Online Frequency Assignment in Wireless Communication Networks

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A presentation especially for COCOON 2007, Banff

This is Hong Kong!

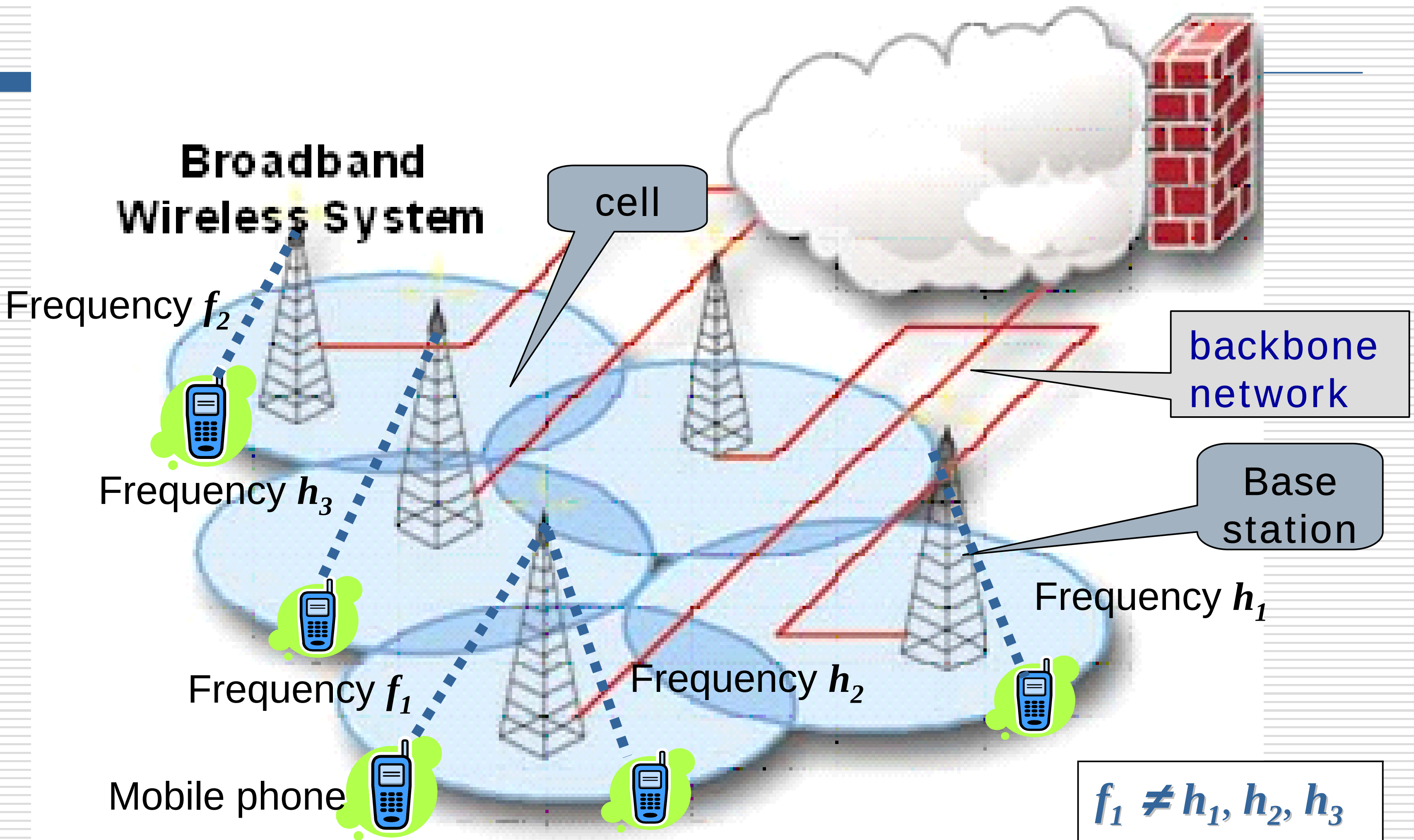
Frequency Assignment Problem (FAP) Outline

- Background
- FAP without reassignments
  - Absolute competitive ratio
  - Asymptotic competitive ratio
  - With deletion
- FAP with reassignments
- FAP with OVSF code

Frequency Assignment Problem (FAP)

- Wireless communication has a long history
- Frequency bandwidth is always a scarce resource and FAP has always been an important problem
- New problems come up because of
  - Change of technology
    - Analog to digital
    - MTS, AMPS, GSM, FDMA, CDMA
  - New algorithmic techniques
    - Approximation and online analysis

Wireless Communication Network



The same and neighboring cells should use different frequencies

$f_1 \neq h_1, h_2, h_3$

$f_2 \neq h_3$

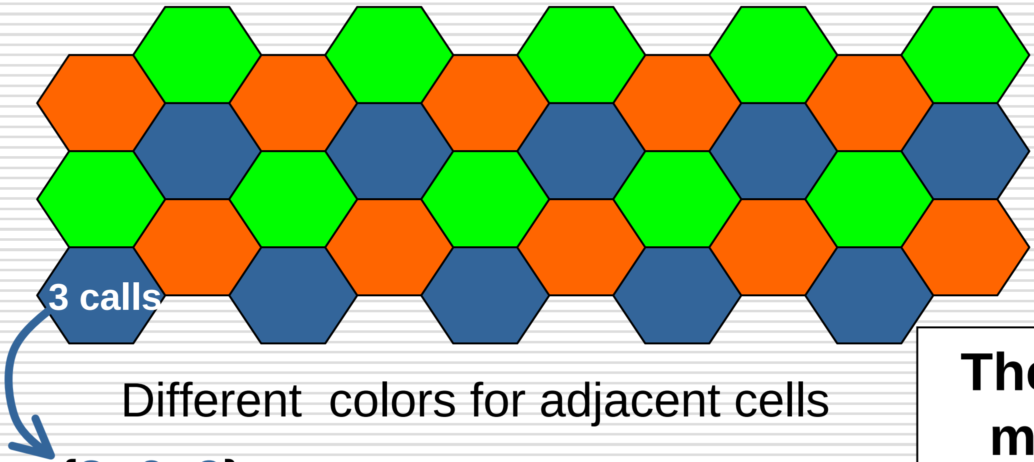
$h_2 \neq h_1, h_3$

Frequency Allocation Problem (FAP)

- Given the hexagon graph
- Frequencies: represented by {1, 2, 3, ...}
- Interference constraint:
  calls from the same or adjacent cells use different frequencies
- Online:
  - calls arrive one at a time
  - assigned frequencies never changed
  - calls never terminate
- Problem:
  minimize the span of frequencies used

