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Project Plan

Project title: A Game-theoretic and Algorithmic study of the toll rates of Hong Kong Road tunnels

Introduction

With the assistance of GPS system and audio broadcast offering real-time traffic information, congestion still seems an insurmountable urban issue prevailing in most metropolises around the world. Specifically, we could notice a pattern that those bridges or tunnels that locates at crucial position in a city, connecting two major areas for example, are more likely under a constant congestion throughout daytime.[1]

Here in Hong Kong, three major tunnels in parallel connects Hong Kong Island to Kowloon. Specifically, the Cross-Harbour Tunnel (abbreviated as CHT) and the Eastern Harbour Crossing (abbreviated as EHC) are more likely suffered from congestion than the Western Harbour Crossing (abbreviated as WHC). From the perspective of theoretic computer science, this system has a relatively low efficiency as the capacity of WHC is not effectively used while CHT and EHC are exploited. This phenomenon could be ascribed to the relatively higher toll rates issued by WHC, twice as the rate issued by EHC and three times as the one issued by CHT. [7]

The difference in toll rates contributes to the uneven flow distribution among harbor tunnels. This motivates us to model the actual congestion problem as the congestion game in the field of Algorithmic Game Theory.[2][5] Specifically, we might draft a start-destination flow network based on geographical reality of harbor tunnels in Hong Kong, see the drivers as rational players and treat the toll rates as a controllable factor among all the factors that incurs a cost onto the players. By manipulating the toll rates, the modeling could potentially lead us to a theoretical solution of congestion, an impartial distribution of driving flows consequently.

Previous researchers have made remarkable effort in exploring the relevant problems. Richard Cole and his colleagues developed a theory on the modeling of pricing network problems, which suggested a solution in the general form and proved its feasibility provided with certain conditions.[4] This result provides a theoretical base and motivates the feasibility of our project to a certain extent. Besides, Hai YAN and his colleague from civil engineering field discussed the

possibility of applying road tolls as a general strategy in tackling queuing and congestion problems, and they also proposed a reasonable algorithm in computing the road tolls and proved the stability of their algorithm under some simple cases.[1][3] While these works have suggested from theoretical perspective the feasibility of controlling congestion traffic with toll rates, it should be noticed that few work was found in applying the relevant theories to a specific congestion problem with the modeling approach, probably as the consequence of the complexity incurred by modeling a real routing problem into a flow network with agents.[6]

In this project, we will focus on the specific case, the congestion problem and uneven driving flow occurred among three harbor tunnels connecting Kowloon and Hong Kong Island. Especially we wish to devote our effort in the design and enhancement of the modeling of the flow network. The reason is two-fold. On the one hand, the three parallel tunnels together with their peripheral areas are of a geographically simple pattern compared with any specific road network of a city. As the tunnels stand out to be our focus, we could probably enhance the quality of our model without incurring more complexity to the model itself. On the other hand, we wish to deliver a solution based on an actual congestion problem, the one in harbor tunnels in this case. That is, by concentrating on the application of algorithmic game theory on reality, we hope to refine our model as closed to the actual behavior as possible, and probably design a reasonable toll-rate computing algorithm proved theoretically to be feasible and effective in our certain case.

The remainder of this project plan as follow. The project objective will be first specified. It will unfold into categories, of which we hope to achieve and present the corresponding improvements of our modeling as well as the solutions. Subsequently the methodology of our project will be delivered. The structure of our project will be revealed, together with the approaches and probable limitation of our project. In the last part a tentative schedule of our project will be presented. The milestones will be set up based on our current knowledge of the problem and anticipation of our progress.

Project Objective

In the generalized sense, our objective is to solve the congestion problem in Hong Kong harbor tunnels with the idea and knowledge of congestion games in the field of algorithmic game theory. Specifically, we wish to develop a sophisticated model which could effectively represent the features of three road tunnels and their

peripheral areas, and reasonably reflect the behavioral pattern of driving flows passing on these tunnels. Based on this model we hope to develop an efficient algorithm that could compute reasonable toll rates for tunnels based on their current situation and we hope these toll rates are strategically useful in controlling the flow among each tunnel and consequently prevent or alleviate the congestion.

From the perspective of the project progression, there are targets that we hope to achieve with regards to the modeling, and the algorithm designing and verification as follow:

Modeling objectives

- 1 Toll rates
 - 1.1 Model without toll rates
 - 1.2 Model introducing fixed rates
 - 1.3 Model introducing demand-dependent rates
- 2 Design of network
 - 2.1 A simplified single-source-single-target graph
 - 2.2 A many-source-many-destination graph
 - 2.3 Taking the actual traffic status into consideration
- 3 Agents (drivers) behavioral pattern
 - 3.1 All agents are rational and using same strategy (making identical choices under same condition)
 - 3.2 Agents with variant choice-making strategy
- 4 Factors that affect agents' choice
 - 4.1 Toll rate
 - 4.2 Estimated passing time (current situation of congestion in another way)
 - 4.3 Drivers' own demand

Solution-generating objectives

For each stage of modelling:

- 1 Prove the existence of ideal toll rates under the given circumstances
- 2 Design an algorithm to compute desired toll rates
 - 2.1 Enhance the algorithm
- 3 Verify the effectiveness of algorithm

Above are the tentative objectives throughout the project. Due to the limitation of resource and ability, adjustment could possibly be made during the progress.

Project Methodology

In this part we will present the rationale for the approaches of our project. Specifically, in accordance with the objectives set up in the previous section, the implementations for a concrete model will be discussed, followed by a brief reasoning of our choice.

