COMP2121A: Discrete Mathematics
Problem Solving Session 1 Sep 11, 2018

Review Propositional logic

1. proposition: a statement that either true or false, but not both
2. logical operators: $\neg$(negation), $\wedge$(and), $\vee$(or), $\oplus$(exclusive or), $\rightarrow$(implication), $\leftrightarrow$(biconditional)
3. truth table: used to determine the truth value of propositions

Predicates and Quantifiers

1. predicates: a statement that may be true or false depending on the values of its variables
2. domain: collection of values a variable would take
3. quantifiers: $\forall$(universal, for all), $\exists$(existential, there exists)

Questions

1. Express these system specifications using the propositions p "The message is scanned for viruses" and q "The message was sent from an unknown system" together with logical connectives (including negations).
   (a) "The message is scanned for viruses whenever the message was sent from an unknown system."
   (b) "The message was sent from an unknown system but it was not scanned for viruses."
   (c) "It is necessary to scan the message for viruses whenever it was sent from an unknown system."
   (d) "When a message is not sent from an unknown system, it is not scanned for viruses."

2. Show that $\neg p \to (q \to r)$ and $q \to (p \vee r)$ are logically equivalent.

3. Let $P(x)$ be "x can speak English." and $Q(x)$ be "x knows C++.", where the universe of discourse is the set of all students in our class. Use quantifiers to express the following statements.

   (a) There is a student in our class who can speak English but does not know C++.

   (b) Every student in our class speaks English.

   (c) Every one that knows C++ in our class can speak English.

4. The following four cards sit on a table:

Each card has a digit on one side and a letter on the other side. Which cards should you turn around to test the following statement: “whenever there is a vowel on one side of a card, there is an even digit on the other side”? (In English, vowels are letters A, E, I, O, U.)

5. Given two statements p and q, we say that p is a stronger statement than q, if $p \to q$. Prove that if p_1 is stronger than p_2, then $(p_2 \to q)$ is stronger than $(p_1 \to q)$.