Fixed Allocation

- Cells are colored by R, G, and B
- Frequencies are partitioned into 3 disjoint sets
  - $F_R = \{1, 4, 7, \ldots\}$ ← For cells with color R
  - $F_G = \{2, 5, 8, \ldots\}$ ← For cells with color G
  - $F_B = \{3, 6, 9, \ldots\}$ ← For cells with color B



Different colors for adjacent cells

{3, 6, 9}

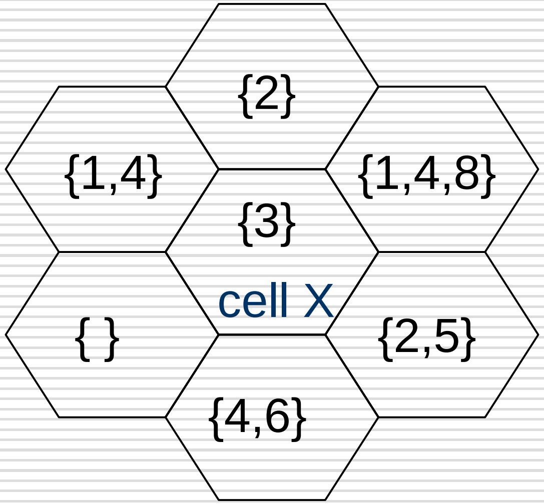
Frequency allocation rule:
Use frequencies according to the color of the cells, starting from the lowest frequency

The span of frequencies used is at most 3 times that of the optimal.
(3-competitive)

Greedy Allocation

Frequency allocation rule:

- Assign the lowest available frequency



E.g., frequency **7** is assigned to call X

The span of frequencies used is at most 17/7(≈2.43) **times that of the optimal.**
[Chan,Chin,Ye,Zhang,Zhu, IPL 2006]

The ratio is tight [Caragiannis, et al, SPAA 2000]

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Hybrid Allocation (Chan et al SPAA2007)

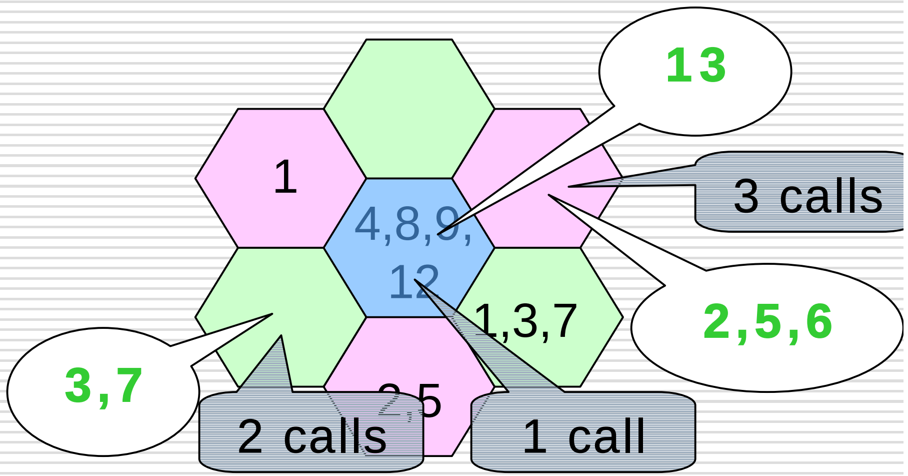
Color the cells by R, G, and B

Partition the frequencies into 4 disjoint sets ← Shared by all cells

- $F_S = \{1, 5, 9, 13\ldots\}$ ← For cells with color R
- $F_R = \{2, 6, 10, \ldots\}$ ← For cells with color G
- $F_G = \{3, 7, 11, \ldots\}$ ← For cells with color B
- $F_B = \{4, 8, 12, \ldots\}$

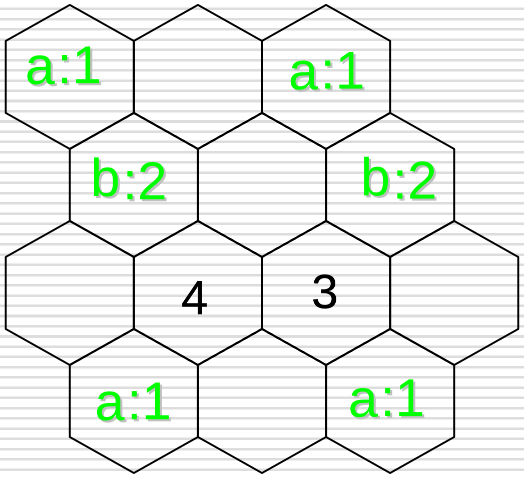
Frequency allocation rule

- Assign the lowest available frequency in $F_S \cup F_x$

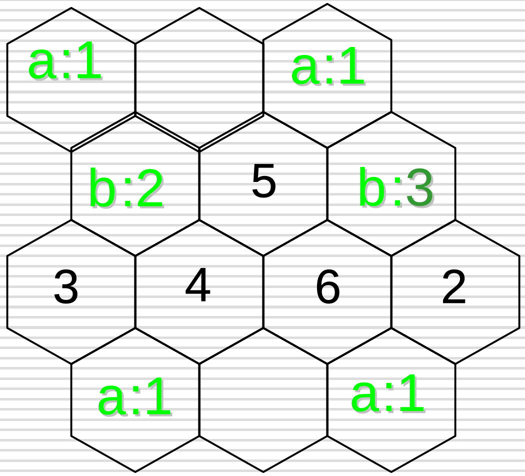


FAP – lower bound=2 (by adversary)

- A call from each cell a
- A call from each cell b



optimal needs 2.



