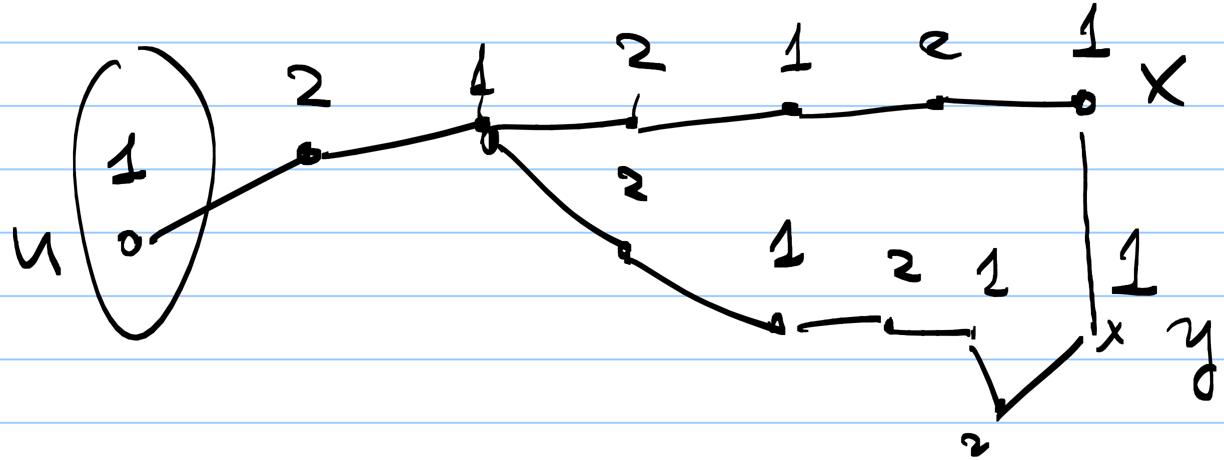
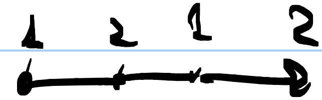
Note Title 11/11/16

Th If a graph has no odd cycles,
then the "coloring procedure" has no conflicts.

Pf Assume otherwise.
How can a conflict occur? Suppose (a) 1-1 conflict

(b) 2-2 conflict?





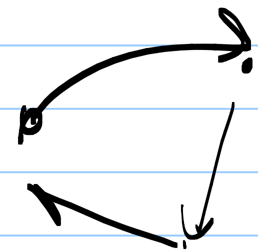
Path from u to x: even #. of edges P_1
Path from u to y. even #. of edges. P_2

Circuit : start from $u \xrightarrow{P_1} x \xrightarrow{1} y \xrightarrow{P_2} u$

#. of hops in circuit is odd.

Does this imply the graph has an odd cycle? Induction on #. of vertices in circuit.

