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Synthesis Reasoning and Its Application in Chinese Calligraphy Generation[1,2)]

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Abstract In this paper, we address the demanding task of developing intelligent systems equipped with machine creativity that can perform design tasks automatically. The main challenge is how to model human beings′ creativity mathematically and mimic such creativity computationally. We propose a "synthesis reasoning model" as the underlying mechanism to simulate human beings′ creative thinking when they are handling design tasks. We present the theory of the synthesis reasoning model, and the detailed procedure of designing an intelligent system based on the model. We offer a case study of an intelligent Chinese calligraphy generation system which we have developed. Based on implementation experiences of the calligraphy generation system as well as a few other systems for solving real-world problems, we suggest a generic methodology for constructing intelligent systems using the synthesis reasoning model.

Key words Synthesis reasoning model, intelligent system design, intelligent Chinese calligraphy generation, system design methodology

1 Introduction

What is creative thinking? What is the mechanism that underlines human beings′ creative thinking? How can one experiment with the purportedly biological creative thinking process through a computational approach? These questions pose challenges for researchers in the fields of AI and cognitive science. Researchers in intelligent CAD (ICAD) systems in particular have a strong interest in these problems. This paper can be considered a step towards understanding human creative process – we use a computer-based automatic reasoning system to mimic artistic creativity. Such a task for the computer is highly demanding as the implemented system has to satisfy both the theoretical soundness of machine intelligence and performance benchmarks.

There is a large body of existent work on simulating creative thinking for solving real-world problems. In 1975, Simon[1] pointed out that design and creation is a class of problems based on synthesis of existing ideas. Qian[2] argued that the synthesis process (using qualitative or quantitative approaches) is an important aspect of brain activities. Hall[3] simulated analogous reasoning using a computational approach. Kapur[4] explored the application of artificial intelligence in geometrical reasoning. Pan[5∼10] has researched in modeling visual information for intelligent computer aided design.

The structure of the paper is as follows. Section 2 briefly surveys the theory and concepts of synthesis reasoning model. Section 3 to Section 5 describe how an intelligent Chinese calligraphy generation system is designed and developed; the system serves as an example of the steps and procedures to develop a synthesis based intelligent system for solving some realistic problem. Section 3 presents the main ideas behind the design of our Chinese calligraphy generation system. Section 4 describes the knowledge representation used in our intelligent system for modeling Chinese characters, which is hierarchical and parametric. Section 5 reveals more details on how we construct our intelligent system using the general principle of synthesis reasoning. Section 6 gives the experimental results. Section 7 summarizes the work and proposes a generic methodology to design intelligent systems using the synthesis reasoning model. Section 8 concludes the paper.

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2 Theory of synthesis reasoning model

The synthesis reasoning model[5], or simply synthesis reasoning, is a model we propose to simulate human beings′ creative thinking activities when performing design tasks involving images of one kind or another. In the following, we briefly survey the basic theory and concepts of the synthesis reasoning model.

2.1 Theory of synthesis reasoning model

Synthesis reasoning is a generation oriented reasoning mechanism for simulating human creative thinking. Concepts essential to synthesis reasoning include: synthesis reasoning source, reasoning source intensity field, synthesis reasoning space, synthesis reasoning viewpoint and synthesis reasoning process. It is an attempt to relax the constraints of traditional reasoning mechanisms in artificial intelligence to solve problems by using a more flexible reasoning method. Because of the flexibility, synthesis reasoning is particularly suitable for reasoning tasks on shape design as in intelligent computer aided design system (ICAD system). Essentially, synthesis reasoning searches the feasible synthesis reasoning space to identify satisfying viewpoints in the space. A core step to apply the synthesis reasoning model therefore is to establish a synthesis reasoning space, usually by superimposing several input synthesis reasoning sources.

2.2 Key concepts of synthesis reasoning model

In the following, we briefly overview some key concepts employed in the synthesis reasoning model.

Synthesis reasoning source: A synthesis reasoning source $\boldsymbol{S}$ is a structure of the form: $\boldsymbol{S} = \{\boldsymbol{P}, \boldsymbol{m}, \boldsymbol{F}\}$. $\boldsymbol{P}$ is a collection of n components $\boldsymbol{P} = \{\boldsymbol{P}_1, \boldsymbol{P}_2, \cdots, \boldsymbol{P}_n\}$. $\boldsymbol{m}$ is a structure, which describes how the above n components can be combined together into a reasoning source. And $\boldsymbol{F}$ is a reasoning source intensity field.

Reasoning source intensity field: A reasoning source intensity field $\boldsymbol{F}$ describes the intensity distribution of different reasoning sources during a synthesis reasoning process. $\boldsymbol{F}$ is composed of two parts: $\boldsymbol{F} = \{\boldsymbol{FP}, \boldsymbol{Fm}\}$. $\boldsymbol{FP}$ is a collection of reasoning intensities, each of which is associated with one component of the reasoning source, *i.e.*, $\boldsymbol{FP} = \{\boldsymbol{FP}_1, \cdots, \boldsymbol{FP}_n\}$. $\boldsymbol{Fm}$ is a structure, which records how multiple components can be combined together. The intensity field $\boldsymbol{F}$ can be classified into two broad types: discrete intensity field where any reasoning intensity is either 0 or 1, and continuous intensity field where intensity can be an arbitrary real number, possibly negative or bigger than 1. With a discrete intensity field, a reasoning source will be either adopted (intensity=1) or ignored (intensity=0).