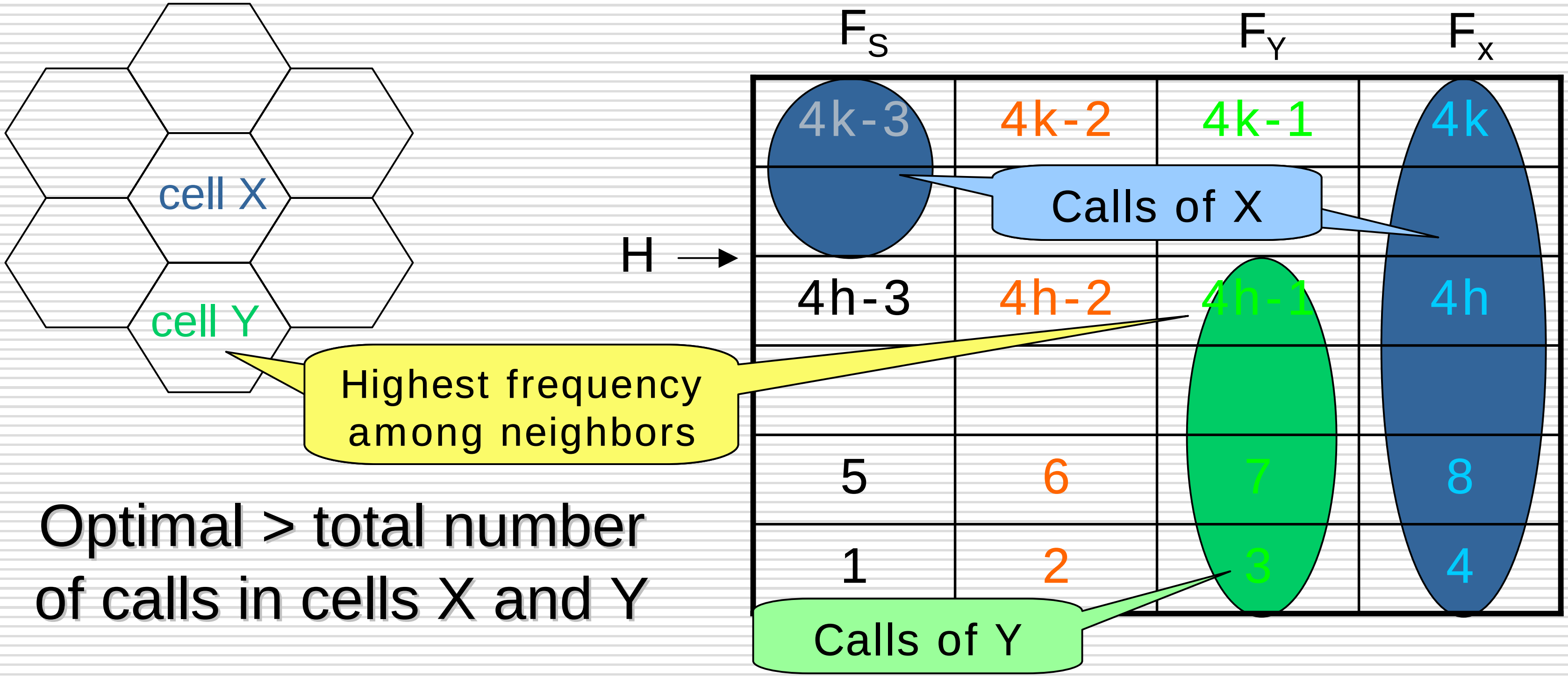
optimal needs 3.

2 is the smallest ratio that we can do for any online algorithm

Proof of the 2-competitive bound

The span of frequencies used is at most <u>twice</u> that of the optimal.

Why? **Suppose cell X has the highest frequency, 4k**



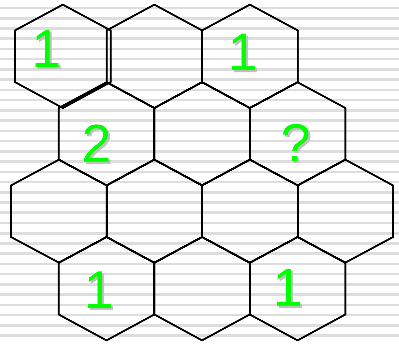
Optimal > total number of calls in cells X and Y

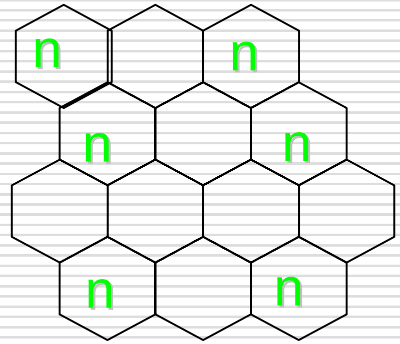
More to be done?

- Hybrid is 2-comptitive
- Lower bound is also 2
- Optimal only when the number of calls is small.
- When the number of calls tends to ∞, we can do better than 2-competitive.

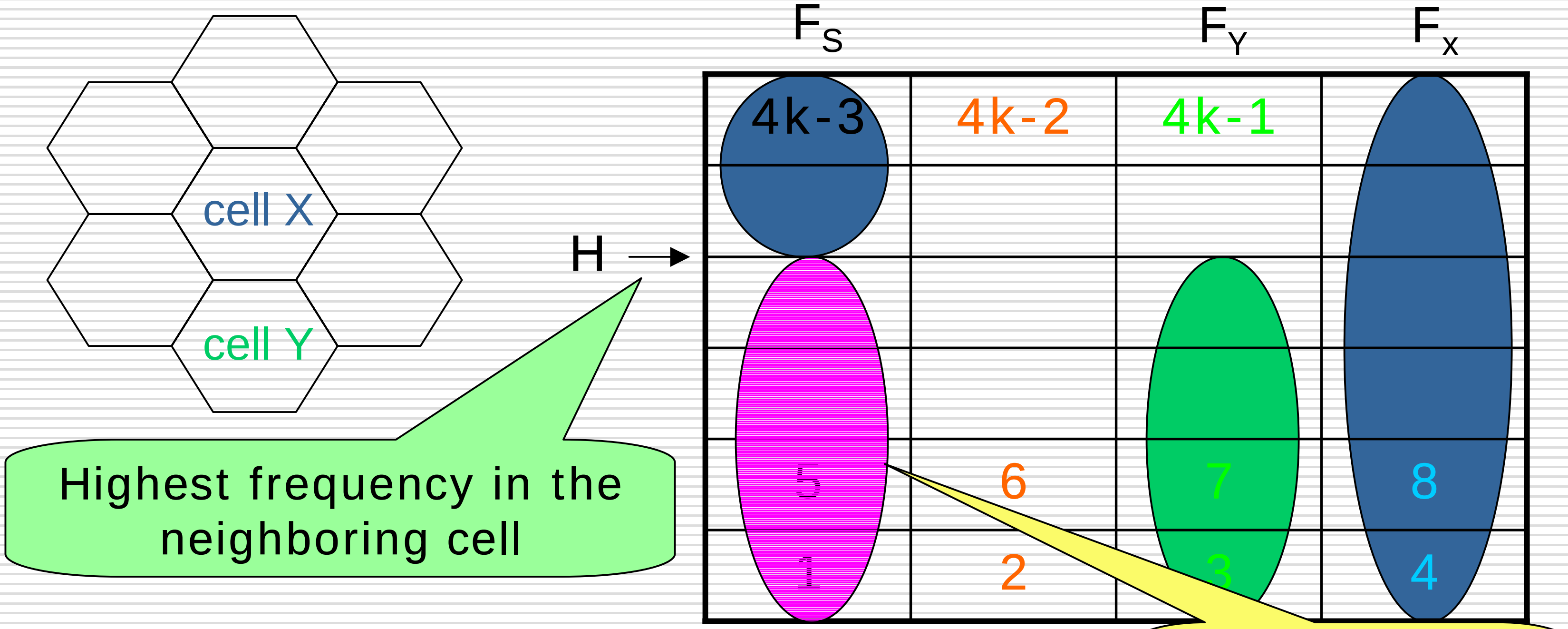
Asymptotic bounds for FAP

- In the lower bound proof
  - Optimal requires 2 or 3 frequencies
  - Adversary uses 4 or 6 frequencies with 2 discrete decisions.
  - When the number of calls are large, decisions may be not discrete.
  - Some the same and some different.





