COMP2121A: Discrete Mathematics
Tutorial 4 4 Oct, 2016

(In this tutorial, you can use axiom of choice, explicitly or implicitly.)

1. Define three functions from $\mathbf{Z}$ to $\mathbf{Z}$: $f(x) = x^2$, $g(x) = -x + 1$ and $h(x) = \lceil \frac{x}{2} \rceil$. Determine whether they are injection, surjection, or bijection. (For any number y, $\lceil y \rceil$ denotes the smallest integer which is at least y. For example, $\lceil 0.5 \rceil = 1$, $\lceil 2 \rceil = 2$, $\lceil -1.5 \rceil = -1$.)
2. Prove that every infinite set has a countably infinite subset.
3. Prove that if there is a surjection from B to A, then there is an injection from A to B.
4. Prove that if there is an injection from A to B and an injection from B to A, then there is a bijection from A to B.
5. (Just for fun, yon don't need to prove it.) The following statements are equivalent to axiom of choice (under some assumptions[1] in set theory).
   - For any infinite set A, there is a bijection between A and $A \times A$.
   - Connected (infinite) graph has a spanning tree.

[1] To be specific, the assumptions are axioms 1 to 8 in Zermelo-Fraenkel set theory, for more information see `https://en.wikipedia.org/wiki/Zermelo-Fraenkel_set_theory`.