Synthesis reasoning space: The synthesis reasoning space $\boldsymbol{SS}$ is the result of superimposing multiple reasoning sources. Each position selected in a synthesis reasoning process is a potential synthesis reasoning result, denoted as $\boldsymbol{SS}(x, y, z)$. A synthesis reasoning space, which is composed of m reasoning sources, can thus be defined as

$$\boldsymbol{SS}(x, y, z) = \sum_{j=1}^{m} \sum_{i=1}^{n} (\boldsymbol{FP}_{ij}(x, y, z) \cdot \boldsymbol{P}_{ij}, \boldsymbol{Fm}_j \cdot \boldsymbol{m}_j)$$

Synthesis reasoning process: In general, we need to carry out two steps to set up a synthesis reasoning model. First, we need to construct a synthesis reasoning space using multiple synthesis reasoning sources. And then we need to identify a certain viewpoint/viewpoints in the synthesis reasoning space. There are two inputs to the synthesis reasoning model: one is the multiple reasoning sources and another is requirements on synthesis reasoning process, if any. Specifically, a two-valued synthesis reasoning process is a process that reasons simply by component replacement. For example, for a language being the experiment target, the word selection and sentence touchup process by rhymists forms a two-valued synthesis reasoning process. Two-valued synthesis reasoning is a degenerate case of the more general and sophisticated continuous valued synthesis reasoning process.

2.3 Computational model of synthesis reasoning model

Definition. If there is a correspondence between $\boldsymbol{T}$ and $\boldsymbol{B}$, then $\boldsymbol{T}$ is similar to $\boldsymbol{B}$ (in a broad sense), which can be denoted as $\boldsymbol{T} \sim \boldsymbol{B}$.

Let $\boldsymbol{B}_1, \cdots, \boldsymbol{B}_N$ be N pieces of synthesis reasoning source knowledge and $\boldsymbol{T}$ be the reasoning result, where each of $\boldsymbol{B}_i$ and $\boldsymbol{T}$ is composed of M components (parts). We can then represent the synthesis reasoning process and its result using the form: $\begin{cases} \boldsymbol{B}_i = \{b_{ij} | j = 1, \cdots, M\} \\ \boldsymbol{T} = \{t_j | j = 1, \cdots, M\} \end{cases} (i = 1, \cdots, N)$.

If the synthesis reasoning process is of one source, *i.e.*, $N=1$, the reasoning equation is simplified to be: $\begin{cases} t_1 = f_1(b_{11}) \\ \vdots \\ t_M = f_M(b_{1M}) \end{cases}$.

If there are multiple pieces of synthesis reasoning source knowledge, the reasoning process can be defined mathematically as follows.

Let

$$\boldsymbol{A} = \begin{pmatrix} a_{11} & \cdots & a_{1M} \\ a_{21} & \cdots & a_{2M} \\ \vdots & \cdots & \vdots \\ a_{N1} & \cdots & a_{NM} \end{pmatrix}_{N\times M} = \begin{pmatrix} \vec{\boldsymbol{a}}_1 \\ \vec{\boldsymbol{a}}_2 \\ \vdots \\ \vec{\boldsymbol{a}}_N \end{pmatrix}, \ \boldsymbol{B} = \begin{pmatrix} b_{11} & \cdots & b_{N1} \\ b_{12} & \cdots & b_{N2} \\ \vdots & \ddots & \vdots \\ b_{1M} & \cdots & b_{NM} \end{pmatrix}_{M\times N} = (\boldsymbol{B}_1 \quad \boldsymbol{B}_2 \quad \cdots \quad \boldsymbol{B}_N)$$

where $\vec{\boldsymbol{a}}_i$ is the similarity metric vector of $\boldsymbol{T}$ to the i-th reasoning source. Then the general form of synthesis reasoning equation $\boldsymbol{T} = \boldsymbol{F}(\boldsymbol{B}_1, \boldsymbol{B}_2, \cdots, \boldsymbol{B}_N)$ can be instantiated as (1)$\sim$(3).

$$\begin{cases} \boldsymbol{T} = \{t_1, t_2, \cdots, t_M\} = \sum\limits_{i=1}^{n} \boldsymbol{A}_i \boldsymbol{B}_i, \text{ where } \boldsymbol{A}_i = \begin{pmatrix} a_{i1} & 0 & \cdots & 0 \\ 0 & a_{i2} & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & a_{iM} \end{pmatrix}_{M\times M} & (1) \\ \sum\limits_{i=1}^{n} \boldsymbol{A}_i = \boldsymbol{I}_{M\times M} & (2) \\ \begin{cases} a_{ij} \in \{0_F, 1_F\} \subset F \\ a_{ij} \in [0_F, 1_F] \subset F \end{cases} & (3) \end{cases}$$

This equation can be expanded for ease of interpretation as

$$\begin{cases} \boldsymbol{T} = \boldsymbol{B}\boldsymbol{A} = (\boldsymbol{B}_1 \quad \boldsymbol{B}_2 \quad \cdots \quad \boldsymbol{B}_N) \begin{pmatrix} a_{11} & \cdots & a_{1M} \\ a_{21} & \cdots & a_{2M} \\ \vdots & \ddots & \vdots \\ a_{N1} & \cdots & a_{NM} \end{pmatrix}_{N\times M} \\ \begin{cases} a_{ij} \in \{0_F, 1_F\} \subset F \\ a_{ij} \in [0_F, 1_F] \subset F \end{cases} \end{cases}$$

In the above, (1) defines the degree of similarity between $\boldsymbol{T}$ and $\boldsymbol{B}_i$. (2) guarantees that the scale of $\boldsymbol{T}$ is synchronized with that of $\boldsymbol{B}_i$. And (3) ensures that $\boldsymbol{T}$ is more or less similar to $\boldsymbol{B}_i$ (similarity in a narrow sense).