Proof of Upper Bound revisited



- When all *n* frequencies are in a single cell X, the bound of Hybrid is still 2
- The component of greedy should be increased.

Family of Hybrid Algorithms

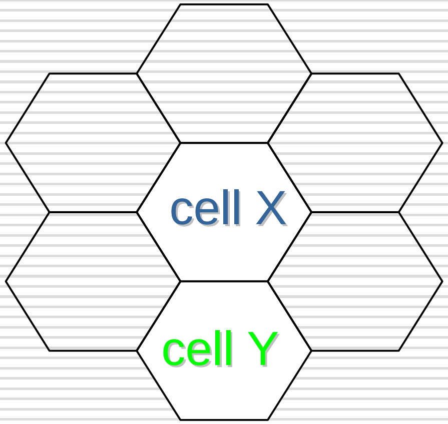
- Frequencies partitioned into groups of $\Delta = \alpha + 3\beta$

  greedy (α) — Fixed allocation (3β)

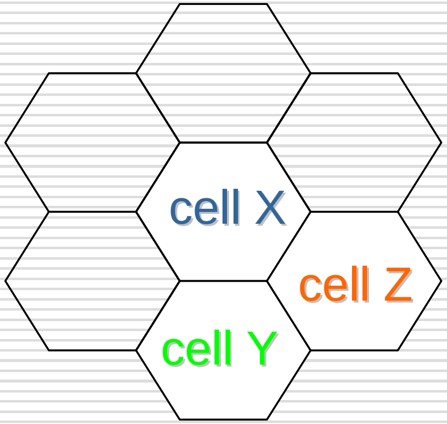
- From each group:
  - F_S is assigned α frequencies
  - Each of F_R, F_G & F_B is assigned β frequencies
- Define two parameters α and β
  - $\alpha=0$ ➔ purely fixed allocation
  - $\beta=0$ ➔ purely greedy
  - $\alpha=\beta$ ➔ ordinary hybrid (described before)

Asymptotic Upper Bound

- In the absolute case,
  we consider cell X and cell Y
  - Competitive ratio ≤
    (highest frequency in cell X) /
    (total no. of calls in X & Y)



- For the asymptotic case,
  we consider cell X and <u>two</u> of its neighbors.
  - Competitive ratio ≤
    (highest frequency in cell X) /
    (total no. of calls in X, Y & Z)



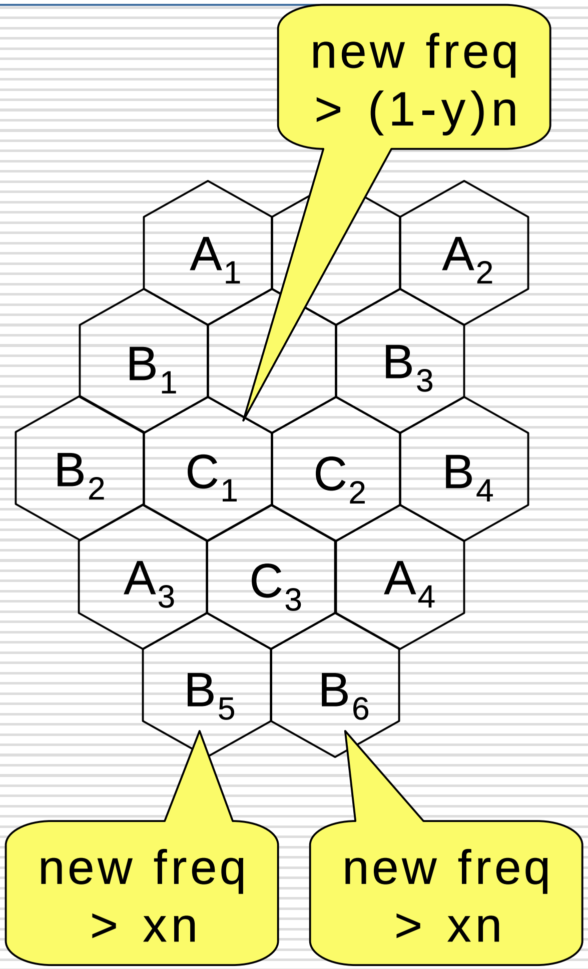
Asymptotic and Absolute Upper Bounds

- <u>Theorem</u>:
  If $\beta / \alpha \rightarrow 0.8393$ (more greedy),
  asymptotic competitive ratio $\rightarrow 1.9126$

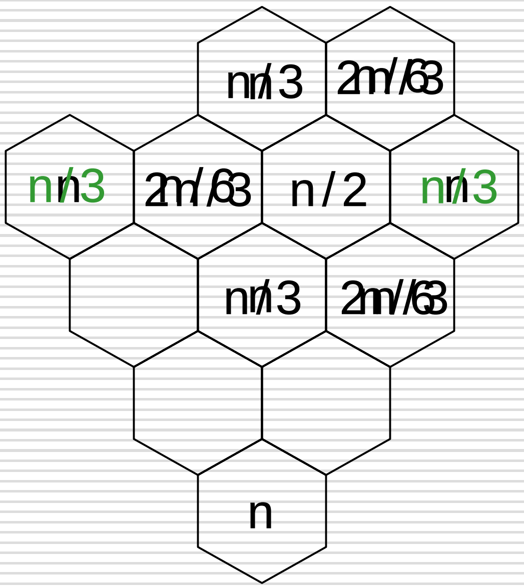
- For $\alpha=13$, $\beta=11$ ($\beta / \alpha \approx 0.8462$),
  asymptotic competitive ratio $\rightarrow 1.9167$
  and absolute competitive ratio = 2.

Asymptotic Lower Bound = 1.5 by Adversary

- n calls from A_1, A_2, A_3 and A_4
  - Common freq between any 2 cells = xn
  - No. of distinct freq > (2-x)n
  - Competitive ratio = (2-x)
  - Adversary stops if x < 0.5 as ratio > 1.5.
- n calls from B_1, B_2, B_3, B_4, B_5, B_6
  - No. of distinct freq = (2+x+y)n [$y \geq 0$]
  - Competitive ratio = (2+x+y)/2,
  - Adversary stops if x+y > 1 [so $x \geq 0.5 \geq y$]
- n calls from C_1, C_2, C_3
  - C_i gets all new freq except yn
  - Each C_i uses $\geq$ (1-y)n new frequencies,
  - The ratio is $(5+x-2y)/3 \geq 1.5$



Asymptotic Lower Bound = 2 (with-deletion)



The optimal takes
n frequencies always

- If no of frequency > 2n, stop. Otherwise
  - Choose 2 with > n/3 common frequencies.
- At least 2n/3 "new" frequencies
- At least n/3 "new" frequencies
- At least n/3 "new" frequencies
- Must be totally different from the surrounding cells.
- Optimal uses only n frequencies.

