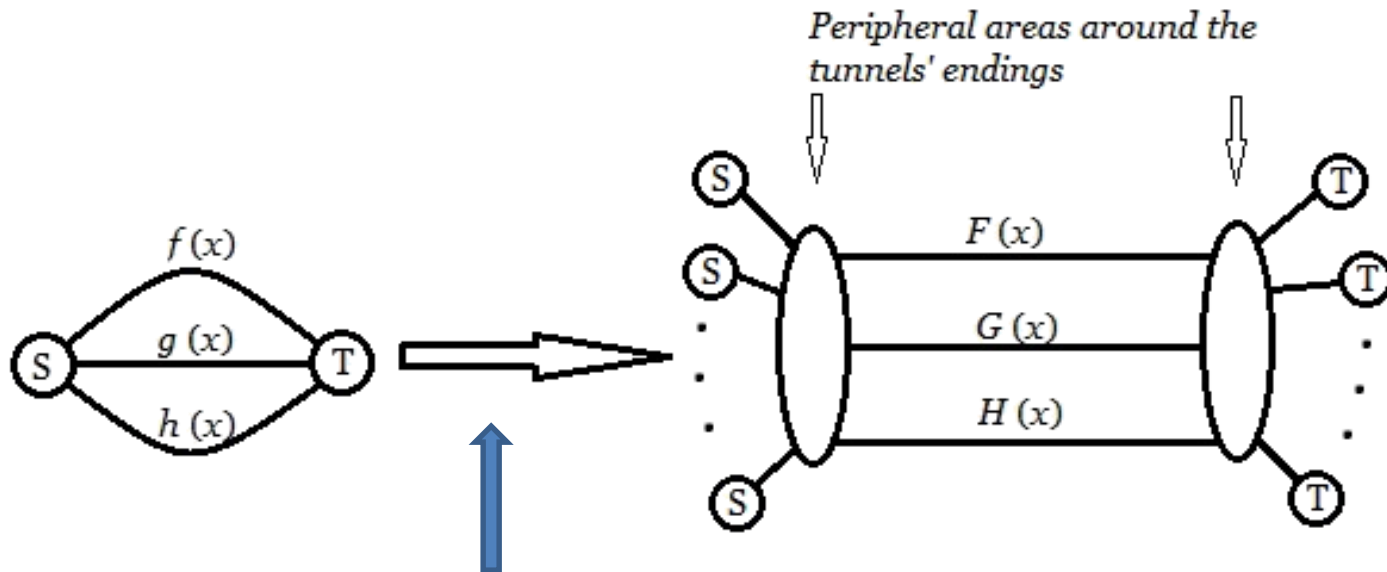
# Progress

- Model development
  - Resource
  - Player



# Progress

- Model development
  - Resource



- Current Progress

# Progress

- Model development
  - Resource
  - Player



Pure strategy



Mixed strategy

# Difficulties & Future Plan

- Math
- Evaluation of our model
- What to do next
  - Evaluation
  - Continue with current model
  - Offering mixed strategy to players

# Reference

## Documents

Nisan, Noam, et al., eds. *Algorithmic game theory*. Vol. 1. Cambridge: Cambridge University Press, 2007.

Cole, Richard, Yevgeniy Dodis, and Tim Roughgarden. *Pricing networks with selfish routing*. CORNELL UNIV ITHACA NY DEPT OF COMPUTER SCIENCE, 2003.

## Figures

Figure 1. Cross-Harbour Tunnel, Retrieved from:

[https://upload.wikimedia.org/wikipedia/commons/thumb/a/aa/HK\\_Cross\\_Harbour\\_Tunnel.jpg/1024px-HK\\_Cross\\_Harbour\\_Tunnel.jpg](https://upload.wikimedia.org/wikipedia/commons/thumb/a/aa/HK_Cross_Harbour_Tunnel.jpg/1024px-HK_Cross_Harbour_Tunnel.jpg)

Figure 2. SR 520 Bridge, Retrieved from:

[https://c1.staticflickr.com/8/7796/17033643714\\_ca5ec0bb39\\_b.jpg](https://c1.staticflickr.com/8/7796/17033643714_ca5ec0bb39_b.jpg)

Figure 3. SR 520 Bridge Toll rates, Retrieved from:

<https://www.wsdot.wa.gov/Tolling/520/520tollrates.htm>

Thank you.