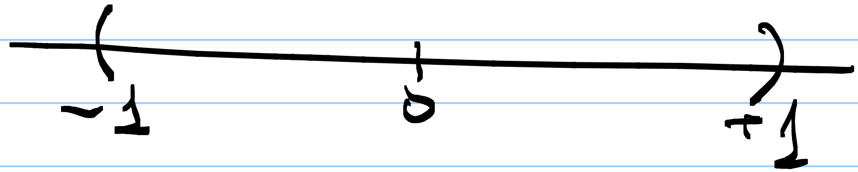
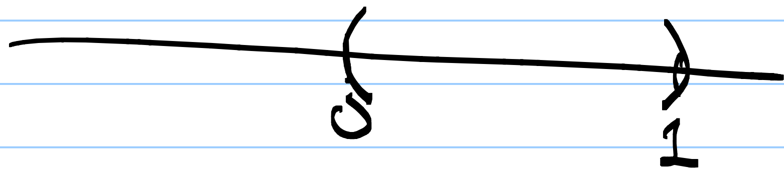
Note Title 30/9/16

More Examples.

$A = (-1, 1)$



$B = (0, 1)$



Is there a bijection ?

$f : A \to B$

$$f(x) = \frac{X+1}{2}.$$

$$f(x) \in (0, 1)$$

Suppose $x \in (-1, 1)$

$$-1 < x < 1$$

$$0 < \frac{x+1}{2} < 1$$

$$\parallel$$

(i) Injection? $x_1, x_2 \in (-1, 1)$

$f(x_1) = f(x_2) \Rightarrow \frac{x_1+1}{2} = \frac{x_2+1}{2}$

$\Rightarrow x_1 = x_2$

(ii) Surjective? (1) let $y \in B = (0, 1)$

$$\boxed{\begin{aligned} \frac{x+1}{2} &\overset{?}{=} y \\ x &= 2y-1 \end{aligned}}$$

(2)

Pick $x := 2y-1$ $\quad f(x) = \frac{(2y-1)+1}{2} = y.$

Th If B is countable and $S \subseteq B$, then S is countable.

Pf (i) B is finite, and $S \subseteq B$. $\Rightarrow$ S is finite ✓

(ii) B is infinite, B countable

$\exists$ bijection $f: B \to \mathbb{Z}^+$.

→ W.L.O.G., by renaming, can assume $B = \{1, 2, 3, \ldots\}$

$S \subseteq B$ → (i) S finite ✓

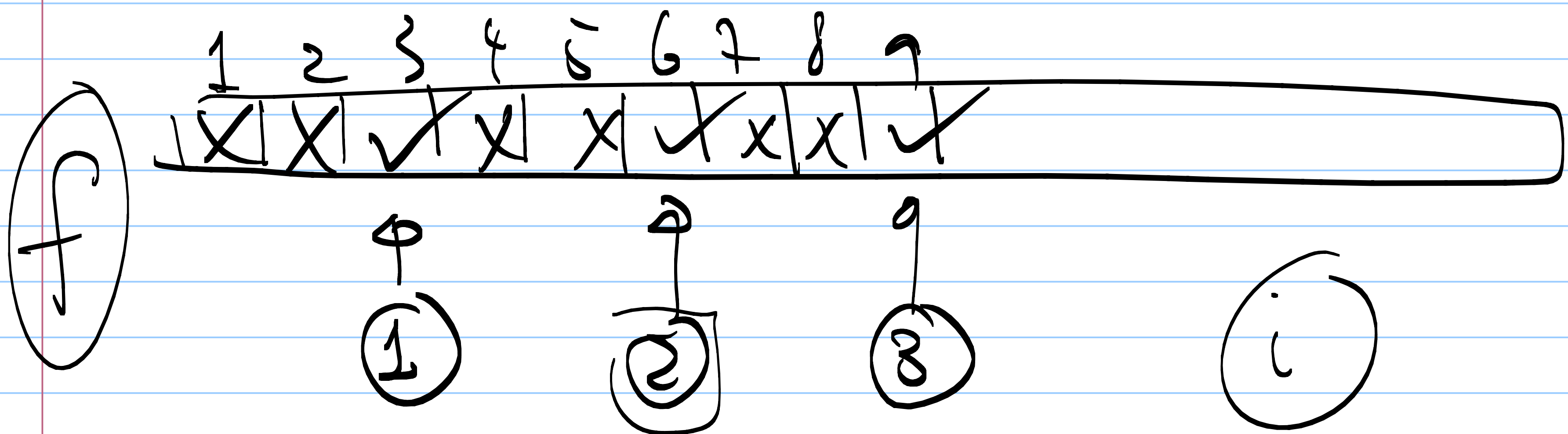
(ii) S is infinite.