Theorem: No online algorithm has the asymptotic competitive ratio less than 2 (with deletion).

Other Variations: With deletion

- Generalized version of the online problem
- A call may terminate at any time
- Performance:
  - Fixed Allocation: 3-competitive
  - Greedy Allocation: 3-competitive [Chan, Chin, Ye, Zhang, Zhu, IPL 2006]
  - Better than 3-competitive? Still open!
- Lower bounds
  - Asymptotic competitive ratio = 2
  - Absolute competitive ratio = ?.

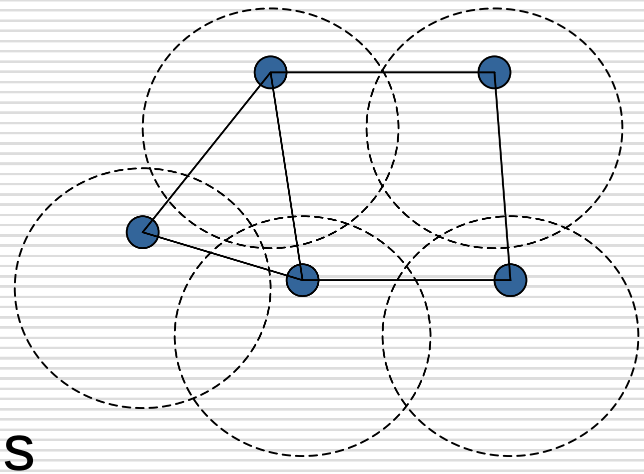
Coverage by Linear Networks

[Chan, Chin, Ye, Zhang, Zhu, ISAAC 2006]

- The geographical coverage area is divided into cells aligned in a line
- Online:
  - Upper bound=lower bound= 1.5
  - Asymptotic upper bound= 1.382
  - Asymptotic lower bound=4/3≈ 1.333
- Dynamic (with deletion):
  - Upper bound=lower bound=5/3≈ 1.667
  - Asymptotic upper bound=5/3≈ 1.667
  - Asymptotic lower bound=14/9≈ 1.556

Other Variations: Different Graphs

- k-colorable graphs
  - Online case: Hybrid algorithm is (k+1)/2-competitive
    - E.g., planar graphs are 4-colorable ➔ 5/2-competitive



- Unit disk graphs
  - Vertices ➔ circles
  - Edge ➔ intersecting circles

No result known for online or dynamic FAP

Other Variations: Reuse Distance

- Current setting ➔ reuse distance=2

- In general, reuse distance=r
  - Offline
    - r=3 ➔ 2-approximation [Kchikech, Togni 2005]
    - r≥4 ➔ 3-approximation [Kchikech, Togni 2005]

  - Online (and Dynamic)
    - By a fixed allocation scheme
    - r=3 ➔ 3-competitive [Jordan, Schwabe, ACM
    - r=4 ➔ 4-competitive J. Wireless Networks1996]

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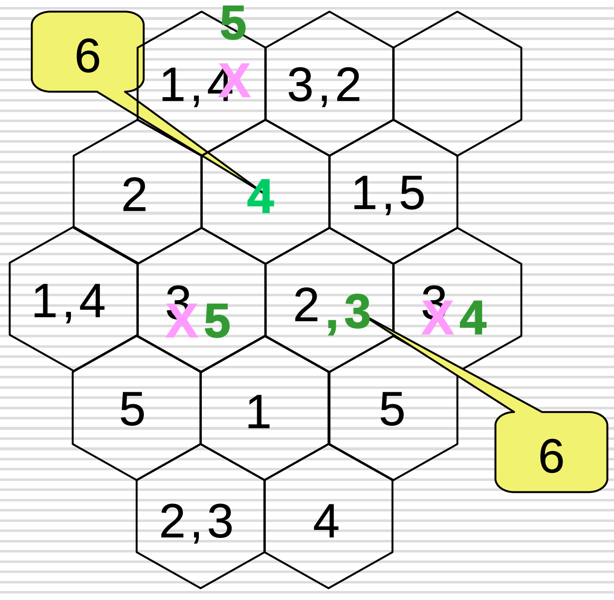
FAP with Reassignments

Maximize bandwidth utilization by reassignment of frequencies



Can we get the optimal utilization?

- NP-complete even for offline
  [McDiarmid & Reed Network 2000)
- NP-complete for an algorithm with approximation ratio < 4/3)
  [3-colorability problem is NP-complete]

Approximation algorithm with ratio 4/3 exists
[Narayanan & Shenda, Algorithmica 2002;
Spare et al, J. Pure App Math 2002]

On-line Distributed Algorithms

How good is the bandwidth utilization by frequency reassignments on the the neighboring cells?

k-locality: neighboring cells with distance $\leq k$.

Fact : *k*-local α-approximation algorithm
$\rightarrow$ *k*-local α-competitive online algorithm
[Janssen, et.al., J. Algorithms 2000]

- 0-local – 3-competitive
- 1-local – 3/2- improved to 13/9-competitive
  [Chin, Zhang, Zhu, this conference]
- 2-local – 17/12- improved to 4/3-competitive
  [Sparl & Zerovnik, J. Algorithm 2005]
- 4-local – 4/3-competitive

Unsolved Problems

- Can we improve the following results?
  - 0-local – 3-competitive
  - 1-local – 13/9-competitive
  - 2-local – 4/3-competitive

- Besides better competitive ratios, do these schemes use the least number of reassignments?

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Background – Third Generation (3G)

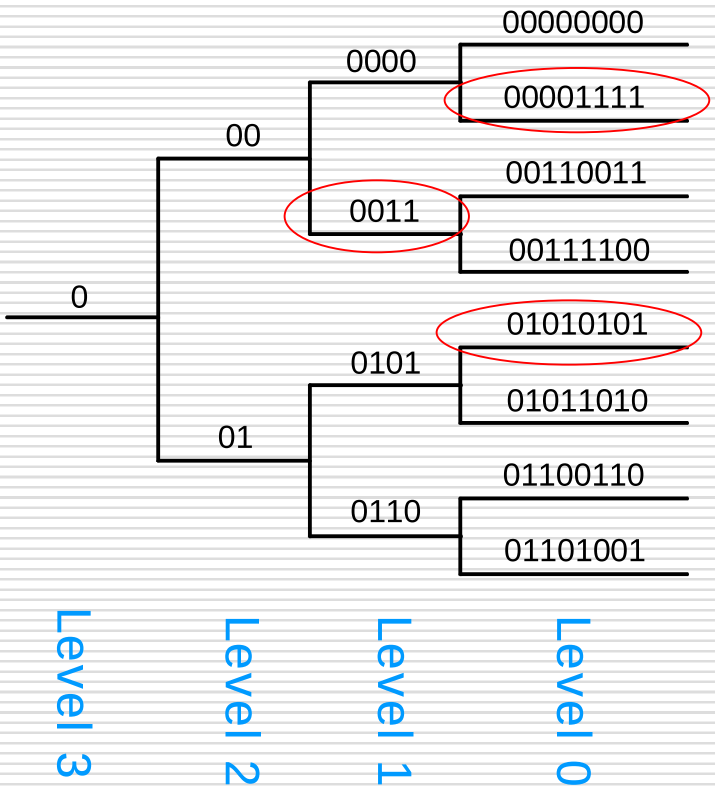
- Multiple services of 3G communication
  - Voice
  - Messaging – email, fax, etc.
  - Medium-rate multimedia – Internet access,
  - High-rate multimedia – file transfer, video
  - High-rate interactive multimedia – video teleconferencing, telemedicine, etc.