If $\boldsymbol{B}_i, \boldsymbol{T} \in V$, $\oplus$ is closed in V, then $(V, \oplus)$ is an algebraic structure and an exchangeable group, where Σ is the abbreviation of the continuous operator $\oplus$ that satisfies the following properties:

$$\begin{cases} 1) \quad t_1 \oplus (t_2 \oplus t_3) = (t_1 \oplus t_2) \oplus t_3 \\ 2) \quad \exists 0 \in V, \exists x \in V \Rightarrow 0 \oplus x = x \oplus 0 = x \\ 3) \quad \forall x \in V, \exists y \in V, \text{ s.t. } x \oplus y = y \oplus x = 0 \\ 4) \quad \forall t_1, t_2 \in V, t_1 \oplus t_2 = t_2 \oplus t_1 \end{cases}$$

Here, V is a vector space on field F, *i.e.*, a) $(V, \oplus)$ is an exchangeable group;
b) $\forall a, b \in F \Rightarrow \begin{cases} (a \oplus b)x = ax \oplus bx \\ a(x \oplus y) = ax \oplus ay \\ (ab)x = a(bx) \\ 1_F x = x \end{cases}$. For simplicity, we will first study situations where there are only two reasoning sources $\boldsymbol{B}_1$ and $\boldsymbol{B}_2$, each of which has only one component, *i.e.*, the synthesis reasoning

process when $M = 1$ and $N = 2$. Under this circumstance, the form is: $\begin{cases} \boldsymbol{T} = \boldsymbol{A}_1\boldsymbol{B}_1 \oplus \boldsymbol{A}_2\boldsymbol{B}_2 \\ \boldsymbol{A}_1 \oplus \boldsymbol{A}_2 = \boldsymbol{I}_M \\ a_{ij} \in F; i = 1, 2; j = 1, \cdots, M \end{cases}$.
To verify such an equation satisfies the properties mentioned above, we assume $\boldsymbol{B}_1$ is unknown while $\boldsymbol{T}$ and $\boldsymbol{B}_2$ are both known. Then, the synthesis reasoning process takes the form:

$$\begin{cases} a_{11}^{-1}a_{11}\boldsymbol{B}_1 = a_{11}^{-1}\boldsymbol{T} - a_{21}a_{11}^{-1}\boldsymbol{B}_2 \\ a_{11} \oplus a_{21} = 1_F \\ a_{11}, a_{21} \in [0_F, 1_F] \end{cases}$$

Letting $k_1 = a_{11}^{-1}, k_2 = -a_{21}a_{11}^{-1}$, we will then have

$$\begin{cases} \boldsymbol{B}_1 = k_1\boldsymbol{T} \oplus k_2\boldsymbol{B}_2 \\ a_{11}^{-1}a_{11} = 1_F, 1_F x = x \\ k_1 \oplus k_2 = a_{11}^{-1}(1_F \oplus (-a_{21})) = a_{11}^{-1}(a_{11} \oplus (a_{21} \oplus (-a_{21}))) = a_{11}^{-1}a_{11} = 1_F \\ k_1, k_2 \in F(k_2 < 0_F) \end{cases}$$

Therefore, the synthesis reasoning process with two one-component reasoning sources is of the form: $\begin{cases} \boldsymbol{T} = a_{11}\boldsymbol{B}_1 \oplus a_{21}\boldsymbol{B}_2 \\ a_{11} \oplus a_{21} = 1_F \\ a_{11}, a_{21} \in F \end{cases}$, which shows that the synthesis reasoning process we defined does satisfy all the properties as described above.

To take a step further, suppose there are N reasoning sources, each of which has M components; the synthesis reasoning process is then of the form:

$$\begin{cases} \boldsymbol{T} = \boldsymbol{A}_1\boldsymbol{B}_1 \oplus \boldsymbol{A}_2\boldsymbol{B}_2 \oplus \cdots \oplus \boldsymbol{A}_N\boldsymbol{B}_N = \sum\limits_{i=1}^{N} \boldsymbol{A}_i\boldsymbol{B}_i & (4) \\ \sum\limits_{i=1}^{N} \boldsymbol{A}_i = \boldsymbol{I}_M & (5) \\ a_{ij} \in F & (6) \end{cases}$$

where $\boldsymbol{A}_i = \begin{pmatrix} a_{i1} & 0 & \cdots & \cdots & 0 \\ 0 & a_{i2} & 0 & \cdots & \vdots \\ \vdots & 0 & \ddots & 0 & \vdots \\ \vdots & \ddots & 0 & \ddots & 0 \\ 0 & \cdots & \cdots & 0 & a_{iM} \end{pmatrix}_{M \times M}$.

In the above equations, a_{ij} is the coefficient of $\boldsymbol{T}'$s similarity metric matrix to the j-th component of the i-th reasoning source. $\boldsymbol{A}_i$ is the similarity metric matrix of $\boldsymbol{T}$ to the i-th reasoning source. (4) defines the similarity of $\boldsymbol{A}_i$ to each $\boldsymbol{B}_i$. So $\boldsymbol{T}$ is analogically generated by all the $\boldsymbol{B}_i'$s. (5) aligns the scale of $\boldsymbol{T}$ with each $\boldsymbol{B}_i$. (6) shows that the similarity meter can be either similar (positive) or dissimilar (negative), *i.e.*, similarity in a broad sense.

3 Building an intelligent calligraphy generation system using the synthesis reasoning model

Chinese calligraphy is among the finest and most important Chinese art forms, and is an inseparable part of Chinese history. It can convey not only what is explicitly in the written message but also the emotions of the writer. The very delicate aesthetic effects achievable by Chinese calligraphy are generally considered to be unique among all calligraphic arts.

The most common use of calligraphic art in the digital world is creating typographic or artistic fonts for display or printing, for which Knuth has done some pioneering work[11]. In [12], the authors gave a detailed analysis of the writing effects that hairy brushes can produce. For artistic rendering, researchers have tried to model the brush used in calligraphy, such as [13] where the brush was modeled as a collection of bristles which evolved over the course of the stroke.

In this section and the two sections that follow, we present the design of an intelligent Chinese calligraphy generation system, which can generate aesthetic novel Chinese calligraphic artwork fully automatically based on a small number of input training samples. Our intelligent calligraphy generation system is a typical synthesis reasoning model based intelligent system. For such an intelligent system, how to devise an effective and complete knowledge representation according to the specific application problem is one of the key issues during system construction time. Another equally important issue is to devise suitable and efficient operators to instantiate the synthesis reasoning model as a concrete computational model for real-world problem solving.

