A Truthful Online Mechanism for Location-Aware Tasks in Mobile Crowd Sensing

Ruting Zhou, Zongpeng Li, Chuan Wu

Abstract—Effective incentive mechanisms are invaluable in mobile crowd sensing, for stimulating participation of smartphone users. Online auction mechanisms represent a natural solution for such sensing task allocation. Departing from existing studies that focus on an isolated system round, we optimize social cost across the system lifespan, while considering location constraints and capacity constraints when assigning sensing tasks to users. The winner determination problem (WDP) at each round is NP-hard even without inter-round coupling imposed by user capacity constraints. We first propose a truthful one-round auction, comprising of an approximation algorithm for solving the one-round WDP and a payment scheme for computing remuneration to winners. We then propose an online algorithm framework that employs the one-round auction as a building block towards a flexible mechanism that makes on-spot decisions upon dynamically arriving bids. Through both theoretical analysis and trace-driven simulations, we demonstrate that our online auction is truthful, individually rational, computationally efficient, and achieves a good competitive ratio.

Index Terms—Mobile Crowd Sensing; Mechanism Design; Approximation Algorithms

1 INTRODUCTION

CROWDSOURCING, a recent term coined in 2006 [1], refers to a creative process of serving business goals, designing products or solving problems by leveraging the collective intelligence from an online community. With the proliferation of smartphones over the past decade, mobile crowd sensing has emerged as a new crowdsourcing paradigm that collects distributed sensory data from smartphone users [2]. It has the capacities to provide various sensing services, as smartphones are usually equipped with many sensing facilities (e.g., accelerometer, digital compass, GPS and camera) and can be easily embedded with professional sensors (e.g., poisonous chemical detection and air quality sensors) [2].

As shown in Fig. 1, a typical mobile crowd sensing system consists of a cloud-based platform and mobile smartphone users. These smartphone users act as sensing service providers, and the platform recruits them to provide sensing services. The recent years have witnessed applications of mobile crowd sensing in a number of areas. For example, Waze [3] is one of the well-known use cases of mobile crowd sensing on the market; it takes live traffic information from users on the road to generate real-time reports on road conditions. Mobile applications like Weathermob [4] and Sunshine [5] improve the accuracy of weather forecasting by using real-time weather reports from users in specific locations. Air condition monitoring is another typical use case of mobile crowd sensing. In such systems, a large number of users employ air quality sensors portal to their smartphones to measure and upload air quality data to the platform, helping generating and predicting air quality profiles of a city [6].

Existing mobile crowd sensing applications often assume voluntary user participation [3], [4], [5], which is not always practical in the real world. Smartphone users consume their own resources such as battery, CPU and manpower when performing sensing tasks. Moreover, they are subject to potential privacy threats by sharing location-based data [7]. Therefore, smartphone users may not be ready to contribute to crowd sensing tasks unless satisfying remuneration is received to compensate for their cost/risk. What price to offer as remuneration can be a tricky question. If the price offered is too low, participation is sparse and the data collected may be insufficient; if the price offered is too high, the platform may incur an unnecessarily high operational cost. Auction mechanisms represent a natural and efficient approach to incentivize smartphone users to participate in mobile crowd sensing. It automatically discovers the right market price that helps select low-cost users to jointly

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accomplish a sensing job.

As compared to crowdsourcing jobs, a mobile crowd sensing job is often marked by location awareness that is central to most sensing tasks. A sensing task typically specifies the locations from which desired information are to be collected. Sensing data such as traffic condition or air quality are most valuable only when they are measured at designated locations. Such data become less valuable or even invalid if gathered from irrelevant locations. Unfortunately, most existing incentive mechanisms do not address the importance of location information [8][9]. Another observation is that a smartphone user’s level of participation is limited in practice [6][10]. It is often unrealistic to assume that a smartphone user can participate in all tasks at all times in a crowd sensing process. Battery consumption, available work time and specific locations are a few examples of practical concerns that may impose limitations on when and how a user participates. An abstract notion of capacity can be associated to a user to characterize such limitations on the degree of participation [6][10], when designing crowd sensing mechanisms.

This work explores the space of modelling and incentive mechanism design in mobile crowd sensing applications, and proposes an online auction such that: i) it runs over multiple rounds in the system lifespan; user bids for undertaking sensing jobs can arrive at any round and is subject to a capacity constraint over a given period; ii) the auction is computationally efficient and executes in polynomial time; iii) the auction is truthful, i.e., bidding true cost represents a dominate strategy for each smartphone user; and iv) social cost is approximately minimized when assigning location-aware sensing tasks to smartphone users.

We first formulate an integer linear program (ILP) that characterizes the winner determination problem (WDP) for social cost minimization in the online mobile crowd sensing auction. Solving the ILP to determine winners at each round requires complete knowledge over the entire system lifespan, which is infeasible in practice. We instead first consider a one-round WDP, by relaxing user capacity constraints that impose inter-round coupling, and develop the online auction later by employing the one-round auction as a building block.

We reformulate the one-round WDP and its dual problem by introducing a new scaled cost for each bid. The scaled cost acts as a key technique that connects a series of one-round auctions into an online auction, and will be explained later. We show that the one-round WDP with location awareness is NP-hard. We propose an efficient approximation algorithm that iteratively selects winners based on a greedy strategy, solving the primal and dual programs simultaneously. We analyze the approximation ratio of the algorithm and prove a small bound. Furthermore, we prove that the winner determination process is monotone, and can be combined with a critical value based payment scheme to form a truthful auction. The end result is a truthful and individually rational one-round auction that can be executed in polynomial time and achieves a good approximation ratio in social cost.

We proceed to propose an online primal-dual auction framework that connects a series of one-round auctions into an online auction. The user’s capacity constraint is the main concern during such a process. The online auction strategically protects a user’s potential of future participation by avoiding depleting its capacity too soon. This is realized by introducing a dual variable for each user to augment its cost based on the remaining level of its capacity. The new scaled cost is adjusted judiciously, serving as input to the one-round auctions, which serves as building blocks in the online auction framework. Through theoretical analysis, we prove that there is only a small additive loss in the competitive ratio in connecting the one-round auctions. Our online auction further guarantees truthfulness, individual rationality and computational efficiency.

We evaluate the performance of the online and one-round crowd sensing mechanisms through extensive simulations based on real-world traces. We demonstrate that both mechanisms perform well in minimizing social cost with a low performance ratio (< 1.3), and investigate other properties such as bounded overall payment, winner satisfaction, time complexity and individual rationality.

In the rest of the paper, we review related work in Sec. 2, and discuss our system model in Sec. 3. Sec. 4 and Sec. 5 present the one-round and online auctions, which are evaluated in Sec. 6. Sec. 7 concludes the paper.

2 Related Work

Crowdsourcing has been extensively studied with different focuses. Poetz et al. [11] investigate the value of crowdsourcing and Ipeirotis et al. [12] present quality management algorithms for crowdsourcing services. Recently, a series of incentive mechanisms are designed for encouraging users to participate in crowdsourcing. Singla et al. [13] design an incentive mechanism for online procurement in crowdsourcing systems, using a regret minimization approach to handle the buyer’s budget constraint. Chen et al. [14] propose a new extrinsic reward mechanism when considering the network effects as a contributing factor. All of these work study general crowdsourcing rather than mobile crowd sensing. Moreover, some of them [15] focus on the network effect for one-round auctions, and some rely on a limiting assumption that user cost is sampled from a given distribution [13].

With the rapid development of mobile devices and embedded sensors, mobile crowd sensing has been a focal point of latest crowdsourcing research. Many researchers investigate auction mechanism design in mobile crowd sensing systems. Feng et al. [6] take into consideration location information when assigning sensing jobs to smartphones, and propose a truthful auction tailored for such a mobile crowd sensing system. Jin et al. [8] incorporate user’s quality of information into the design of incentive mechanisms for mobile crowd sensing systems. Han et al. [9] study incentive mechanisms for a scheduling problem in mobile crowd sensing. The above two studies [8][9] fail to address location awareness when assigning sensing tasks to smartphone users. They make an impractical assumption that any user can perform any task. Furthermore, the optimization problem with location considered in our work is a special set cover problem, which requires different solution techniques. Although Feng et al. [6] acknowledge the importance of location information, they focus on one-round auction design only. Moreover, their one-round auction model is
structured different from ours, in that each sensing task needs to be covered only once and each smartphone user can be accepted up to $r_i$ bids where $r_i$ is a positive integer. Thus, to solve our one-round winner determination problem, we propose a primal-dual algorithm with a different greedy strategy and analyze the approximation ratio by applying the LP-duality based approach. Different objectives of incentive mechanism design for mobile crowd sensing have also been considered widely. Yang et al. [16] aim to maximize the crowd source’s utility from both the crowd source’s and users’ perspectives. Koutsopoulos et al. [17] introduce a randomized incentive mechanism to minimize the total payment to the participating users, while delivering a certain quality of service to subscribers. Luo et al. [18] develop an all-pay auction based incentive mechanism, targeting expected profit maximization and individual rationality. The above literature considers only one-round auction that happens at a single time spot. Many practical sensing tasks, as exemplified by weather and traffic information collection, require continuous and realtime participation. Consequently, a multiple-round online auction over a desired time domain constitutes a natural incentive mechanism.