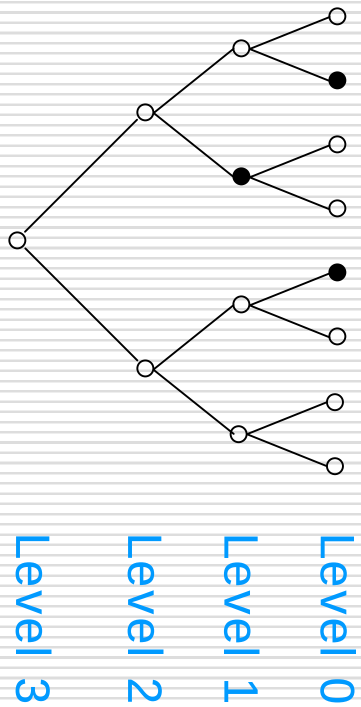
Background – OVSF Codes

Wideband Code-Division Multiple-Access (WCDMA) with OVSF code

- Structure: a complete binary tree
- Assigned nodes must be *mutual orthogonal*, i.e., each root-to-leaf path contains at most one assigned code (the 2 nodes are neither ancestor nor descendant of each other).

An Example of OVSF Codes





Data rate of level-i code is twice that of level-(i-1) code

Problem Background

Given a code operation sequence $\delta = \{C_1, C_2, \ldots, C_t, \ldots, C_n\}$

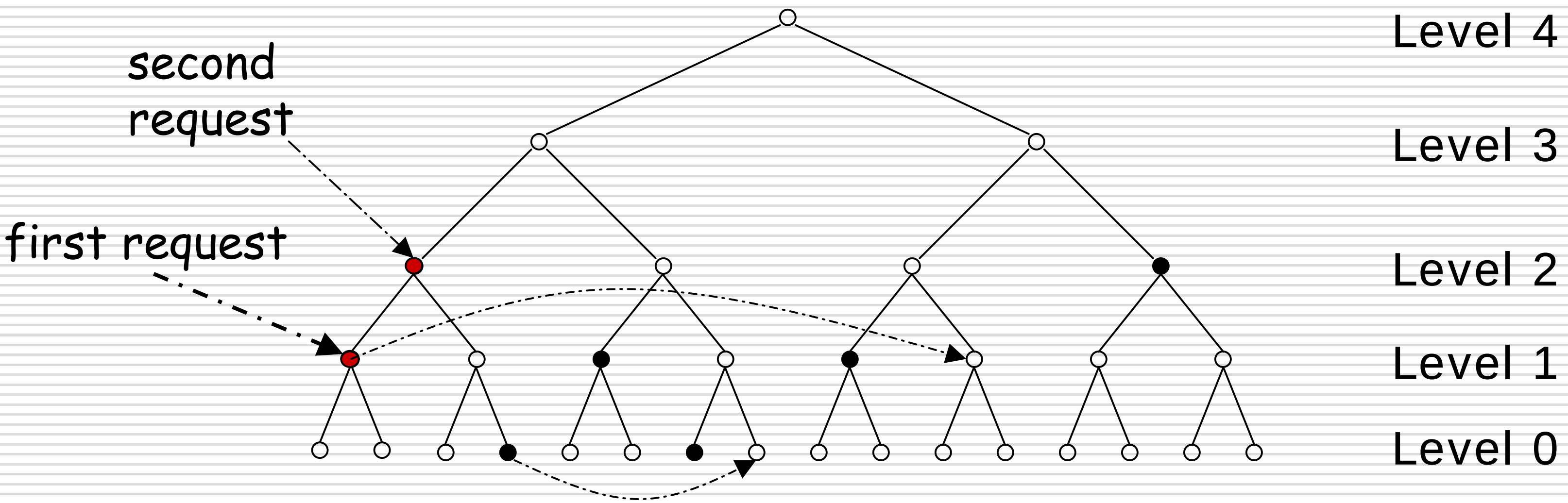
$$C_t = \begin{cases} \text{a code request for a particular level (cost = 1)} \\ \text{an assigned code release (cost = 0).} \end{cases}$$

Problems with a code request

- Code blocking or Code-tree fragmentation
- Reassignments of assigned codes might be needed.
- Each reassignment (code request + code release) costs 1 unit.

Problem Statement – an example

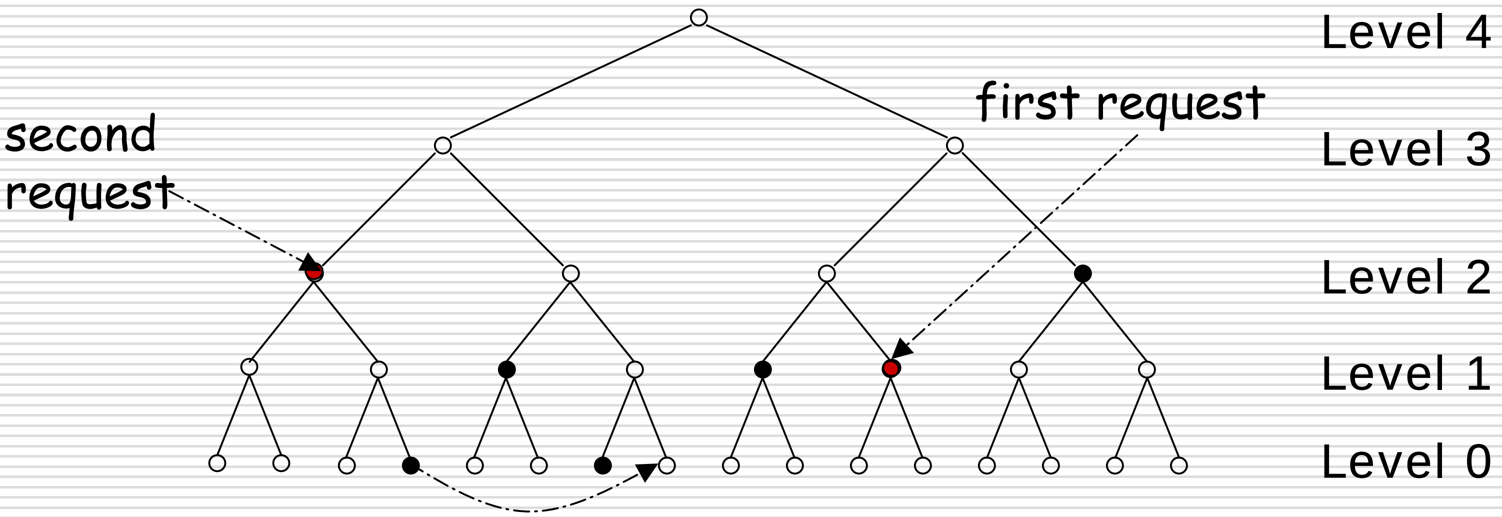
Request : a level-1 code and a level-2 code.