In terms of knowledge representation, our system adopts a hierarchical and structural representation for Chinese calligraphic artwork, in which each synthesis reasoning source (in our case, it is one of the sample calligraphy pieces scanned from copybooks) is parameterized and represented in the synthesis reasoning space. After that we devise a series of operators to define the knowledge operations necessary for implementing the synthesis reasoning model. Finally, based on the constructed synthesis reasoning space and the defined knowledge operators, a certain viewpoint in the reasoning space is chosen, with which a new calligraphic artwork can be inferred.

To demonstrate the feasibility of the proposed methodology, we have implemented a prototype system, which is able to generate brand new Chinese calligraphic artwork fully automatically when given as input a small training set (typically below 10). To the best of our knowledge, there has not been any published work on the same approach. A related research is Ramalho and Ganascia′s work[14] on the simulation of creativity in jazz performance, in the sense that it is also an attempt to model artistic activities using analogous reasoning.

4 Knowledge representation of intelligent Chinese calligraphy generation system

The earliest Chinese characters are pictographs, which project meanings through shapes and images in an intuitive fashion. Over time, these characters gradually became symbols. It can be easily observed that many basic features frequently occur in different Chinese characters. To take advantage of this representation redundancy, we employ a hierarchical representation for Chinese characters. The hierarchical approach extends also to the level of calligraphy artwork.

4.1 Representation of Chinese calligraphy

Like many intelligent systems, knowledge representation is one of the key decisions to be made during system construction time. In this paper, we treat Chinese characters and calligraphic artwork as images that are in a parametric form. Mathematically, the "image" of the calligraphic artwork $\boldsymbol{C}$ is represented as the zonary image area covered by a series of ellipses. Denote this series $\boldsymbol{C}$ as composed of num_0 ellipses $\boldsymbol{P}_1, \boldsymbol{P}_2, \cdots, \boldsymbol{P}_{num_0}$. Let $\boldsymbol{F}_0 = \{1, 2, \cdots, num_0\}$, (x_i, y_i) be the center and a_i, b_i the lengths of the ellipse $\boldsymbol{P}_i'$s major and minor axes respectively. We can then define Chinese calligraphy as:

$$Area \triangleq \{(x, y) \in R^2 | \exists i \in \boldsymbol{F}_0, \frac{(x - x_i)^2}{a_i^2} + \frac{(y - y_i)^2}{b_i^2} \leqslant 1\}$$

The above knowledge representation is inspired by the method in [11], in which a zonary area is defined through an ellipse moving along a predefined curve.

4.2 Hierarchy of Chinese calligraphic artwork

Once all the input Chinese calligraphy data have been parameterized, they form a space, which is the feasible space of synthesis reasoning. It can be easily observed that many local features frequently recur in many different Chinese characters. To capitalize on this image information redundancy, we introduce a hierarchical representation of Chinese calligraphy. We decompose a Chinese calligraphy artwork into six levels: constructive ellipse level, primitive stroke level, compound stroke level, radical level, single-character level, and complete artwork level. Via this six-level framework, a Chinese calligraphy artwork is represented parametrically. The parametric representations of the calligraphic artwork at all levels altogether form the parameter space for the ensuing reasoning process to generate aesthetically acceptable novel calligraphy artwork.

We now turn to the concept of equivalent relationship for the formal definition of hierarchical representation of Chinese calligraphy. If $\boldsymbol{R}$ is the equivalent relationship defined over the field of $\boldsymbol{A} = \{a_1, a_2, \cdots, a_n\}$, *i.e.*, $\boldsymbol{R}$ is (1) self-reflective, (2) symmetrical, and (3) transitive; field $\boldsymbol{A}$ can be divided into a collection of sub-sets $\boldsymbol{A}_1, \boldsymbol{A}_2, \cdots, \boldsymbol{A}_m$ under $\boldsymbol{R}$, which satisfy the following properties: a) if $i \neq j,\ 1 \leqslant i, j \leqslant m$, then $\boldsymbol{A}_i \cap \boldsymbol{A}_j = \Phi$; b) $\forall a_i \in \boldsymbol{A}, \exists j, 1 \leqslant j \leqslant m$, s.t. $a_i \in \boldsymbol{A}_j$. Under the equivalent relationship $\boldsymbol{R}$, if $(a_i, a_j) \in \boldsymbol{R}, 1 \leqslant i, j \leqslant n$, we say that a_i is equivalent to a_j, which is denoted as $a_i \xleftrightarrow{R} a_j$.

To set up the hierarchy of calligraphy representation, we introduce four kinds of equivalent relationship, R_1, R_2, R_3, R_4:

R_1: all the constructive ellipses that compose the same primitive strokes are equivalent to each other;

R_2: all the primitive strokes that compose the same compound strokes are equivalent to each other;

R_3: all the compound strokes that compose the same radical are equivalent to each other;

R_4: all the radicals that compose the same Chinese character are equivalent to each other.

In our prototype system, we selected five typical and most frequently occurring primitive strokes (horizontal strokes, vertical strokes, left slanting strokes, right slanting strokes, point strokes), 24 typical and most frequently occurring compound strokes (Fig. 1(a)) and 36 radicals (Fig. 1(b)).