Kang et al. [10] study quality-aware online assignment of location-based tasks. They assume voluntary user participation, and aim to optimize the overall quality of task-completion. Zhao et al. [19] focus on an online auction model where users arrive randomly and bid tasks within their service coverage. Their online mechanisms aim to maximize the values of services under a platform’s budget constraint, targeting different optimization objectives. Wei et al. [20] investigate two-sided online interactions among mobile crowdsourcing service providers and users. Their online double auction framework can work with different price schedules. Two practical concerns were overlooked in the above incentive mechanism [20], which we address in this work: location awareness and user capacity limits. Gao et al. [21] study the long-term sensor selection problem in a location-aware sensing system. Their online auction aims to maximize the expected social welfare over all possible information realizations. Compared with the above existing literature, we target social welfare maximization, which is equivalent to social cost minimization in our setting. In addition, we have a different model for task coverage and user constraints. Our online mechanism design contains a non-trivial online set cover problem with capacity constraints, bringing more technical challenges.

Auction mechanisms have been been exclusively studied in many other network systems, such distributed systems [22], multi-agent systems [23], cloud computing systems [24]. However, none of them can be directly applied to the mobile crowd sensing systems, which have their unique characteristics: covering requirements since each sensing task needs to be covered at least once, location awareness and user capacity limits, leading to different optimization problems.

The primal-dual method with LP-duality based analysis is a powerful algorithmic technique that has been applied to solve many NP-hard problems. Briest et al. [25] design primal-dual greedy algorithms to solve packing integer programs like single-minded combinatorial auctions, unsplittable flow routing and multicast routing. For covering integer programs, Vazirani [26] uses the LP-duality approach to analyze a greedy algorithm for the minimum set multi-cover problem. Our one-round winner determination problem is a special set multi-cover problem, which involves both conventional constraints (covering requirement) and unconventional constraints (XOR-only bidding). The latter further lead to unconventional dual variables that are hard to update by the LP-duality approach we will leverage. With the help of auxiliary variables, we develop a new primal-dual greedy algorithm to solve the primal and dual programs simultaneously, achieving a good, provable approximation ratio.

## 3 System Model and Preliminaries

### 3.1 System Model

We consider a mobile crowd sensing system consisting of a large number of smartphone users and a cloud-based platform. The platform publicizes $K$ sensing tasks $S = \{s_1, s_2, \ldots, s_K\}$ for execution by smartphones, with remuneration. A sensing task specifies the desired service (e.g., collecting and reporting traffic or air quality information) and the location where the sensing data are to be collected. The platform acts as the auctioneer who procures sensing service and data from smartphone users through a reverse auction.

Let $[X]$ denote the integer set $\{1, 2, \ldots, X\}$. Assume there are at most $I$ smartphone users bidding for sensing tasks, and the set of users is $[I] = \{1, 2, \ldots, I\}$. As illustrated in Fig. 2, each smartphone user $i \in [I]$ is aware of its own location through a location service such as GPS, and is confined by a geographical service coverage. Only sensing tasks within its service coverage can be performed by user $i$. The service coverage naturally varies for different users, depending on their locations and the configuration of their smartphones.

![Fig. 2. An illustration of service coverage of smartphones. The blue circle denotes the service coverage of smartphone 1, within which three sensing requests (tasks 2, 3 and 4) fall. Thus, smartphone 1 can provide sensing service over any of the three tasks.](image)

The system evolves in a time-slotted fashion over $T$ time slots (a.k.a. rounds). A sensing task $s_k$ needs $Q_{k}^{(t)}$ participants to achieve desired quality (e.g., frequency, precision and fidelity) in the sensing data at round $t \in [T]$. At the beginning of round $t$, the platform publishes a set of sensing tasks to be performed at the current round. If no sensing data is needed for task $s_k$ in round $t$, $Q_{k}^{(t)}$ is set to zero. A smartphone user may arrive in any round. Assume user $i \in [I]$ arrives at the $t_i$-th round. It first submits an integer pair $(t_i, \Gamma_i)$, where $\Gamma_i$ is user $i$’s capacity, i.e.,
the maximum number of tasks user $i$ can accommodate from now ($t_i$-th round) to a future time ($t_i$+$th$ round) it specifies. Without loss of generality, we assume that each sensing task consumes one unit of the user’s capacity. Such a user sensing capacity constraint is generally considered in the mobile crowd sensing literature [6], [10], and the values of $\Gamma_i$ and $t_i+$ depend on a number of factors, e.g., the smartphone’s battery level and the user’s available service period. Then within $[t_i, t_i+]$, at the beginning of round $t_i$, user $i$ may submit up to $J$ alternative bids, each of which is a tasks-bid pair $(s_{ij}^{(t)}, b_{ij}^{(t)})$: $S_i \subseteq S$ is a set of sensing tasks that user $i$ is willing to perform within its service coverage in $t_i$, and $b_{ij}^{(t)}$ is the claimed cost that user $i$ wants to charge for the service. We further adopt the XOR-bidding language [27] and assume that each user can win at most one bid (including one set of sensing tasks) during each round.

### 3.2 Truthful Mechanism Preliminaries

In each round $t_i$, after receiving user bids, the platform computes and announces the auction result. A binary variable $x_{ij}^{(t)}$ equals 1 if the platform accepts user $i$’s $j$-th bid in slot $t_i$, and 0 otherwise. The platform also calculates the remuneration $p_{ij}^{(t)}$ for a winner $i$. It pays $p_{ij}^{(t)}$ to the winner $i$ after verifying the data (e.g., check whether the desired tasks are completed within the time limit). Let $v_{ij}^{(t)}$ denote the true cost of user $i$ to perform tasks in set $S_{ij}^{(t)}$, the utility of that bid with bidding price $b_{ij}^{(t)}$ is:

\[
  u_{ij}(v_{ij}^{(t)}) = \begin{cases} 
  p_{ij}^{(t)} - v_{ij}^{(t)} & \text{if } x_{ij}^{(t)} = 1 \\
  0 & \text{otherwise}
\end{cases}
\]

In terms of strategic behaviour, users are assumed to be rational but selfish, with a natural goal of maximizing their respective utilities. They may choose to submit a falsified bid $b_{ij}^{(t)} \neq v_{ij}^{(t)}$, if doing so may lead to a higher utility. We instead value the “honesty” of the entire crowd sensing ecosystem, and pursue highest social welfare possible, for which it is important to elicit truthful bids from users. Important notations are listed in Table 1 for easy reference.

#### TABLE 1

<table>
<thead>
<tr>
<th>Notation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$I$</td>
<td># of users</td>
</tr>
<tr>
<td>$T$</td>
<td># of time slots</td>
</tr>
<tr>
<td>$K$</td>
<td># of sensing tasks</td>
</tr>
<tr>
<td>$S$</td>
<td>task set $S = {s_1, s_2, \ldots, s_K}$</td>
</tr>
<tr>
<td>$S_{ij}^{(t)}$</td>
<td>set of tasks in user $i$’s $j$-th bid</td>
</tr>
<tr>
<td>$b_{ij}^{(t)}$</td>
<td>asking price of user $i$ to execute tasks in $S_{ij}^{(t)}$</td>
</tr>
<tr>
<td>$v_{ij}^{(t)}$</td>
<td>true cost of user $i$ to execute tasks in $S_{ij}^{(t)}$</td>
</tr>
<tr>
<td>$x_{ij}^{(t)}$</td>
<td>user $i$’s $j$-th bid wins (1) or not (0) at slot $t$</td>
</tr>
<tr>
<td>$Q_k^{(t)}$</td>
<td># of participants required for task $s_k$ in slot $t$</td>
</tr>
<tr>
<td>$\Gamma_i$</td>
<td>user $i$’s capacity from round $t_i$ to round $t_i+$</td>
</tr>
<tr>
<td>$P_{ij}^{(t)}$</td>
<td>payment to user $i$ in slot $t$</td>
</tr>
</tbody>
</table>

**Definition (Dominant Strategy):** A strategy is dominant if, the strategy earns a user a larger utility than any other strategies, regardless of what any other users do.