- Code blocking occurs.
- 2 code reassignments are needed.
- Total cost = 4 units (two assignments and two reassignments)

Another Appraoch

Request : a level-1 code and a level-2 code



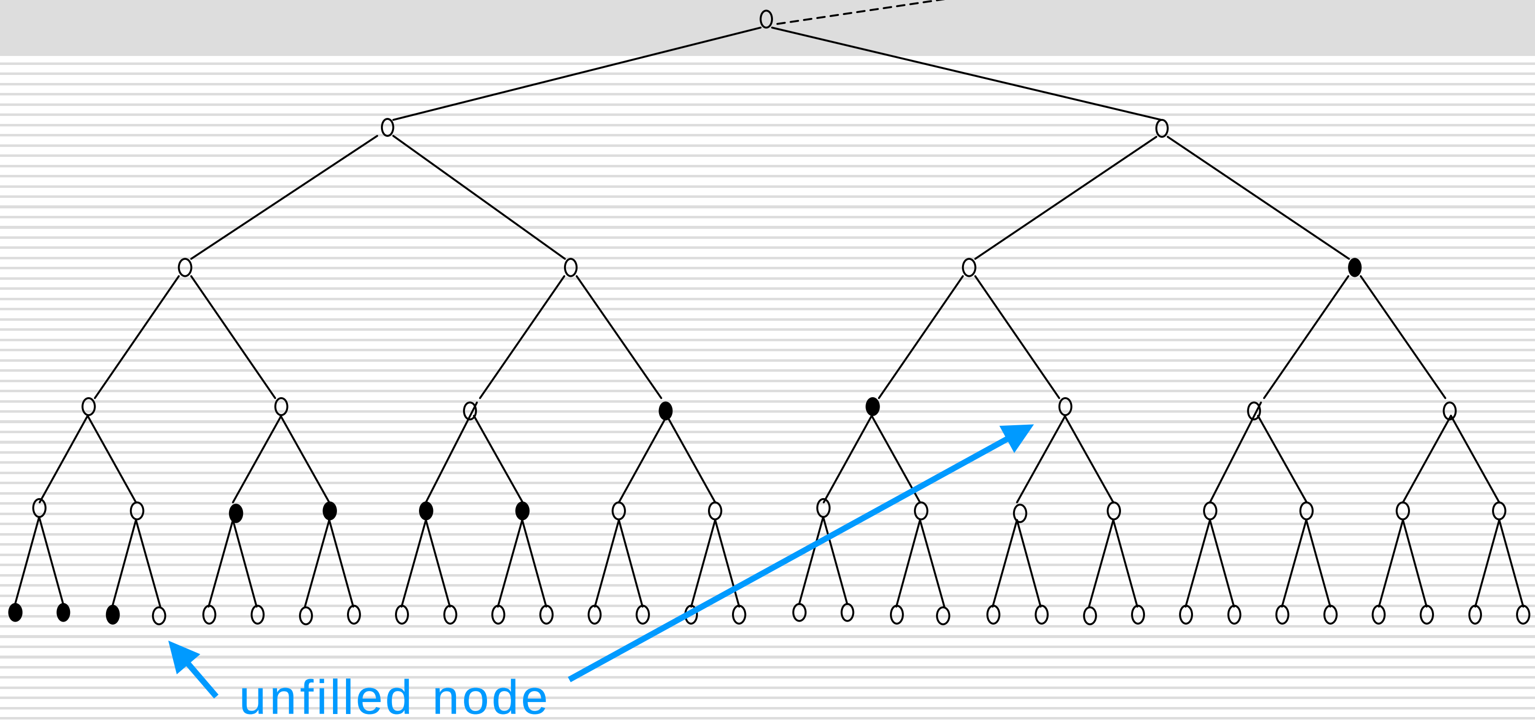
- One code reassignment is needed
- Total cost = 3 units (2 assignments and 1 reassignment)

Problem: Minimize the total cost, i.e., minimize the total number of assignments and reassignments.

An O(h)-competitive algorithm

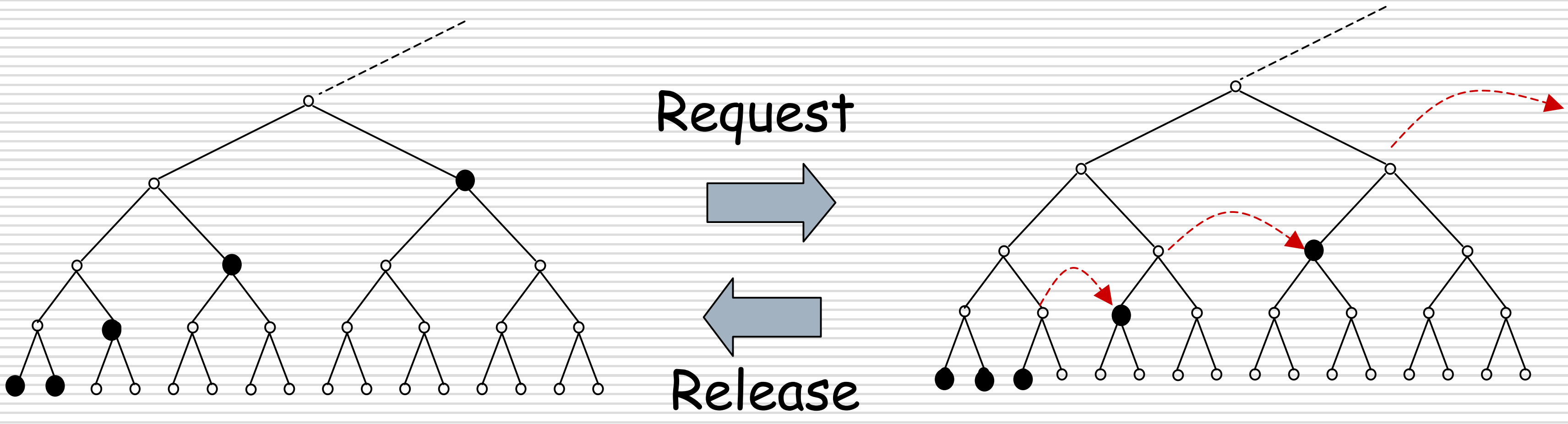
(Erlebach et al. STACS2004)