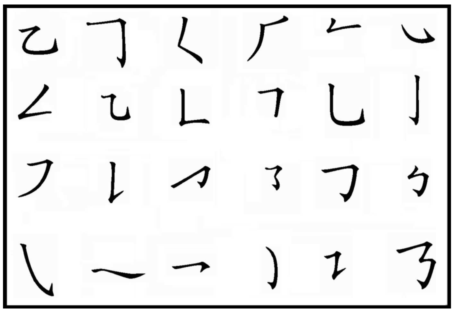

Fig. 1 (a) 24 implemented compound strokes

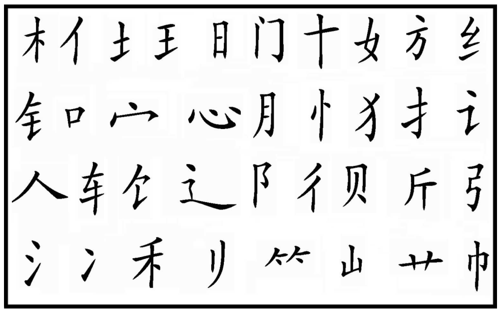

Fig. 1 (b) 36 implemented radicals

Suppose the family of constructive ellipses $\boldsymbol{C} = \{\boldsymbol{P}_t | t \in F_0\}$ is divided into num_1 equivalent classes (classes of primitive strokes) under the equivalent relationship R_1, which is denoted as $\boldsymbol{F}_1 \triangleq \{1, 2, \cdots, num_1\}$. These num_1 primitive strokes are divided into num_2 equivalent classes (classes of compound strokes) under the equivalent relationship R_2, which is denoted as $\boldsymbol{F}_2 \triangleq \{1, 2, \cdots, num_2\}$. These num_2 compound strokes are divided into num_3 equivalent classes (classes of radicals) under the equivalent relationship R_3, which is denoted as $\boldsymbol{F}_3 \triangleq \{1, 2, \cdots, num_3\}$. These num_3 radicals are divided into num_4 equivalent classes (classes of single characters) under the equivalent relationship R_4, which is denoted as $\boldsymbol{F}_4 \triangleq \{1, 2, \cdots, num_4\}$. That is, in a certain Chinese calligraphic artwork $\boldsymbol{C}$, there are num_1 primitive strokes $\boldsymbol{P}_{1,i}, i \in \boldsymbol{F}_1$. And $\boldsymbol{C}$ contains num_2 compound strokes $\boldsymbol{P}_{2,i}, i \in \boldsymbol{F}_2$. Or we can say that $\boldsymbol{C}$ has num_3 radicals $\boldsymbol{P}_{3,i}, i \in \boldsymbol{F}_3$. Also we can say that $\boldsymbol{C}$ comprises num_4 single characters $\boldsymbol{P}_{4,i}, i \in \boldsymbol{F}_4$. Obviously, when we apply synthesis reasoning using the granularity of single characters, $num_4 \equiv 1$.

Now the hierarchical representation of Chinese calligraphic artwork can be formally stated as:

$$\begin{cases} \boldsymbol{P}_{k,1} = \left\{ \boldsymbol{P}_{k-1,s} | \boldsymbol{P}_{k-1,s} \xleftrightarrow{R_k} \boldsymbol{P}_{k-1,1}, s \in F_{k-1} \right\} \\ \boldsymbol{P}_{k,l} = \Big\{ \boldsymbol{P}_{k-1,s} | \boldsymbol{P}_{k-1,s} \xleftrightarrow{R_k} \boldsymbol{P}_{k-1,q}, \text{ in which } q = \\ \qquad \min(t | \boldsymbol{P}_{k-1,t} \notin \bigcup_{i=1}^{l-1} \boldsymbol{P}_{k,i} \ t \in \boldsymbol{F}_{k-1}) \ s \in \boldsymbol{F}_{k-1} \Big\}, \quad l \in \boldsymbol{F}_k / \{1\} \quad k = 1, 2, 3, 4 \\ \boldsymbol{P}_{0,l} = \boldsymbol{P}_l, \quad l \in F_0 \end{cases}$$

Denote the number of elements in the set M as $|M|$; then the following relationship holds within the hierarchy of Chinese calligraphic artwork representation:

$$\begin{cases} \displaystyle\sum_{s=1}^{|\boldsymbol{F}_k|} |\boldsymbol{P}_{k,s}| = |\boldsymbol{F}_{k-1}| = num_{k-1}, & k = 1, 2, 3, 4 \\ |\boldsymbol{P}_{0,s}| = 1, & s = 1, \cdots, |\boldsymbol{F}_0| \end{cases}$$

The hierarchical representation describes how an artwork is composed from constructive ellipses at the lowest level. Each higher level describes how to generate its level of representation from the information at one level down. It is essentially a tree-like knowledge representation. Fig. 2 shows the hierarchical representation of the Chinese character "*zhe*"(浙).

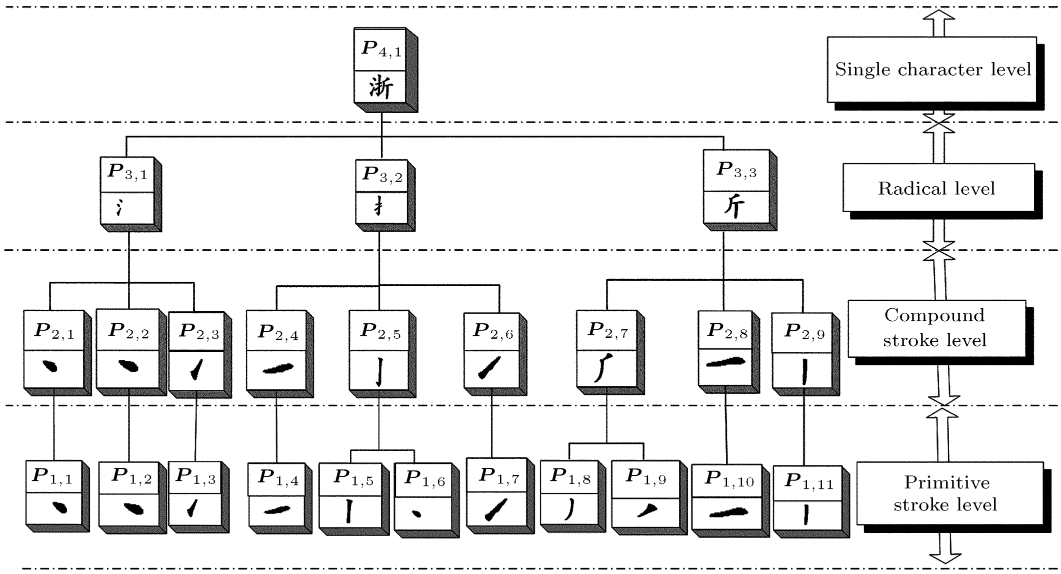

Fig. 2 Hierarchical representation of the character "*zhe*(浙)"