The project is mainly based on theoretical modeling of actual cases in Hong Kong harbor tunnels. In the introduction part it has already been shown that the variances toll rates among different paths result in the uneven distribution of driving flows. This indicates a potential availability of controlling driving flows' distribution with meticulously calculated toll rates, and motivates us to have toll rates as the major variable in our model.

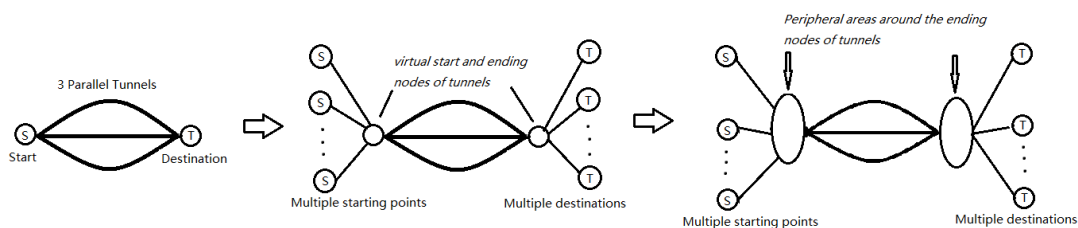


Figure 1: models with different graphical design

As for the graphical network design, a cascading illustration is given in Figure 1. The left graph presents the initial model with three parallel paths barely representing three tunnels, and it also assumes that three tunnels and all the drivers share the exact same start and ending points. Based on this, we will improve the model to the middle, in which drivers have individual starts and destinations, and will discuss the influence on drivers' choices incurred by this modification. Eventually we will take the areas around the ending points of harbor tunnels into consideration, as the traffic of these areas is inevitably an important factor contributing to drivers' selection of tunnel. And in this case, our model is fairly similar to the actual reality.

Regarding the agents in this congestion game modeling, the drivers will first be assumed as rational player exerting the same strategy. In the meantime, toll rates and degree of congestion would be the major factors affecting their choice. Subsequently, an enhanced version of the agents is expected to represent a relative randomness of their choice-making. It is expected to show a reasonable distribution of variant strategies applied among all the drivers, instead of an unanimity which is ideal yet

deviates from reality. And meanwhile we hope to deliver more factors into consideration, apart from the previous two.

The establishment of solution to the congestion game is on full account of the model we build. Previous research has proposed some toll-rate computing algorithm based on their generalized model of the problem. ^ Their solution will be evaluated and possibly referred. Meanwhile we hope to deliver an effective algorithm on full basis of our own model. Although the solution designing and verification constitute a major part of our project, it should be admitted that this is as much as we can envision and deliver at the current stage.

Project Schedule and Milestones

The project will proceed at the same pace as our model improves. As the modeling objectives of different stages have been proposed in the previous section, they will also underlie our tentative schedule and milestones setup. The timeline presented as follow (number notations represent the corresponding objective mentioned in ‘Project Objective’):

- *Current to Oct 15, 2017:*
 1. Previous research work collection and reference
 2. Analysis of simple model without toll rate, single-s-single-t network with ideal agents – Obj. 1.1, 2.1 and 3.1
- *Oct 15, 2017 to Nov 15, 2017:*
 1. Introducing fixed rates – Obj. 1.2
 2. Enhance the graph to many-source-many-destination – Obj. 2.2
 3. Agents concerning toll rates and time delay – Obj. 4.1, 4.2
- *Nov 15, 2017 to Dec 31, 2017:*
 1. Algorithm design for previous model
 2. Introducing demand-dependent rates – Obj. 1.3
- *Jan 1, 2018 to Feb 1, 2018:*
 1. Refinement of previous model – Obj. 1.3
 2. Algorithm design for the current model
(First presentation on *Jan 8, 2018*; Phase 2 delivery on *Jan 21, 2018*)
- *Feb 1, 2018 to Mar 15, 2018:*
 1. Introducing agents with variant strategies – Obj. 3.2
 2. Considering other factors affecting drivers’ strategy – Obj. 4.3
 3. Improving algorithm
- *Mar 15, 2018 to Apr 15, 2018:*

1. (Optional) Considering real-time traffic status – Obj. 2.3
 2. Finalizing the project
(Phase 3 delivery on *Apr 15, 2018*)
- *Apr 16, 2018 – After:*
 1. Final presentation and exhibition

Reference:

- [1] Yan, Hai, and William HK Lam. "Optimal road tolls under conditions of queueing and congestion." *Transportation Research Part A: Policy and Practice* 30.5 (1996): 319-332.
- [2] Bell, Michael GH. "A game theory approach to measuring the performance reliability of transport networks." *Transportation Research Part B: Methodological* 34.6 (2000): 533-545.
- [3] Lo, Hong K., and Anthony Chen. "Traffic equilibrium problem with route-specific costs: formulation and algorithms." *Transportation Research Part B: Methodological* 34.6 (2000): 493-513.
- [4] Cole, Richard, Yevgeniy Dodis, and Tim Roughgarden. *Pricing networks with selfish routing*. CORNELL UNIV ITHACA NY DEPT OF COMPUTER SCIENCE, 2003.
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- [6] Zhou, Bojian, et al. "A trial-and-error congestion pricing scheme for networks with elastic demand and link capacity constraints." *Transportation Research Part B: Methodological* 72 (2015): 77-92.
- [7] "Toll Rates of Road Tunnels and Lantau Link." Transport Department of HKSAR. 27 September 2017.
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