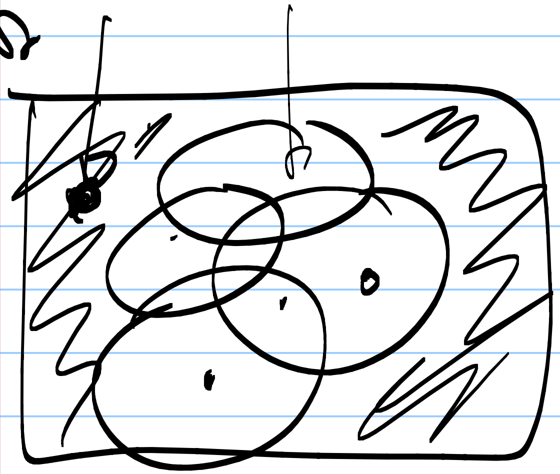
Note Title 7/10/16

$$\left|\bigcap_{i=1}^{n} \bar{A}_i\right| = \sum_{S \subseteq [n]} (-1)^{|S|} |A_S|$$

Ω



contribution of each object

Case 1 $x \in \bigcap_{i=1}^{n} \bar{A}_i$. x makes contribution only for $S = \phi$

contribution is 1 ✓

Case 2 $x \notin \bigcap_{i=1}^{n} \overline{A_i}$.

x is in k of the $A_1, A_2, \ldots, A_n$.

$I := \{ i \in [n] : x \in A_i \}$.

$|I| = k$.

x makes a contribution only $S \subseteq I$.

$x \in A_1, A_3, A_4$

$|A_2 \cap A_4|$

$$\sum_{\phi \subseteq S \subseteq I} (-1)^{|S|} \qquad |I| = k$$

$$= \sum_{r=0}^{k} (-1)^r \binom{k}{r} \qquad |S| = r.$$

$$= 0$$

$$\binom{k}{0} - \binom{k}{1} + \binom{k}{2} - \dots$$

$$\boxed{(1+x)^k = \sum_{r=0}^{k} \binom{k}{r} x^r \qquad x = -1}$$