- All assigned codes are sorted and compact
  - Sorted – by level, from left to right
  - Compact – left shift as much as possible
    (at most one unfilled code at each level)



Example for O(h) reassignments

Request or release of a level-0 code



Recent Results

Code Assignment with Extra Resources

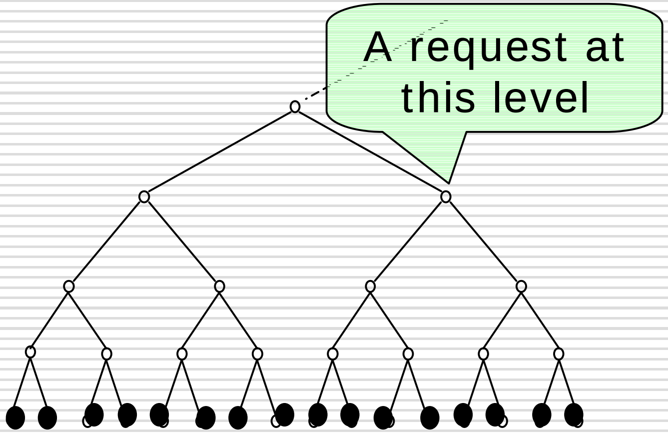
- 4-competitive with twice bandwidth
  (Erlebach et al. STACS2004)
- 5-competitive with 9/8 bandwidth
  (Chin , Zhang, Zhu, AAIM 07)

Code Assignment without Extra Resources

- Constant competitive (amortized)
  (Forisek et al.,ESA2007)
- 10-competitive by grouping levels
  (Chin, Ting, Zhu, manuscript)
- 8-competitive and 6-competitive (amortized) by a skew configuration (Chin and Zhu, manuscript)

Lower Bound

- No online algorithm can be better than 1.5-competitive [Erlebach et al, STAC 2004]
- Outline proof for the 1.5 bound
  - Start with *n* assigned leaves
  - Release alternative assigned nodes
  - Request a level-(h-2) node which requires *n*/4 reassignments

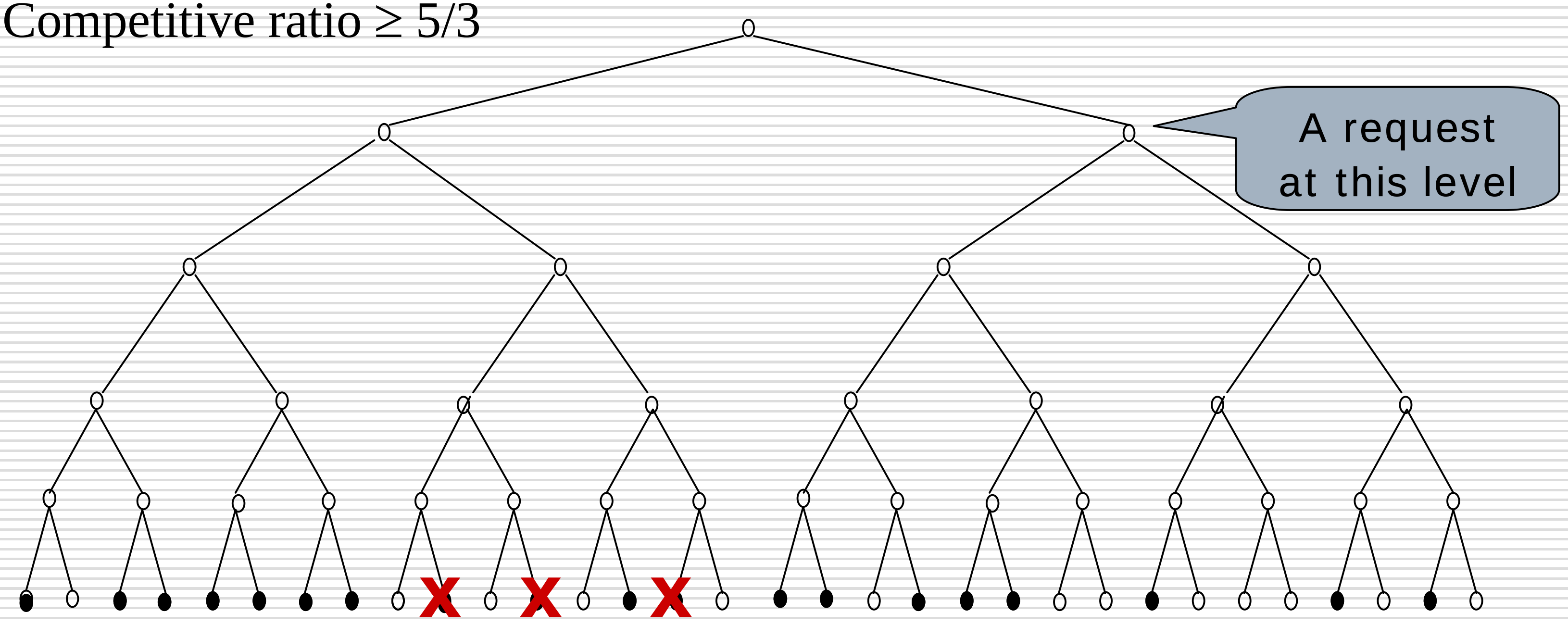


Request level	No. of leaves	No of reassign
0	16	4
1	8	2
2	4	1

- The cost by adversary $= n + n/2 - 1 + \log n - 1 = 3n/2 + \log n - 2$
- Optimal cost $= n + \log n$
- The competitive ratio is at least 3/2

An Improved Lower Bound to 5/3

- Each of the two subtrees has at least $n/4$ assigned leaves (at most 3n/4 code requests)
- The remaining steps are similar …
- The cost by adversary = $3n/4 + n/2 + \log n - 2 = 5n/4 + \log n - 2$
- Optimal cost = $3n/4 + \log n$
- Competitive ratio $\geq 5/3$



Open problem (OVSF code)

- The best algorithm is 6-competitive for single cell, lower bound is 1.75.

  Can we narrow the gap?
- What is the code assignment for cellular network, instead of single cell?
  - A 5-competitive algorithm with 19/8 resource augmentation (Chin et al, AIMM 2007)

HKU Theory Group at 2006

Thanks

COCOON 2007!

Questions?