At the 0-th level of the hierarchical representation, the artwork is viewed as a collection of ellipses, denoted as $\boldsymbol{P}_0$. These ellipses are called the "constructive ellipses" of the artwork. For each constructive ellipse denoted as $\boldsymbol{P}_{0,i}$, let (x_i, y_i) be the center and ai and bi the lengths of its major and minor axes respectively. Then, the "image" of the artwork $\boldsymbol{C}$ will be rendered as the regions in the image space that are covered by the collection of the constructive ellipses (please refer to Section 4.1 for the detailed mathematic definition).

Lower-level elements will be promoted to the next upper level if they cannot be combined with other elements at the same level. For example, the primitive stroke $\boldsymbol{P}_{1,1}$ in Fig. 2 becomes the compound stroke $\boldsymbol{P}_{2,1}$ at the next upper level. Similarly, it is possible for a radical to degrade to a compound stroke, and then to a primitive stroke. With this promotion and degradation arrangement, a uniform six-level hierarchy can be obtained, which will be needed in the calligraphy generation phase.

4.3 Topological constructive information in the hierarchy of Chinese calligraphy representation

We denote the s-th constructive element at the k-th level of Calligraphy as $\boldsymbol{P}_{k,s}$, and its corresponding topological constructor as $\boldsymbol{T}_{k,s}$, where $k = 1, 2, 3, 4,\ s \in F_k$. $\boldsymbol{T}_{k,s}$ contains the topological relationship by which $\boldsymbol{P}_{k-1,1+l}, \boldsymbol{P}_{k-1,2+l}, \cdots, \boldsymbol{P}_{k-1,|\boldsymbol{P}_{k,s}|+l}$ $(l = \sum_{i=1}^{s-1} |\boldsymbol{P}_{k,i}|)$ compose $\boldsymbol{P}_{k,s}$.

We denote the bounding box of the image space that the element $\boldsymbol{P}_{k,s}$ occupies as $\boldsymbol{C}_{k,s} = [CX_{k,s} \quad CY_{k,s} \quad CH_{k,s} \quad CW_{k,s}]$, $k = 1, 2, 3, 4,\ s \in \boldsymbol{F}_k$, in which $CX_{k,s}, CY_{k,s}, CH_{k,s}, CW_{k,s}$ are the x, y coordinates of the lower-left corner of the bounding box $\boldsymbol{C}_{k,s}$ and its height and width, respectively. All the coordinates are in the world coordinate system. Then, we have

$$\boldsymbol{T}_{k,s} \triangleq (\boldsymbol{TCR}_{k,s}, \boldsymbol{TCS}_{k,s}) \triangleq \left(\begin{bmatrix} \boldsymbol{TR}_{k,1+l} \\ \boldsymbol{TR}_{k,2+l} \\ \vdots \\ \boldsymbol{TR}_{k,|\boldsymbol{P}_{k,s}|+l} \end{bmatrix}, [\boldsymbol{TS}_{k,1+l} \quad \boldsymbol{TS}_{k,2+l} \quad \cdots \quad \boldsymbol{TS}_{k,|\boldsymbol{P}_{k,s}|+l}] \right)$$

in which $l = \sum_{i=1}^{s-1} |\boldsymbol{P}_{k,i}|$; $s \in F_k$; $k = 1, 2, 3, 4$. $\boldsymbol{TCR}_{k,s}, \boldsymbol{TCS}_{k,s}$ are the scale and transition matrices respectively. The strict mathematical definition for the matrices $\boldsymbol{TR}_{k,z}, \boldsymbol{TS}_{k,z}$ are as follows:

$$\begin{cases} \boldsymbol{TR}_{k,z} \triangleq \begin{bmatrix} \boldsymbol{RW}_{k,z} & 0 \\ 0 & \boldsymbol{RH}_{k,z} \end{bmatrix} \triangleq \begin{cases} \begin{bmatrix} \dfrac{CW_{k,s}}{CW_{k-1,z}} & 0 \\ 0 & \dfrac{CH_{k,s}}{CH_{k-1,z}} \end{bmatrix}, \ (k=2,3,4) \\ \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \ (k=1) \\ z = 1+l, 2+l, \cdots, |\boldsymbol{P}_{k,s}|+l \end{cases} \\ \boldsymbol{TS}_{k,z} \triangleq [\boldsymbol{RX}_{k,z} \quad \boldsymbol{RY}_{k,z}]^{\mathrm{T}} \triangleq \begin{cases} \begin{bmatrix} \dfrac{CX_{k-1,z}-CX_{k,s}}{CW_{k,s}} & \dfrac{CY_{k-1,z}-CY_{k,s}}{CH_{k,s}} \end{bmatrix}^{\mathrm{T}}, \ (k=2,3,4) \\ [0 \quad 0]^{\mathrm{T}}, \ (k=1) \end{cases} \end{cases}$$

If $k \geqslant 1$, $\boldsymbol{P}_{k,i}$ must be composed of one or more constructive elements in one level down; we call the latter sub-constructive elements. All the information needed for the composition of $\boldsymbol{P}_{k,i}$ is stored in $\boldsymbol{T}_{k,i}$, the topological constructor of $\boldsymbol{P}_{k,i}$.