**Definition (Truthfulness in Bidding Price):** A mobile crowd sensing auction is truthful in bidding price if for any user $i$, bidding its true cost $v_{ij}^{(t)}$ forms its dominant strategy, i.e., for all $b_{ij}^{(t)} \neq v_{ij}^{(t)}$, $u_{ij}(v_{ij}^{(t)}) \geq u_{ij}(b_{ij}^{(t)})$.

**Definition (Individual Rationality):** Users always obtain nonnegative utility, $u_{ij}^{(t)}(b_{ij}^{(t)}) \geq 0$.

**Definition (Social Welfare, Social Cost):** The social welfare in the online mobile crowd sensing auction is the aggregate utility of the platform $-\sum_{i \in [I]} \sum_{j \in [J]} \sum_{s \in [S]} p_{ij}^{(t)} x_{ij}^{(t)}$ and smartphone users $\sum_{i \in [I]} \sum_{j \in [J]} \sum_{s \in [S]} (y_{ij}^{(t)} - v_{ij}^{(t)}) x_{ij}^{(t)}$.

Towards design of such an online auction, we first relax its true cost $v_{ij}^{(t)}$ forms its dominant strategy, i.e., for all $b_{ij}^{(t)} \neq v_{ij}^{(t)}$, $u_{ij}(v_{ij}^{(t)}) \geq u_{ij}(b_{ij}^{(t)})$.

**Definition (Individual Rationality):** Users always obtain nonnegative utility, $u_{ij}^{(t)}(b_{ij}^{(t)}) \geq 0$.

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### 3.3 Online Auction Problem

Under truthful bidding, the winner determination problem for social cost minimization can be modelled by the following integer linear program (ILP):

\[
\begin{align*}
\text{minimize} & \quad \sum_{i \in [I]} \sum_{j \in [J]} \sum_{s \in [S]} b_{ij}^{(t)} x_{ij}^{(t)} \\
\text{subject to:} & \quad \sum_{i \in [I]} x_{ij}^{(t)} \leq 1, \quad \forall i \in [I], \forall t \in [T], \\
& \quad \sum_{j \in [J]} \sum_{s \in [S]} |s_{ij}^{(t)}| x_{ij}^{(t)} \leq \Gamma_i, \quad \forall i \in [I], \\
& \quad x_{ij}^{(t)} \in \{0, 1\}, \quad \forall i \in [I], \forall j \in [J], \forall t \in [T].
\end{align*}
\]

Constraint (1a) guarantees that each sensing task $s_k \in S$ is covered by at least $Q_k^{(t)}$ participants in round $t_i$ for quality control. Constraint (1b) specifies that each user wins at most one bid in each round. Each user’s capacity constraint is modelled in (1c), to practically limit user $i$’s participation. Note that a user may choose not to submit any bid in the current round, due to its remaining capacity, preference or work schedule. That can be equivalently modelled by setting its bidding prices to infinity.

We relax the integrality constraints of $x_{ij}^{(t)} \in \{0, 1\}$ to $x_{ij}^{(t)} \geq 0$ and add one more set of constraints $x_{ij}^{(t)} \leq 1, \forall i \in [I], \forall j \in [J], \forall t \in [T]$ to the above ILP. By introducing dual variables $y_{ij}^{(t)}$, $z_{ij}^{(t)}$ and $\lambda_i$ to constraints (1a), $x_{ij}^{(t)} \leq 1$, (1b) and (1c) respectively, the dual of the resulting linear program relaxation, which will be used in the online primal-dual auction design in Sec. 5.1, can be formulated as:

\[
\begin{align*}
\text{maximize} & \quad \sum_{i \in [I]} \sum_{k \in [K]} (\sum_{i \in [I]} \sum_{j \in [J]} \sum_{s \in [S]} Q_{ik}^{(t)} y_{ik}^{(t)} - \sum_{i \in [I]} \sum_{j \in [J]} \sum_{s \in [S]} z_{ij}^{(t)} - \sum_{i \in [I]} \sum_{s \in [S]} \Gamma_i \lambda_i) \\
\text{subject to:} & \quad \sum_{k \in [K]} k \in [S] \left| S_{ij}^{(t)} \right| z_{ij}^{(t)} - \left| S_{ij}^{(t)} \right| \lambda_i - z_{ij}^{(t)} - \lambda_i \leq b_{ij}^{(t)} + \sum_{k \in [K]} k \in [S] \left| S_{ij}^{(t)} \right| y_{ik}^{(t)} \\
& \quad \forall i \in [I], \forall j \in [J], \forall t \in [t_i, t_i+].
\end{align*}
\]
users’ capacities constraints (1c) that impose temporal correlation in auction decision making, and design a truthful auction that is carried out at each round to determine the winners. Then based on the one-round auction, we present an online auction framework that can decompose the long term auction to a series of one-round auctions by modifying the bidding prices according to remaining available capacity, with a proven small loss factor in competitive ratio.

**Definition** (Competitive ratio): The competitive ratio is the upper-bound ratio of the social cost generated by our online auction to the social cost produced by an optimal solution of the offline WDP in (1).

## 4 One-Round Auction Mechanism

In this section, we first formulate the one-round WDP in Sec. 4.1. We then introduce our one-round auction in Sec. 4.2, which will be used later for developing the online mechanism in Sec. 5. Theoretical analysis of the one-round auction is presented in Sec. 4.3.

### 4.1 The One-Round Winner Determination Problem

The one-round WDP is formulated as follows, which includes the same constraints related to the current time slot from (1), and excludes user capacity constraints (1c). We modify the cost $b_{ij}^{(t)}$ to a scaled cost $w_{ij}^{(t)}$ according to the level of remaining capacity, and use it in the objective function. Although we delete constraints (1c), they are actually considered in the design of the scaled cost (which will be discussed later in Sec. 5.1).

\[
\begin{align*}
\text{minimize} & \quad \sum_{i \in [I]} \sum_{j \in [J]} w_{ij}^{(t)} x_{ij}^{(t)} \\
\text{subject to:} & \quad \sum_{j \in [J]} x_{ij}^{(t)} \geq Q_{ij}^{(t)}, \forall k \in [K], \quad (3a) \\
& \quad \sum_{j \in [J]} x_{ij}^{(t)} \leq 1, \forall i \in [I], \quad (3b) \\
& \quad x_{ij}^{(t)} \in \{0, 1\}, \forall i \in [I], \forall j \in [J]. \quad (3c)
\end{align*}
\]

By relaxing integrality constraints of $x_{ij}^{(t)}$ into $0 \leq x_{ij}^{(t)} \leq 1, \forall i \in [I], \forall j \in [J]$, the dual of the above ILP’s LP relaxation can be formulated as follows. Here $y_k^{(t)}, z_{ij}^{(t)}$ and $\gamma_i^{(t)}$ are the same dual variables as in the dual of (1), corresponding to constraints (3a), $x_{ij}^{(t)} \leq 1$ and (3b), respectively.

\[
\begin{align*}
\text{maximize} & \quad \sum_{k \in [K]} Q_{ij}^{(t)} y_k^{(t)} - \sum_{i \in [I]} \sum_{j \in [J]} x_{ij}^{(t)} - \sum_{i \in [I]} \gamma_i^{(t)} \\
\text{subject to:} & \quad \sum_{k \in [K], s_k \in S_{ij}^{(t)}} y_k^{(t)} - z_{ij}^{(t)} - \gamma_i^{(t)} \leq w_{ij}^{(t)}, \forall i \in [I], \forall j \in [J] \quad (4a) \\
& \quad y_k^{(t)}, z_{ij}^{(t)}, \gamma_i^{(t)} \geq 0, \forall i \in [I], \forall j \in [J], \forall k \in [K]. \quad (4b)
\end{align*}
\]

Consider a special case of the primal problem in (3) by letting $Q_{ij}^{(t)} = 1, \forall k \in [K]$, and removing constraint (3b). The resulting problem is an instance of the minimum weighted set cover problem, known to be NP-hard [28]. Because the minimum weighted set cover problem is a special case of WDP (3), WDP (3) is also NP-hard. We resort to an efficient approximation algorithm for solving WDP (3) instead, and tailor a payment scheme to work in concert with the algorithm to form a truthful auction.

### 4.2 One-Round Auction Design

We first briefly introduce the main idea of our approximation algorithm. We say a sensing task $s_k$ is alive in $t$ until the number of participants at $t$ reaches the requirement $Q_k^{(t)}$. The algorithm iteratively selects winning bids based on a greedy strategy. In each iteration, a bid is picked to cover alive tasks at the least average cost.