$f : S \to \mathbb{Z}^+$

S is an infinite subset of $\mathbb{Z}^+$.

Idea : "sort" elements in S in ascending order,

and assign index from 1 . . .



(i) injection. because each i used as "name" by at most 1 person

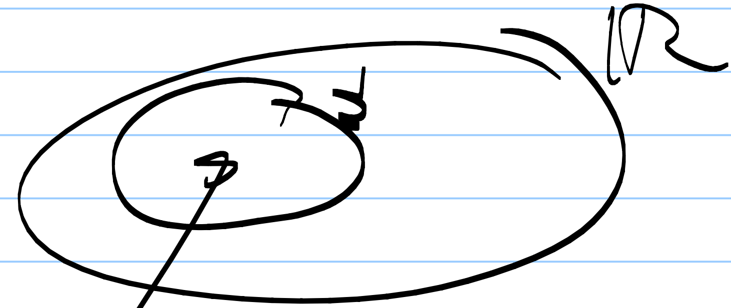
(ii) <u>surjective</u>: because S is infinite.

Hence, every integer $i \in \mathbb{Z}^+$ will be used as a name eventually

Alternative Proof that $\mathbb{R}$ is uncountable

$0.\underbrace{\qquad\qquad\qquad\qquad}_{\text{digits}}$

S only use digits 0 or 1



Suppose S is countable.

		digits							
1	.	0	1	0	1	1	1	1	1
2	.	1	0	1	1	1	1	0	0
3	.			1					
4	.				0				
5	.					1			
6	.								

Diagonalization

Create a real no.:

(i) For digit i,
look at row i,
flip the corresponding digit i

$.11010.$

Can this no. appear in row j ?

j^{th} digit of no. constructed ?

$\to$ flipped from j^{th} digit of row j.

$\mathbb{Z}^+$

$\mathbb{P}(\mathbb{Z}^+)$

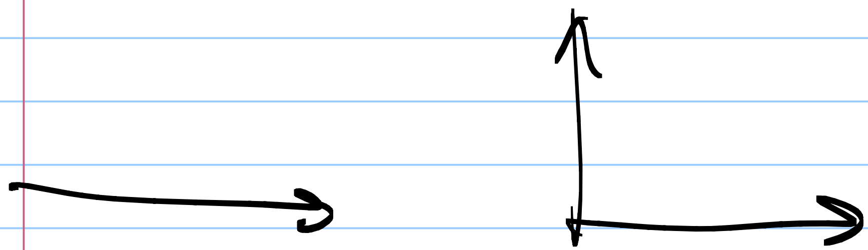
$\mathbb{P}(\mathbb{P}(\mathbb{Z}^+))$

$\vdots$

no bijection S to $\mathbb{P}(S)$

Examples

$\mathbb{R}$ $\qquad$ $\mathbb{R} \times \mathbb{R}$. same cardinality.



$.014\ldots \longleftrightarrow (.04\ldots, .0\ldots)$.

$.1\,a_1 a_2 a_3 a_4 \ldots$ $\qquad$ $\mathbb{R}$

$\underline{.a_1 a_3 a_5}$, $\underline{.a_2 a_4 a_6 \ldots}$ $\qquad$ $\mathbb{R}\times\mathbb{R}$