With the topological constructors, we can set up a one to one mapping between pixels on different levels of the hierarchical representation. That is, any pixel $[x_{k,s} \quad y_{k,s}]^{\mathrm{T}}$ that belongs to the s-th constructive element $\boldsymbol{P}_{k,s}$ on the k-th level of the hierarchical representation is uniquely mapped to the pixel $[x_{l,t} \quad y_{l,t}]^{\mathrm{T}}$ which is the t-th constructive element $\boldsymbol{P}_{l,t}$ on the l-th level of the hierarchical representation. Here $l > k$, $l, k = 0,1,2,3,4$, and $\exists m_j \in \boldsymbol{F}_{k+j}$ $(j = 1,2,\cdots,l-k-1)$ s.t. $\boldsymbol{P}_{k,s} \in \boldsymbol{P}_{k+1,m_1} \in \cdots \in \boldsymbol{P}_{k+(l-k-1),m_{l-k-1}} \in \boldsymbol{P}_{l,t}$.

If we introduce the operator $\nabla^b_{m,n} : \nabla^b_{m,n} \begin{bmatrix} x \\ y \end{bmatrix}_{2\times 1} \triangleq \boldsymbol{TR}_{m+1,n} \begin{bmatrix} x \\ y \end{bmatrix}_{2\times 1} + \boldsymbol{TS}_{m+1,n}$, then there exists the relationship $[x_{l,t} \quad y_{l,t}]^{\mathrm{T}} = \nabla^b_{l-1,m_{l-k-1}}(\cdots(\nabla^b_{k+1,m_1}(\nabla^b_{k,s}[x_{k,s} \quad y_{k,s}]^{\mathrm{T}})))$.

4.4 Parametric representation of calligraphic artwork

Denote the s-th constructive element in the k-th level of the hierarchy as $\boldsymbol{P}_{k,s}$, whose parametric representation is $E_{k,s}$, $k = 0,1,\cdots,4$, $s \in \boldsymbol{F}_k$. We now introduce a matrix operator ∇^c_n, which can produce an $m \times \sum_{l=1}^{n} d_l$ dimensional matrix $M = (a_{i,j})_m \times \sum_{l=1}^{n} \mathrm{d}l$ according to n $m \times d_l$ $(l = 1,2,\cdots,n)$ dimensional matrices $M_l = (a_{l,i,j})_{m\times d_l}$ $(l = 1,2,\cdots,n)$. That is, if $\nabla^c_n(M_1, M_2, \cdots, M_n) = M$, then we have

$$a_{i,j} = \begin{cases} a_{z+1,i,j} - \sum_{t=1}^{z} d_t \text{ in which } \sum_{t=1}^{z} d_t < j \leqslant \sum_{t=1}^{z+1} d_t, \ z = 1,2,\cdots,n-1 \\ a_{1,i,j}, \ j \leqslant d_1 \end{cases}, \ i = 1,2,\cdots,m$$

We further define a matrix operator ∇^d_n: $\nabla^d_n(A) \triangleq \nabla^c_n \underbrace{(A, A, \cdots, A)}_{n}$.

Then the hierarchy of Chinese calligraphic artwork can be formally expressed as:

$$\begin{cases} \boldsymbol{E}_{0,i} = [x'_i \quad y'_i \quad a'_i \quad b'_i]^{\mathrm{T}}, \ i \in F_0 \\ \boldsymbol{E}_{k,s} = \nabla^c_{|\boldsymbol{P}_{k,s}|}(\nabla^e_{k-1,1+l} E_{k-1,1+l}, \nabla^e_{k-1,2+l} E_{k-1,2+l}, \cdots, \nabla^e_{k-1,|\boldsymbol{P}_{k,s}|+l} E_{k-1,|\boldsymbol{P}_{k,s}|+l}) \end{cases}$$

in which $l = \sum_{i=1}^{s-1} |\boldsymbol{P}_{k,i}|, k = 1,2,3,4, \ s \in F_k$.

Here, the operator $\nabla^e_{n,m}$ is defined as:

$$\nabla^e_{n,m} E_{n,m} \triangleq \nabla^c_2((\nabla^c_2(\boldsymbol{TR}_{n,m}, \mathbf{0}_{2\times 2}))^{\mathrm{T}}, (\nabla^c_2(\mathbf{0}_{2\times 2}, \boldsymbol{TR}_{n,m}))^{\mathrm{T}}) E_{n,m} + \nabla^d_{col(E_{n,m})}((\nabla^c_2((\boldsymbol{TS}_{n,m})^{\mathrm{T}}, [0 \quad 0]))^{\mathrm{T}})$$

where $col(E_{n,m})$ is the number of columns of the matrix $E_{n,m}$, and $\mathbf{0}_{2\times 2}$ is a 2×2 dimensional zero matrix.

In the above equation, each $E_{0,i}$ represents the area covered by a constructive ellipse, $\frac{(x-x_i')^2}{a_i'^2}+\frac{(y-y_i')^2}{b_i'^2}\leqslant 1$. This ellipse is the normalized version of the original constructive ellipse, $\frac{(x-x_i)^2}{a_i^2}+\frac{(y-y_i)^2}{b_i^2}\leqslant 1$, against its bounding box. That is, assuming $\boldsymbol{P}_{0,i}\in\boldsymbol{P}_{1,j}$, $i\in\boldsymbol{F}_0, j\in\boldsymbol{F}_1$, we have:

$$\begin{bmatrix} x_i' \\ y_i' \\ a_i' \\ b_i' \end{bmatrix}=\begin{bmatrix} \dfrac{x_i-CX_{1,j}}{CW_{1,j}} \\ \dfrac{y_i-CY_{1,j}}{CH_{1,j}} \\ \dfrac{a_i}{CW_{1,j}} \\ \dfrac{b_i}{CH_{1,j}} \end{bmatrix}$$

5 Synthesis reasoning and intelligent calligraphy generation

In this subsection, we reveal more details on how we construct our system based on the general principle of calligraphic model generation as discussed above.