We next define some variables. Consider a set $A^{(t)} = \{(i_1, j_1), (i_2, j_2), \ldots\}$ indicating a subset of bids submitted at the $t$th round, e.g. $(i_1, j_1)$ denotes the $j_1$th bid submitted by user $i_1$. Let $\rho_k^{A^{(t)}} = \sum_{(i,j) \in A^{(t)}, s_k \in S_{ij}^{(t)}} 1$ be the total number of participants to task $s_k$ among all bids in $A^{(t)}$. The utility of set $A^{(t)}$ is defined as $U(A^{(t)}) = \sum_{k \in [K]} \min(\rho_k^{A^{(t)}}, Q_k^{(t)})$, representing the valid contribution of bids in $A^{(t)}$ to all tasks in $S$. The marginal contribution of user $i$’s $j$th bid is the increased utility from adding $i$ into $A^{(t)}$, defined as:

\[
\begin{align*}
U_{ij}(A^{(t)}) &= U(A^{(t)} \cup (i, j)) - U(A^{(t)}) \\
&= \sum_{k \in [K]} (\min(\rho_k^{A^{(t)} \cup (i, j)}, Q_k^{(t)}) - \min(\rho_k^{A^{(t)}}, Q_k^{(t)})). \quad (5)
\end{align*}
\]

We present our one-round auction ORA in Algorithm 1. ORA selects one winning bid per iteration, and adds it to the set of winning bids $A^{(t)}$. Let $U_{ij}^{(t)}$ be a set that records current alive tasks in user $i$’s $j$th bid. Then $U_{ij}(A^{(t)})$ also denotes the number of alive tasks in that bid. To analyze the approximation ratio of ORA, which is the upper-bound ratio of the social cost generated by ORA to the social cost produced by an optimal solution of WDP (3), we seek the help of its dual problem (4). By LP duality [29], the bound between the primal and dual objective values also bounds the approximation ratio.

In ORA, $F^{(t)}(t)$ is a grand set of all bids in $t$, and $C^{(t)}$ is the candidate set of all valid bids, i.e., bids satisfying XOR-bidding (3b) and user capacity constraints (1c). Line 1 initializes the set of winning bids as $\emptyset$. All primal and dual variables are initialized to zero. The While loop in lines 2-11 adopts a greedy strategy to select winning bids, calculates payments and updates primal variables. Specifically, a winning bid with minimum average cost is determined in line 3, with the corresponding primal variable $x_{ij}^{(t)}$ updated to 1 in line 4. When a tasks set $S_{ij}^{(t)}$ is accepted, its cost $w_{ij}^{(t)}$ is ascribed to pairs $(k, S_{ij}^{(t)})$ for each alive task $s_k$ it can cover, i.e., for each such pair, define a cost for task $s_k$:

\[
c(k, S_{ij}^{(t)}) = \frac{w_{ij}^{(t)}}{\sum_{(i,j) \in A^{(t)}, k \in [K]}}. \forall k \in S_{ij}^{(t)}. \quad (6)
\]

In line 5-6 compute the payment to winner $i^*$ based on the critical value rule [30], [31], ensuring that if the winner reports a smaller cost, it must win (see Lemma 3 for details). Line 7 determines the winning bid if we relax the XOR-bidding constraint (3b) and user capacity constraint (1c). Another cost variable $c(k, S_{ij}^{(t)})$ is recorded in line 8 to help compute dual variables. Other bids from user $i^*$ are removed from candidate set $C^{(t)}$, for implementing XOR-bidding. Line 10 adds the winning bid to set $A^{(t)}$ and removes it from the bid set $F^{(t)}$. We update dual variables based on the cost of the task to bound the approximation ratio. Lines 12-15 compute values of dual variables $y_k^{(t)}$, where $H_K = \sum_{k=1}^{K} \frac{1}{k}$ is the $K$th harmonic number. The for loop in lines 16-19 assign values to dual variables $z_{ij}^{(t)}$ corresponding to winning bids, and announces the auction result.
Algorithm 1 One-Round Auction ORA

Input: \((w_{ij}^{(t)}, S_{ij}^{(t)}), Q_k^{(t)}, C^{(t)}, F^{(t)}\), \(i, j, k\)
Output: \(A^{(t)}\)

1. Define \(A^{(t)} = \emptyset\); Initialize \(x_{ij}^{(t)} = 0, p_i^{(t)} = 0, y_k^{(t)} = 0, z_i^{(t)} = 0, \gamma_i^{(t)} = 0, \forall i \in [I], j \in [J], k \in [K];\)
2. \(\text{while } U(A^{(t)}) < \sum_{k \in [K]} Q_k^{(t)}\) \(\text{do}
\quad 3. \ (i^*, j^*) = \arg \min_{(i,j)\in C^{(t)}} \frac{w_{ij}^{(t)}}{U_{ij}(A^{(t)})};
\quad 4. \ x_{ij}^{(t+1)} = 1; \ c(k, S_{ij}^{(t+1)}) = \frac{w_{ij}^{(t)}}{U_{ij}(A^{(t)})}, \forall k \in L_{ij}^{(t)};
\quad 5. \ (i^-, j^-) = \arg \min_{(i,j)\in C^{(t)} \setminus \{(i^*, j^*)\}} \frac{w_{ij}^{(t)}}{U_{ij}(A^{(t)})};
\quad 6. \ p_i^{(t)} = U_{ij}^{(t+1)}(A^{(t)}) \cdot \frac{w_{ij}^{(t)}}{U_{ij}^{(t)}(A^{(t)})};
\quad 7. \ (i^+, j^+) = \arg \min_{(i,j)\in C^{(t)} \setminus \{(i^*, j^*)\}} \frac{w_{ij}^{(t)}}{U_{ij}(A^{(t)})};
\quad 8. \ c(k, S_{ij}^{(t+1)}) = \frac{w_{ij}^{(t+1)}}{U_{ij}(A^{(t)})}, \forall k \in L_{ij}^{(t)};
\quad 9. \ C^{(t)} = C^{(t)} \setminus \{(i^*, j^*)\};
\quad 10. \ A^{(t)} = A^{(t)} \cup \{(i^*, j^*)\}; F^{(t)} = F^{(t)} \setminus \{(i^*, j^*)\};
\quad \text{end while}
12. \ max(k) = \max_{S_{ij}^{(t)}} \left\{\left(c(k, S_{ij}^{(t)}) + \sum_{i,j} c(k, S_{ij}^{(t)})\right), \forall k \in [K]\right\};
13. \ min(k) = \min_{S_{ij}^{(t)}} \left\{\left(c(k, S_{ij}^{(t)}) + \sum_{i,j} c(k, S_{ij}^{(t)})\right), \forall k \in [K]\right\};
14. \ \epsilon_k = max(k) - min(k), \forall k \in [K]; \ \epsilon = max_k \epsilon_k;
15. \ m_c(k) = max_{S_{ij}^{(t)}} \left\{\left(c(k, S_{ij}^{(t)}) + \sum_{i,j} c(k, S_{ij}^{(t)})\right), \forall k \in [K]\right\};
16. \text{for all } x_{ij}^{(t)} = 1 \text{ do}
\quad 17. \ \gamma_i^{(t)} = \sum_{k \in L_{ij}^{(t)}} (m_c(k) - \gamma_i^{(t)}); \ \forall k \in [K];
18. \text{Accept user } i^*\text{'s } j^*\text{th bid; Collect and verify sensing data; Pay } p_i^{(t)} \text{ to user } i^*;
19. \text{end for}
20. \text{Return } A^{(t)};

We next use a simple example to illustrate the winner determination and the payment calculation process in ORA. Here we ignore the superscript \((t)\). Assume in the current round, the platform publishes two tasks: \(s_1\) and \(s_2\) with \(Q_1 = Q_2 = 2\). Three smartphone users participate in the auction. User 1 submits 2 bids: \((\$2, (s_1, s_2)), (\$2, b_{s_1})\). User 2 also submits 2 bids: \((\$2, (s_1, s_2)), (\$1, s_1)\). User 3 submits 1 bid: \((\$3, (s_1, s_2))\). Assume the candidate set \(C\) includes all five bids and \(w_{ij} = b_{ij}\).

