Note Title 11/2/2018

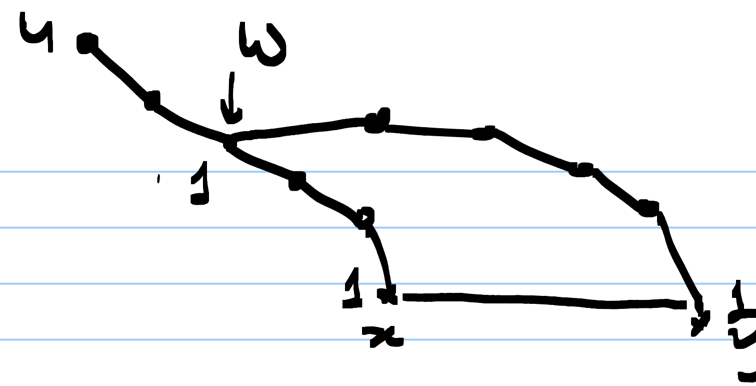
To prove: If a graph has no odd cycle, then it is bipartite.

Pf. By procedure/construction:

- start from any vertex u, label it "1".
- assign adjacent vertex with opposite label
- repeat until every vertex has been labeled, or conflict is detected.

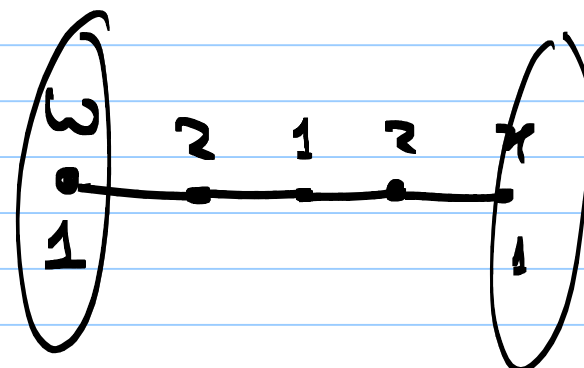
It suffices to show that no conflict occurs.

Suppose a conflict occurs between x and y.



There must be a <u>simple</u> path from u to x; there must be a simple path from u to y.

Let w be the last common point

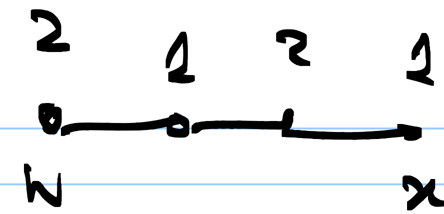


<u>What is the label of w?</u>

Label 1: path from w to x has even length; path from w to y has even length } ⇒ odd cycle

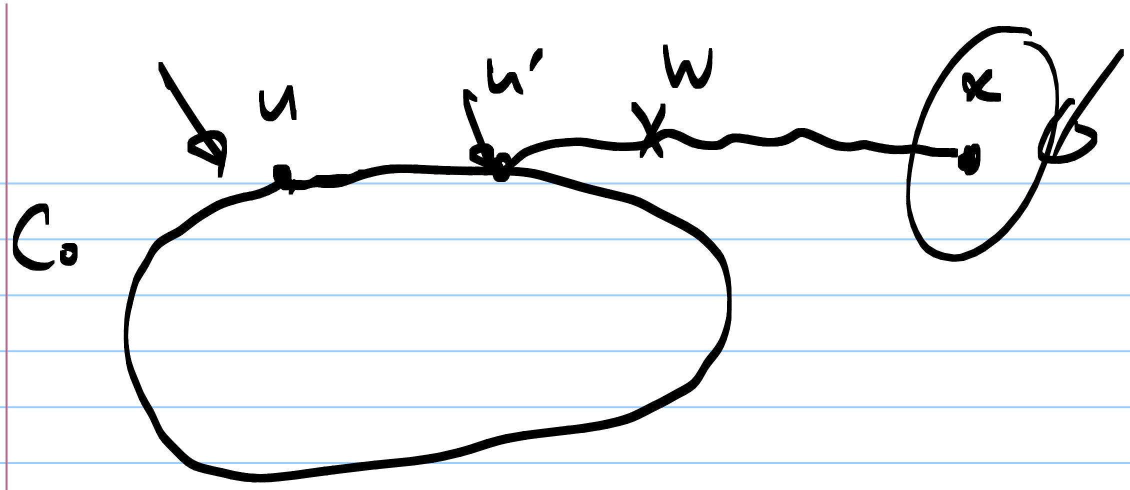
Label 2: path from w to x
and path from u to y
both have odd no. of edges.

$\Rightarrow$ odd cycle.



<u>Claim</u> Suppose C_0 is a circuit in a connected graph G and some edge $\{x, y\}$ is not visited by C_0.
Then, $\exists u'$ in C_0 s.t. u' touches some unvisited edge.

Pf. Start from $u \in C_0$.
Since graph G is connected, there is a (simple) path from u to x.
If x or $y \in C_0$, then done ✓;
both x and y are not on C_0.



Travel along the path from u to x,
there must be a first point u' on G_0 before
leaving the circuit.
$\{u', w\}$ is not in G_0
We have found the point u'.