COMP2121: Discrete Mathematics
Problem Solving Session 6 16 Nov, 2018

1. Assume that a digital lock opens anytime a correct sequence of 3 digits $(0, ..., 9)$ is entered. The brute force approach is to try out all possibilities, i.e., $000, 001, 002, ..., 999$ and $10^3 \times 3$ digits have to be entered. However, note that if 135286 is entered, effectively the following 3-digit sequences have been tried: 135, 352, 528 and 286. Does there exist a method to construct a sequence of $10^3 + 4$ digits so that the sequence contains all possible 3-digit sequences as its subsequences. Prove or give arguments to support your claim. (Hint: Transform this problem into a graph problem, and then define the vertices and edges of this graph.)

2. We draw some circles on the plane (say, n in number). These divide the plane into a number of regions. Figure 0.1 shows such a set of circles, and also an "alternating" coloring of the regions with two colors. Now our question is: can we always color these regions this way?

   You may follow the following steps to prove that the graph G representing regions is bipartite (hence can be two-colored).

   (a) Encode each region by $0, 1$ strings such that the code of adjacent regions differ by exactly one bit.

   (b) Argue that any cycle on the graph G is of even length.

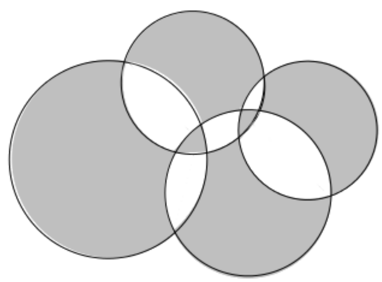

Figure 0.1: Two-coloring the regions formed by a set of circles.

3. If a planar graph has all degrees even, prove that the faces can be colored with two colors in such a way that any two faces with a common edge on their boundary get different colors.