- Initialization: \(A = \emptyset; U(A) = 0\).
- First iteration: because \(U(A) < 4\), we compute \(u_{11}(A) = 1, u_{12}(A) = 1.2, u_{21}(A) = 2, u_{22}(A) = 1.8, u_{31}(A) = 1.5\). As \(u_{11}(A)\) is the minimum, user 1’s first bid is selected. Next we compute the payment to user 1. User 1’s second bid is the threshold bid as it has the second minimum average cost, thus the payment is calculated as \(p_1 = U_{11}(A) \times \frac{w_{11}^{(t)}}{U_{11}(A)} = 2.4\). We exclude user 1’s bids from the candidate set \(C\) and update \(A = \{(1, 1), A(U) = 2\).
- Second iteration: Since \(U(A) = 2 < 4\), we continue to compute \(u_{21}(A) = 2, u_{22}(A) = 1.8, u_{31}(A) = 1.5\). As \(u_{21}(A)\) is the minimum, user 3’s bid wins. User 2’s second bid is the threshold bid and the payment to user 2 is \(p_2 = U_{21}(A) \times \frac{w_{21}^{(t)}}{U_{21}(A)} = 3.6\). We exclude user 3’s bid from the candidate set \(C\) and update \(A = \{(1, 1), (3, 1), A(U) = 4\). Now \(U(A) = \sum_k Q_k\), ORA terminates.

4.3 Theoretical Analysis

We next analyze the performance of ORA, in terms of correctness, polynomial running time, approximation ratio in social cost, truthfulness and individual rationality.

i) Correctness and Polynomial Time

Lemma 1. ORA in Algorithm 1 generates a feasible solution to dual problem (4).

Proof: Case 1: First, suppose user i’s jth bid is not picked by ORA. Let’s order the tasks in \(S_{ij}^{(t)}\) by the reverse order in which their participation requirement \((Q_k^{(t)})\) is satisfied. When task \(s_k\)’s participation requirement is satisfied, \(S_{ij}^{(t)}\) has at least \(k\) unsatisfied tasks. Therefore, it can cover the alive tasks at a cost no larger than \(w_{ij}^{(t)}\). Recall that \(max(k)\) and \(min(k)\) represent the maximum and minimum costs, respectively, of \(s_k\) when we consider all bids and ignore constraints (3b) and (1c). When \(s_k\)’s requirement is satisfied, its cost \(m_c(k)\) should be at most \(\frac{w_{ij}^{(t)} \max(k)}{k \min(k)}\).

\[
\sum_{k \in [K], s_k \in S_{ij}^{(t)}} y_k^{(t)} - z_i^{(t)} - \gamma_i^{(t)} = \sum_{k \in [K], s_k \in S_{ij}^{(t)}} y_k^{(t)} - z_i^{(t)} = \frac{1}{H_k \epsilon} \sum_{k \in [K], s_k \in S_{ij}^{(t)}} \frac{1}{k \min(k)} \frac{w_{ij}^{(t)} \max(k)}{k \min(k)} \\
\leq \frac{w_{ij}^{(t)}}{H_k \epsilon} H_{S_{ij}^{(t)}} \epsilon \leq \frac{w_{ij}^{(t)}}{H_k \epsilon} H_{S_{ij}^{(t)}} = w_{ij}^{(t)}.
\]

Thus, constraint (4a) holds when user i’s jth bid fails.

Case 2: Next assume user i’s jth bid wins in the one-round auction ORA, Then

\[
\sum_{k \in [K], s_k \in S_{ij}^{(t)}} y_k^{(t)} - z_i^{(t)} - \gamma_i^{(t)} = \sum_{k \in [K], s_k \in S_{ij}^{(t)}} y_k^{(t)} - z_i^{(t)} = \frac{1}{H_k \epsilon} \sum_{k \in [K], s_k \in S_{ij}^{(t)}} \frac{1}{k \min(k)} \frac{w_{ij}^{(t)} \max(k)}{k \min(k)} \\
= \frac{1}{H_k \epsilon} \left( \sum_{k \in [K], s_k \in S_{ij}^{(t)} \setminus L_{ij}^{(t)}} m_c(t) + \sum_{k \in [K], s_k \in L_{ij}^{(t)}} c(k, S_{ij}^{(t)}) \right) \\
= \frac{1}{H_k \epsilon} \left( \sum_{k \in [K], s_k \in S_{ij}^{(t)} \setminus L_{ij}^{(t)}} m_c(t) + w_{ij}^{(t)} \right)
\]

Order tasks in \(L_{ij}^{(t)}\) first, and the remaining tasks \((S_{ij}^{(t)} \setminus L_{ij}^{(t)})\) in the reverse order of their participation requirement satisfaction. Then \(\forall s_k \in S_{ij}^{(t)} \setminus L_{ij}^{(t)}\), when \(s_k\)’s requirement is satisfied, its cost \(m_c(k)\) should be at most \(\frac{w_{ij}^{(t)} \max(k)}{k \min(k)}\).

Therefore, the right hand side of constraint (4a) is bounded by \(w_{ij}^{(t)}\). □

Theorem 1. ORA in Algorithm 1 returns a feasible solution to ILP (3) and (4) in polynomial time.

Proof: (Polynomial running time): Line 1 initializes all the variables in linear time, within \(O(IJK)\) steps. The while loop runs at most \(I\) times since it selects one winner during each iteration and there are at most \(I\) users. The body of the while loop (lines 3-10) takes \(O(IJK)\) steps to determine the winner, calculates the payments and updates variables. Thus, the computation complexity of the while loop is \(O(I^2JK)\). The values of \(max(k), min(k), \epsilon_k, \epsilon, m_c(k),\) and \(y_k^{(t)}\)
can be computed in $O(IJK)$ steps (lines 12-15). The \texttt{for} loop (lines 16-19) iterates $IJ$ times to update the dual variable $z_{ij}^t$, and its running time is $O(IJK)$. The last step in line 20 takes one step to return the winner set. In summary, the running time of ORA in Algorithm 1 is $O(I^2JK)$.

(Primal feasibility): If the one-round WDP (3) is solvable, there exists at least a feasible solution by selecting one bid per user. Therefore, Algorithm 1 terminates either before or when the candidate set $C^t$ becomes empty. When it terminates, $U(A^{(t)}) \geq \sum_{k \in [K]} Q_k^{(t)}$ and $p_k^{(t)} \geq Q_k^{(t)}, \forall k \in [K]$, satisfying constraints (3a). Constraints (3b) hold because line 3 and line 9 guarantee each user can have at most one bid accepted. Constraints (3c) will not be violated as values $x_{ij}^t$ are initialized to 0 (line 1) and updated to 1 (line 4).

(Dual feasibility): Lemma 1 proves that ORA returns a feasible solution to LP (4).

\textbf{ii) Approximation Ratio}

\textbf{Theorem 2.} Let $p$ and $d$ be the primal objective value in (3) and the dual objective value in (4) returned by ORA, respectively. ORA guarantees $\alpha d \geq p$ with $\alpha = H_K \epsilon$, where $H_K = \sum_{k=1}^{K} \frac{1}{K}$ and $\epsilon$ is defined in line 14 of Algorithm 1. The approximation ratio of ORA is $\alpha$.

Proof: Upon termination of ORA, the objective value of the dual problem (4) is:

$$d = \frac{1}{H_K \epsilon} \sum_{i,j} \sum_{k} Q_k^{(t)} m_{k}(k) - \frac{1}{H_K \epsilon} \sum_{i,j} \sum_{k : s_k \in U_{ij}} (m_{k}(k) - c(k, S_{ij}^{(t)}))$$

$$= \frac{1}{H_K \epsilon} \sum_{i,j} \sum_{k} c(k, S_{ij}^{(t)}).$$

The second equality holds because for each sensing task $s_k$, there are $Q_k^{(t)}$ cost variables ($c(k, S_{ij}^{(t)})$) corresponding to it. Furthermore, $c(k, S_{ij}^{(t)})$ is assigned a value only when $s_k$ belongs to set $U_{ij}$. Consequently, $\sum_{i,j} \sum_{k : s_k \in U_{ij}} (m_{k}(k) - c(k, S_{ij}^{(t)}))$ can be rewritten as $\sum_{k \in [K]} Q_k^{(t)} m_{k}(k) - \sum_{i,j} \sum_{k} c(k, S_{ij}^{(t)}).

The objective value of the primal problem (3) is:

$$p = \sum_{(i,j) \in A^{(t)}} w_{ij}^{(t)} = \sum_{i,j} \sum_{k} c(k, S_{ij}^{(t)}).$$

The above equality follows because when bid $(S_{ij}^{(t)}, w_{ij}^{(t)})$ is selected by ORA, $w_{ij}^{(t)}$ is distributed over variables $c(k, S_{ij}^{(t)})$ of the tasks that are inside $S_{ij}^{(t)}$ and still alive.