Base case #. of hops $= 3$

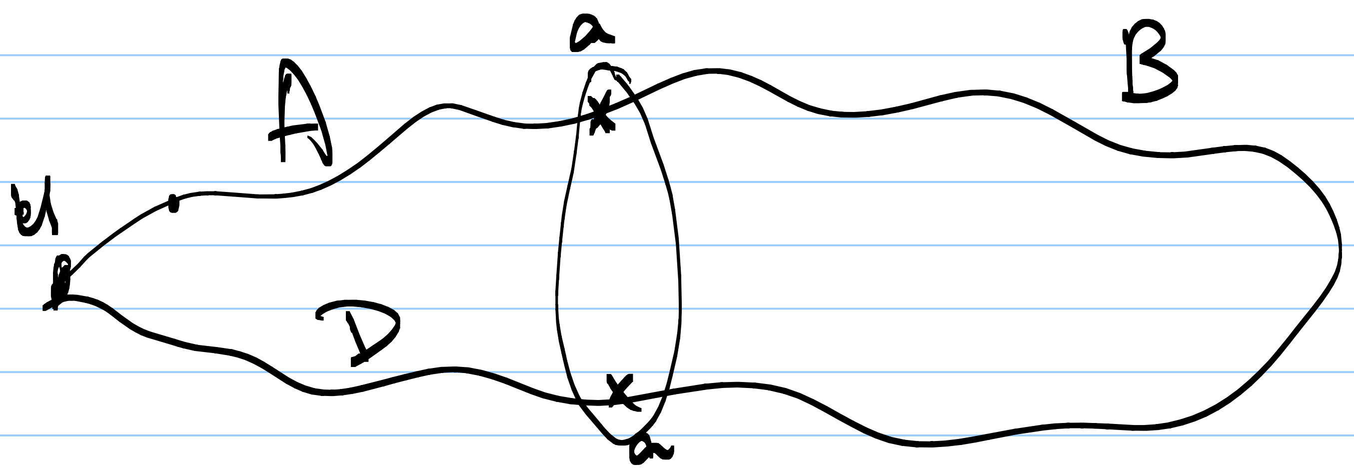


I.H. : For some odd number k,

if the graph has an odd circuit with at most k hops,
then it has an odd cycle.

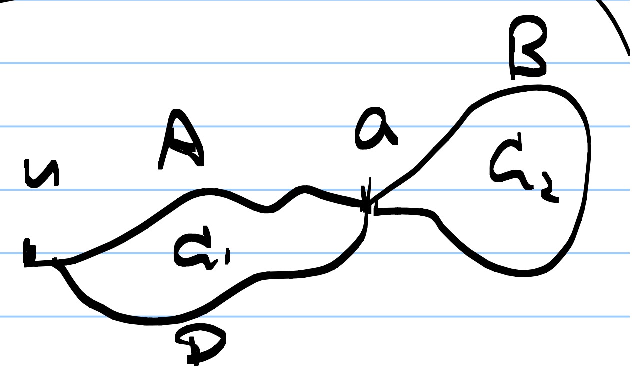
Inductive step

Suppose G is a circuit with $k+2$ loops, k is odd.



(i) If G is a cycle, then done ✓.

(ii) otherwise, there is some vertex a that appears more than once.



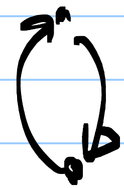
length of G_1 + length of G_2 is odd

$\Rightarrow$ exactly 1 of G_1 and G_2 is odd length of C_1, length of $C_2 \le k$

$\Rightarrow$ by I.H., there is an odd cycle.

- Construct Euler circuit for a connected (multi-) graph with even degrees.

induction on the edges.



(i) # of edges $= 2$

(ii) IH. $\leq m$ edges.

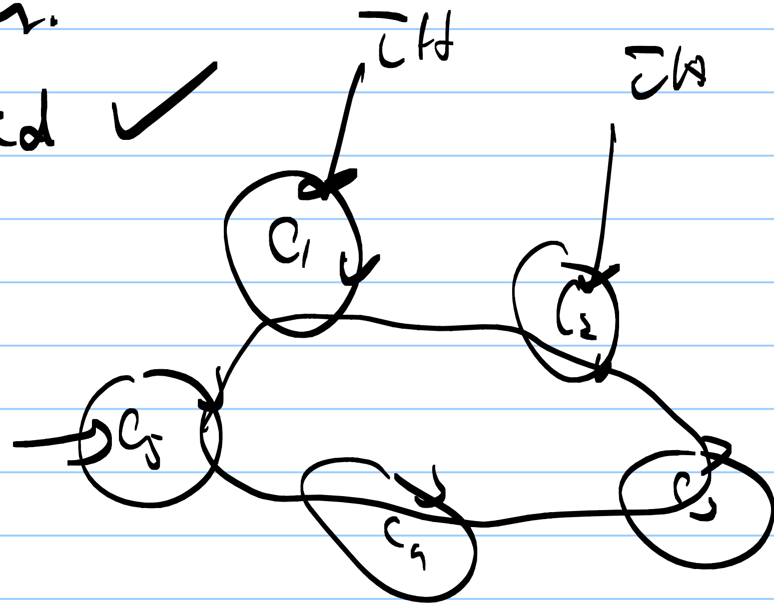
Ind^n step

Start with a graph with $m+1$ edges.

1. Form a circuit C.
   - every edge visited ✓
2. Otherwise, remove C.

IH

- apply IH
  to each connected
  components



Use ς to stitch the circuits together.