Therefore, $H_K \epsilon \cdot d = p$. Let $p^*$ be the optimal objective value of ILP (3). By LP duality [29], $p^* \geq d$, then $p/p^* \leq p/d = H_K \epsilon = \alpha$. The approximation ratio of ORA is $\alpha$. Furthermore, if each user submits only one bid at each round, we can obtain $H_K/d = p$ and the approximation ratio is $H_K$ under this typical scenario.

\textbf{iii) Truthfulness and Individual Rationality}

\textbf{Lemma 2.} ORA is bid-monotonic, i.e., $\forall i \in [I], \forall j, \tilde{j} \in [J]$, if $w_{ij}^{(t)} < w_{i\tilde{j}}^{(t)}$ and $S_{ij}^{(t)} = S_{i\tilde{j}}^{(t)}$, then $x_{ij}^{(t)} = 1$.

Proof: Assume user $i$’s $j$th bid wins, i.e., $x_{ij}^{(t)} = 1$, then this bid has the minimum $\frac{w_{ij}^{(t)}}{U_{ij}(A^{(t)})}$ in the current iteration. If user $i$ reports a smaller cost $w_{ij}^{(t)} < w_{i\tilde{j}}^{(t)}$, then $x_{ij}^{(t)}$ will cover the same set of sensing tasks, as $U_{ij}(A^{(t)}) = U_{i\tilde{j}}(A^{(t)})$ according to the definition in (5), $w_{ij}^{(t)}/U_{ij}(A^{(t)}) < w_{i\tilde{j}}^{(t)}/U_{i\tilde{j}}(A^{(t)})$ implies that bid $(w_{ij}^{(t)}, S_{ij}^{(t)})$ wins in or even before the current iteration by our greedy algorithm in ORA. Therefore, Lemma 2 holds.

\textbf{Lemma 3.} The payments to all winners calculated by ORA are critical in the following sense: assume a winning bid, user $i$’s $j$th bid, reports a new cost $\tilde{w}_{ij}^{(t)}$, instead of $w_{ij}^{(t)}$; user $i$’s $j$th bid will win if $\tilde{w}_{ij}^{(t)} \leq p_i$, and will fail otherwise.

Proof: According to ORA, user $i$’s $j$th bid is the threshold bid for it since when we exclude $(i^*, j^*)$ from the candidate set, user $i$’s $j$th bid is the first bid that is accepted by ORA. Apparently, user $i$’s $j$th bid will win if $\tilde{w}_{i^*j^*}^{(t)}/U_{i^*j^*}(A^{(t)}) \leq \tilde{w}_{ij}^{(t)}/U_{ij}(A^{(t)})$ and will fail otherwise. By setting $p_i^{(t)} = U_{i^*j^*}(A^{(t)}) \cdot \tilde{w}_{ij}^{(t)}/U_{ij}(A^{(t)})$, ORA guarantees that $\tilde{w}_{i^*j^*}^{(t)}/U_{i^*j^*}(A^{(t)}) \leq \tilde{w}_{ij}^{(t)}/U_{ij}(A^{(t)})$ when $\tilde{w}_{i^*j^*}^{(t)} \leq p_i$ and $\tilde{w}_{i^*j^*}^{(t)}/U_{i^*j^*}(A^{(t)}) > \tilde{w}_{ij}^{(t)}/U_{ij}(A^{(t)})$ when $\tilde{w}_{i^*j^*}^{(t)} > p_i$. Thus, we conclude that each winner is paid with a critical value.

\textbf{Theorem 3.} ORA is truthful in bidding price.

Proof: The Myerson’s theorem [30], [31] indicates that a reverse auction is truthful in bidding price if and only if

1. the auction result ($x_{ij}^{(t)}$) is monotonically non-decreasing with the decrease of the reported cost ($w_{ij}^{(t)}$) and
2. each winner is paid with a critical value. Hence, combining Lemma 2 and Lemma 3, we finish the proof.

\textbf{Theorem 4.} The one-round auction ORA achieves individual rationality.

Proof: As indicated in Lemma 3, bid $(w_{i^*j^*}^{(t)}, S_{i^*j^*}^{(t)})$ is the threshold bid for winning bid $(w_{i^*j^*}^{(t)}, S_{i^*j^*}^{(t)})$, and $\tilde{w}_{i^*j^*}^{(t)}/U_{i^*j^*}(A^{(t)}) \leq \tilde{w}_{ij}^{(t)}/U_{ij}(A^{(t)})$. It is clear that the payment $p_i^{(t)}$ to winner $i^*$ is at least $w_{i^*j^*}^{(t)}$, as $p_i^{(t)} = U_{i^*j^*}(A^{(t)}) \cdot \tilde{w}_{ij}^{(t)}/U_{ij}(A^{(t)})$. Furthermore, $w_{i^*j^*}^{(t)} \geq b_i^{(t)}$ (lines 6-7 in OPD), the utility of user $i$’s $j$th bid with bidding price $b_i^{(t)}$ is $u_i^{(t)} = p_i^{(t)} - b_i^{(t)} \geq 0$ as Theorem 3 guarantees $u_i^{(t)} = b_i^{(t)}$. Therefore, smartphone users always obtain non-negative utility.

\section{The Online Mechanism}

We next design the online auction mechanism for task allocation and participation incentivization, based on an online auction framework that decomposes the online WDP into a series of one-round WDFs.

\subsection{Online Auction Mechanism}

The key challenge of online task allocation lies in users’ capacity constraints over their available periods: different overall social costs can be produced when the capacities are spent at different rounds. If a user’s bid with a large set of tasks is accepted at an early stage, that user may lose opportunities to participate later in the crowd sensing
Algorithm 2 The Online Auction Mechanism OPD

Input: \( \{b_{ij}^{(t)}, S_{ij}^{(t)}\}, Q_{ij}^{(t)}, \Gamma, \tau, t, t_-, t_+, \forall i, j, t, k \)

Output: \( A^{(t)}, \forall t \)

1: Initialize \( \lambda_i^{(t)} = 0, \tau_i = 0, \forall i \in [I], \forall t \in [T] \);
2: Define \( F^{(t)} = \bigcup_{(i,j) \in [I] \times [J]} \{i, j\}, C^{(t)} = F^{(t)}, \forall t \in [T] \);
3: for all \( 1 \leq t \leq T \) do
4:   for all \( i \in [I], j \in [J] \) do
5:     if \( t \in [t_-, t_+] \) and \( \tau_i + |S_{ij}^{(t)}| > \Gamma_i \) then
6:       \( w_{ij}^{(t)} = b_{ij}^{(t)}, C^{(t)} = C^{(t)}(i, j) \);
7:     else \( w_{ij}^{(t)} = b_{ij}^{(t)} + |S_{ij}^{(t)}|\lambda_i^{(t-1)} \);
8:   end if
9: end for
10: \( A^{(t)} = \text{ORA}(\{w_{ij}^{(t)}, S_{ij}^{(t)}\}, Q_{ij}^{(t)}, C^{(t)}, F^{(t)}, \forall i, j, k) \);
11: for all \( (i, j) \in A^{(t)} \) do
12: \( \lambda_i^{(t)} = \lambda_i^{(t-1)}(1 + \frac{|S_{ij}^{(t)}|}{\alpha \tau_i}) + \frac{b_{ij}^{(t)}|S_{ij}^{(t)}|}{\alpha \tau_i}; \tau_i = \tau_i + |S_{ij}^{(t)}| \);
13: end for
14: for all \( (i, j) \in A^{(t)} \) do
15: \( \lambda_i^{(t)} = \lambda_i^{(t-1)} \);
16: end for
17: \( \lambda_i = \lambda_i^{(t)}, \forall i \in [I] \); Return \( A^{(t)}, \forall t \in [T] \);

Auction, due to capacity depletion, limiting future decision space of the platform. The platform may be consequently forced to purchase service from more expensive alternatives, leading to a higher social cost. The optimal strategy may intend to let a user \( i \) participate in all rounds between round \( t_- \) and round \( t_+ \) such that the platform can make the best decision among all users.

Along this direction, we should avoid exhausting a user’s capacity prematurely, by adjusting its cost based on remaining capacity. Following this intuition, OPD in Algorithm 2 is our online auction framework. A new variable \( \lambda_i^{(t)} \) is introduced for each user in line 1. It increases with the decrease of a user’s remaining capacity, \( \tau_i \) stores the number of performed tasks for user \( i \) before the current round, and is initialized to zero in line 1. A grand set \( F^{(t)} \) that includes all bids is defined in line 2. A candidate set \( C^{(t)} \) that contains all valid bids for the current round (satisfying constraints (3b) and (1c)) is initialized to \( F^{(t)} \). At each round, the new scaled cost \( w_{ij}^{(t)} \) will be used in the one-round auction ORA. When the number of tasks in the current bid plus the number of previously performed tasks exceed user \( i \)'s capacity (line 5), \( w_{ij}^{(t)} \) remains the same but that bid is excluded from the candidate set (line 6). \( w_{ij}^{(t)} \) equals \( b_{ij}^{(t)} + |S_{ij}^{(t)}|\lambda_i^{(t-1)} \) (line 7) otherwise. In this way, a bid with a smaller remaining capacity will be assigned a higher cost, reducing its chance to win. \( \lambda_i^{(t)} \) is updated carefully for each winning bid in the current round (lines 11-13), consisting of both the proportion of the capacity consumption and the proportion of the corresponding cost increment, and remains unchanged otherwise (lines 14-16). We set the dual variable \( \lambda_i \) to \( \lambda_i^{(t)} \) in line 18. The adjustment of \( \lambda_i^{(t)} \) at each round is the increment of \( \lambda_i \) at each round, which will be used to bound the competitive ratio.

Note that OPD invokes the one-round auction ORA in each round \( t \), and winners are paid during the execution of ORA (line 18 of Algorithm 1).

We next use a simple example to illustrate the idea of our online auction OPD. Suppose the online system spans 3 time slots (a.k.a. rounds). Both user 1 and user 2 arrive at the first round and stay to the last round. They submit one bid at each round and have the same capacity, i.e., \( J = 1, \Gamma_1 = \Gamma_2 = 2 \). For ease of presentation, we assume the one-round auction ORA can output the optimal solution with \( \alpha = 1 \).

- **First round:** The platform publishes one sensing task \( s_1 \) with \( Q_1 = 1 \). User 1 asks \$4 to perform \( s_1 \) and user 2 asks \$6 to perform \( s_1 \). Because \( \lambda_1^0 = \lambda_2^0 = 0 \), the new scaled cost \( w_1^1 = b_1^1 + \lambda_1^0 = 4 \) and \( w_2^1 = b_2^1 + \lambda_2^0 = 6 \). ORA selects user 1 as the winner. We update \( \lambda_1^1 = 4/4 = 1 \) and \( \lambda_2^1 = 0 \).

- **Second round:** The platform publishes \( s_2 \) with \( Q_2 = 1 \). User 1 asks \$6 to perform \( s_2 \) and user 2 asks \$6.5 to perform \( s_2 \). We compute the scale cost: \( w_1^2 = b_1^2 + 1 \times \lambda_1^1 = 7, w_2^2 = b_2^2 + 1 \times \lambda_2^1 = 6.5 \). User 2 is selected by ORA. We continue to update \( \lambda_2^2 = 1, \lambda_2^2 = 6.5/4 = 1.625 \).

- **Third round:** The platform publishes \( s_3 \) with \( Q_3 = 1 \). User 1 asks \$2 to perform \( s_3 \) and user 2 asks \$8 to perform \( s_3 \). We compute the scale cost: \( w_1^3 = b_1^3 + 1 \times \lambda_1^2 = 3, w_2^3 = b_2^3 + 1 \times \lambda_2^2 = 9.625 \). User 1 is selected by ORA. As this is the last round, OPD terminates.

Without the online optimization, a greedy algorithm produces a higher total cost, even if it can output the optimal bid from the candidate set at each round. Because it selects user 1 at the first round, user 1 at the second round and user 2 at the third round (user 1 runs out of capacity). The cost equals \$4 + \$6 + \$8 = \$18 \). At the same time, both the optimal strategy and our online auction generates a lower cost \$12.5 \).

5.2 Theoretical Analysis

We next prove the properties of our online auction.

**i) Correctness and Polynomial Time**

Theorem 5. OPD in Algorithm 2 computes a feasible solution for ILP (1) and its dual (2) in polynomial time.

Proof: *(Polynomial time):* Lines 1-2 define three new variables and initialize one dual variable in \( O(IJT) \) steps. The outer \( for \) loop iterates \( T \) times to determine winners at each round. The first inner \( for \) loop calculates the scaled cost in \( O(IJ) \) steps. Then ORA is executed in line 10 to select winners, and its running time is \( O(I \Gamma^2 JK) \) according to Theorem 1. The second and third inner \( for \) loops can be executed in \( O(I) \) steps to update dual variable \( \lambda_i^{(t)} \). Therefore, the running time of the outer \( for \) loop is \( O(I \Gamma^2 JK) \). The last line takes \( O(IT) \) steps to assign value to \( \lambda_i \) and return winner sets. In summary, the running time of OPD in Algorithm 2 is \( O(I \Gamma^2 JK) \).

*(Primal feasibility):* Constraints (1a), (1b) and (1d) can be guaranteed by constraints in (3). Constraint (1c) holds because if the number of tasks in the current bid exceeds the user’s remaining capacity, that bid is excluded from the candidate set \( C^{(t)} \) by OPD (lines 5-6) and is never accepted by the platform.
Lemma 4. Let \( \Delta P(t) \) and \( \Delta D(t) \) be the incremental increase of the primal and dual objective values after \( t \)th iteration, \( \Delta P(t) = P(t) - P(t-1) \) and \( \Delta D(t) = D(t) - D(t-1) \). For all \( t \in [T] \), \( \Delta P(t) \leq \alpha \frac{\beta}{\beta - 1} \Delta D \) where \( \beta = \min_{i \in [I], j \in [J], t \in [T]} \frac{\Gamma_i}{|S^{(t)}_{ij}|} \).

Proof: Recall that \( p \) and \( d \) are the prilam and dual objective value in (3) and the dual objective value in (4) returned by ORA, respectively. Theorem 2 proves that \( \alpha d \geq p \). At time \( t \), \( \Delta P(t) = \sum_{(i,j) \in A(t)} b_{ij} \).

\[
\Delta D(t) = \sum_{i \in A(t)} \Gamma_i (\lambda_i^{(t-1)} - \lambda_i^{(t)}) + d = d - \sum_{(i,j) \in A(t)} \frac{|S^{(t)}_{ij}| \lambda_i^{(t-1)}}{\alpha} \rho_{ij} - \sum_{(i,j) \in A(t)} \frac{b_{ij}^{(t)}}{\alpha} \frac{|S^{(t)}_{ij}|}{\Gamma_i} \geq \frac{p}{\alpha} - \sum_{(i,j) \in A(t)} \frac{|S^{(t)}_{ij}| \lambda_i^{(t-1)}}{\alpha} - \sum_{(i,j) \in A(t)} \frac{b_{ij}^{(t)}}{\alpha} \frac{|S^{(t)}_{ij}|}{\alpha \beta} \\
\geq \frac{\sum_{(i,j) \in A(t)} b_{ij}^{(t)}}{\alpha} - \frac{\sum_{(i,j) \in A(t)} |S^{(t)}_{ij}| \lambda_i^{(t-1)}}{\alpha} - \sum_{(i,j) \in A(t)} \frac{b_{ij}^{(t)}}{\alpha \beta} = \frac{1}{\alpha} - \frac{1}{\alpha \beta} \sum_{(i,j) \in A(t)} b_{ij}^{(t)} \Delta P(t). \] \( \square \)

**Theorem 5.** OPD in Algorithm 2 is a truthful and individually rational online auction that solves the WDP (1) in polynomial time and is \( \alpha \frac{\beta}{\beta - 1} \)-competitive in social cost.

Proof: (Truthfulness in bidding price): ORA is truthful in bidding price (Theorem 3) running at each round with an input of the scaled cost. The scaled cost is based on the bidding price and is known only to the platform. Therefore, OPD with ORA ensures truthfulness in bidding price.

(Truthfulness in task set \( S^{(t)}_{ij} \)): We need to show that a user will never bid sensing tasks that are not within its service coverage. As indicated by line 18 in ORA, if a winner cannot complete its submitted tasks, it will not receive any payment. Thus, user i has no incentives to lie about the task set.

(Truthfulness in \( t_{i-}, t_{i+} \) and \( \Gamma_i \)): \( t_{i-} \) is the time when user \( i \) starts to submit bids, which is recorded by the platform. Thus, user \( i \) cannot lie about it. If user \( i \) reports a smaller \( t_{i+} \) and \( \Gamma_i \), its utility in the single round remains the same, but it may lose opportunities to win in some rounds, reducing its overall utility. As a result, no user will take the risk to do that. If user \( i \) reports a larger \( t_{i+} \) and \( \Gamma_i \), it may be assigned to perform tasks that it cannot finish, and then it will not be paid and its overall utility cannot be improved by doing so.

Individual rationality, competitiveness in social cost, correctness and polynomial running time of OPD are guaranteed by Theorem 4, Theorem 6 and Theorem 5. \( \square \)

**6 Performance Evaluation**

We evaluate the performance of our one-round and online auctions based on real-world trace data. We further compare our mechanism with two related mechanisms from recent literature [6][19]. A widely-adopted trace [32], which includes GPS coordinates of approximately 320 taxis collected over 30 days in Rome, Italy, is utilized in our simulation study. We select a specific region, as marked in Fig. 3, and a 12-hour period (00:00 am—11:59 am) on 02/01/2014 from the trace. We assume that a smartphone is carried by the driver or the passenger of a taxi.

![Fig. 3. The selected region of Rome, surrounded by red pins.](image)

Inside the selected region, we randomly deploy 50-150 sensing tasks on some streets and identify 150-300 cars that are moving along these streets. By default, the costs of bids are uniformly distributed in the range of [10, 30]. \( Q_k^{(t)} \) is set within the range of [5, 50]. For the one-round auction, we choose a 10-minute trace and assume that if a taxi passes a sensing task along its route within the 10 minutes, then that task is within the service coverage of the smartphone on the taxi. We pick tasks randomly within the user’s service coverage to form the task set \( S^{(t)}_{ij} \). For the online auction, we vary the number of rounds \( T \) from 1 to 20, and assume the length of each round is 30 minutes. \( t_{i-} \) and \( t_{i+} \) are randomly picked between [1, T] and [t_{i-}, T].
respectively. \( \Gamma_i \) is a random number within the range of \([K, KT/2]\). The default value of \( T, J, I \) and \( K \) are 10, 2, 150 and 100, respectively.

### 6.1 Performance of One-round Auction ORA

**Performance Ratio.** We first examine the performance of our one-round auction, measured by the ratio of the objective value of ILP (3) returned by Algorithm 1 to the optimal objective value of (3). Fig. 4 shows the change of the ratio of ORA when we vary the number of tasks (\( K \)) and the number of bids per user (\( J \)). We can observe that it always performs well with a very low ratio (< 1.3). Its performance becomes better with the decrease of \( K \) and \( J \), achieving a close-to-optimal ratio (≈ 1) when there is a small set of tasks and each user submits only one bid. The observation is consistent with the theoretical analysis in Theorem 2, which indicates that the performance ratio is upper bounded by the the value of \( H_{KJ} \). The value of \( H_{KJ} \) increases when \( K \) becomes larger and the value of \( \epsilon \) grows larger when a user submits multiple bids.

Feng et al. [6] consider the location information when designing their one-round auction. We have evaluated their mechanism using the same trace data. Fig. 5 compares the performance of the two one-round auctions. ORA achieves a clearly lower performance ratio than Feng et al.’s mechanism. Furthermore, the ratio of ORA slightly decreases with the increment in the number of participants. This is because ORA is able to select more low-cost bids to cover the same sensing tasks from a larger set of bidders.

**Social Cost and Payment.** We generate the cost of bids using three distributions: uniform distribution (UNM), normal distribution (NORM) and exponential distribution (EXP), and plot the social cost (solid line) and payment (dashed line) returned by ORA under different numbers of tasks in Fig. 6. UNM is the default setting with the cost in [10, 30]. The mean and standard deviation of NORM are 20 and 10, respectively. The mean of EXP is set to 30. When the number of tasks increases, the platform must employ more smartphone users, incurring a higher social cost. The social cost of the exponential distribution is lower than those of the other two distributions, as it generates low cost bids with higher probabilities. The social cost of the uniform distribution is higher than that of the normal distribution, due to the fact that the probability of generating high cost bids (> 15) from the uniform distribution is much larger than that from the normal distribution. In addition, the total payment to winners is always higher than the social cost, as ORA guarantees that the payment to each winner is no smaller than its cost.

**Winner Satisfaction and Time Complexity.** Winner satisfaction of ORA, as measured by the percentage of winning users, is illustrated in Fig. 7. The highest value occurs when there are a small number of users and a large number of tasks. Winner satisfaction drops when a high number of users participate in the auction but only a small number of tasks are to be covered. The reason can be explained as follows: The number of winners remains relatively steady when the number of tasks is fixed. Therefore, only a small percentage of users can win when a large number of users submit bids. Furthermore, the platform does not need to hire many users when there is a small set of tasks, reducing the number of winners.

Fig. 8 illustrates the complexity of ORA under different input scales. We use tic and toc functions in MATLAB to measure the execution time of the main program without counting the initialization stage. We run five tests on our laptop (Intel Core i7-6700HQ/16GB RAM) and present the average result. It can be observed that the running time is less than 100 milliseconds even with a large input size and increases linearly with the increase of the number of tasks and users. We also compare our one-round auction with Feng et al.’s one-round auction, and then observe similar performance.

**Winner Satisfaction and Time Complexity.** Winner satisfaction of ORA, as measured by the percentage of winning users, is illustrated in Fig. 7. The highest value occurs when there are a small number of users and a large number of tasks. Winner satisfaction drops when a high number of users participate in the auction but only a small number of tasks are to be covered. The reason can be explained as follows: The number of winners remains relatively steady when the number of tasks is fixed. Therefore, only a small percentage of users can win when a large number of users submit bids. Furthermore, the platform does not need to hire many users when there is a small set of tasks, reducing the number of winners.

### 6.2 Performance of Online Auction OPD

**Performance Ratio.** We show the ratio computed by dividing the social cost generated by OPD to the offline optimal social cost under different numbers of tasks and users in the
left of Fig. 9. Comparing to the performance ratio of the one-round auction ORA in Fig. 4, we can observe that there is only a small loss. Furthermore, the ratio slightly decreases with user population increases and task number decrease. This is because OPD can select more low-cost bids from a larger candidate set, achieving a better performance ratio. However, when the number of tasks grows, OPD performs worse, which is in line with Theorem 7. Next, we construct an online mechanism, labelled Feng, by running Feng et al.’s one-round auction [6] at each round. Zhao et al.’s two online auctions (OMZ and OMG) [19] are also designed for location-based tasks. We directly use their simulation results which show the performance ratio under a larger budget (2000) and compare it with Feng. The figure on the right in Fig. 9 shows that the performance of Feng becomes worse when the number of tasks increases. Note that although the performance of Zhao et al.’s online auctions is slightly better than Feng, their study in [19] shows that the ratio can reach 3.4 with a small budget. It can be observed that our online auction OPD always outperforms the other three solutions over a wide range of user and task populations.

We further plot the ratio in Fig. 10 with varying number of rounds ($T$) and number of bids per user ($J$). Similar to the observation in Fig. 4, a larger $J$ leads to worse performance. The growth of $T$ has a negative influence on the ratio. This can be explained as follows. We proved that the competitive ratio of OPD depends on $\alpha$ and $\beta$. When the platform allows users to submit multiple bids and the online auction consists of many rounds, $\alpha$ and $\beta$ are more likely to have larger values. Fig. 10 also illustrates that Feng’s performance ratio is larger than ours and grows with the increase of $T$. This is because Feng is built by repeating Feng et al.’s one-round auction [6] without careful online coordination.

**Social Cost and Payment.** We plot two sets of data in Fig. 11 when we vary the number of tasks. The three upper lines represent the social cost generated by OPD, the total payment and the optimal social cost when the number of users is 150; and the three lower lines are for the case of $I = 250$. Again, we can see a downward trend in social cost with the increment of $I$ and the decrease of $K$. The overall cost in the online auction is always lower than the total payment, but slightly higher than the optimal cost.

**Individual rationality and Time complexity.** Finally, we verify individual rationality by comparing each winning bid’s real cost with its payment. The result is shown in Fig. 12, where the payment is plotted by a triangle mark and the cost is labelled by a star mark. It is clear that the payment is always greater than the cost. Fig. 13 shows the average running time of OPD. Again, the execution time is short even when we include 1000 users, 600 tasks and 600 one-round auctions. The running time linearly grows with the increment of $K$ and $T$, confirming our analysis in Theorem 5.

7 Concluding Remarks

We study online mechanism design for mobile crowd sensing systems, for incentivizing user participation and assigning location-aware sensing tasks to dynamically arriving users each subject to a capacity constraint. Our mechanism design consists of a one-round auction and an online algorithm framework. Theoretical analysis and trace-driven simulations demonstrate that our online mechanism achieves truthfulness, individual rationality, computational efficiency and a good competitive ratio. The scope of auction-based incentive mechanism design for mobile crowd sensing is a wide one. For example, there are natural alternatives to the XOR bidding rule. It is also interesting to consider a variation of the online decision making requirement where bids arriving in each decision interval are processed in batches, with the hope of more informed decision making. We will explore these interesting problem settings in our future work.

**References**


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