An element at a level higher than the zero-th level, say $\boldsymbol{P}_{k+1,1}$ for all $k>0$, is composed of $N_{k+1,l}$ elements at the next lower level, from $\boldsymbol{P}_{k,l_1},\boldsymbol{P}_{k,l_2},\cdots,\boldsymbol{P}_{k,l_{N_{k+1,1}}}$. The corresponding shape matrix $E_{k+1,1}$ is derived by concatenating the matrices $E_{k,l_1},E_{k,l_2},\cdots,E_{k,l_{N_{k+1,1}}}$ column by column in sequence. Since the parametric representation of a constructive ellipse is a 4×1 dimensional matrix, the proposed concatenation at the higher levels will produce matrices having exactly four rows. Each row of the matrix formed is called a field of the element′s parametric representation. Different fields of an element can be separately reasoned on.

The weight associated with the s-th field of the i-th source knowledge on the k-th level, *i.e.*, the i-th training example ($\boldsymbol{P}_{k,l}^i$), is denoted as $\boldsymbol{\omega}_{l,s}^i$. We call $\boldsymbol{\omega}_{l,s}^i(s=1,\cdots,4)$ the reasoning intensity of $\boldsymbol{P}_{k,l}^i$ in the synthesis reasoning. Then the ordered set of analogous reasoning intensities, $\boldsymbol{\omega}_{l,s}^i(s=1,\cdots,4)$ forms the "viewpoint sequence" of the current synthesis reasoning process: $\bar{\boldsymbol{\omega}}=\{\omega_{l,s}^i|i=1,\cdots,n;s=1,\cdots,4\}$. It is so called as different orders of presenting the training examples will yield different calligraphy results.

In the synthesis reasoning process for novel calligraphic artwork generation, we assume there are n components of k-th level synthesis source knowledge $\boldsymbol{P}_{k,l_1},\boldsymbol{P}_{k,l_2},\cdots,\boldsymbol{P}_{k,l_n}$ taking part in the reasoning process. And each synthesis source knowledge consists of m components, where $\forall i\neq j; i,j=1,2,\cdots,n\Rightarrow l_i,l_j\in\boldsymbol{F}_k$ and $l_i\neq l_j$. We can therefore use an $m\times 1$ dimensional partitioned matrix to represent a single synthesis reasoning source, and use $m\times\sum_{i=1}^{n}l_i$ dimensional partitioned matrices to represent all the activated sources in the current synthesis reasoning. If we apply synthesis reasoning at the level of single characters, then $m\equiv 4$. Denote the t-th property of the source knowledge as $\boldsymbol{P}_{k,l_i,t}$. It associates an intensity of $\omega_{k,l_i,t}$ in the synthesis reasoning process, and there exists the relationship that $\sum_{i=1}^{n}\omega_{k,l_i,t}=1(t=1,2,\cdots,m)$. In our intelligent calligraphy generation system, this reasoning intensity can be adjusted through a graphical user interface. Denote the new knowledge generated from the synthesis reasoning process as $\boldsymbol{P}_{k,r}$, and its matrix form parametric representation as $E_{k,r}$. Applying the principle of synthesis reasoning process requires that all the source knowledge in the reasoning should possess equal dimensions. That is, if we apply synthesis reasoning onto the source knowledge $\boldsymbol{P}_{k,s}$ and $\boldsymbol{P}_{k,t}$, their respective matrix form parametric representation $(E_{k,s})_{m_1\times n_1}$ and $(E_{k,t})_{m_2\times n_2}$ must satisfy the relationship of $n_1=n_2$. The equalization requirement is a soft constraint of the synthesis reasoning process (please refer to the discussion in Section 7.1). If it is violated, we can introduce the source knowledge equalization operator ∇_t^e to relax this soft constraint. This is similar to the mapping issue in analogous reasoning. In the following, we will introduce several symbols to define ∇_t^e in a strict mathematical way.

We first denote $\boldsymbol{e}_{n,i}\triangleq[\sigma_{(i,1)}\quad\sigma_{(i,2)}\quad\cdots\quad\sigma_{(i,n)}]^{\mathrm{T}}$, in which $\sigma_{(i,j)}=\begin{cases}1, & i=j\\ 0, & i\neq j\end{cases}$.

We can also derive a discrete curve composed of the centers of all the covering ellipses for each constructive element used in our intelligent calligraphy generation system, which is expressed by $\boldsymbol{P}_{k,l_s}$ and parametrically represented by E_{k,l_s}. Thus a discrete curve derivation method can be stated as:

$$C_{k,l_s} = (\nabla_2^c((\boldsymbol{e}_{4,1}^{\mathrm{T}} \times E_{k,l_s})^{\mathrm{T}}, (\boldsymbol{e}_{4,2}^{\mathrm{T}} \times E_{k,l_s})^{\mathrm{T}}))^{\mathrm{T}} = \begin{bmatrix} x_1 & x_2 & \cdots & x_{co} \\ y_1 & y_2 & \cdots & y_{co} \end{bmatrix}, co = col(E_{k,l_s})$$

And then, the concept of key columns of a matrix can be introduced and defined. If the curve C_{k,l_s} has $v+1$ key points, whose coordinates are $C_{k,l_s}\boldsymbol{e}_{co,u_0}, C_{k,l_s}\boldsymbol{e}_{co,u_1}, \cdots, C_{k,l_s}\boldsymbol{e}_{co,u_v}$, then the key columns of E_{k,l_s} can be identified as $E_{k,l_s}\boldsymbol{e}_{co,u_0}, E_{k,l_s}\boldsymbol{e}_{co,u_1}, \cdots, E_{k,l_s}\boldsymbol{e}_{co,u_v}$. As for the extraction of key points for a given planar curve C_{k,l_s}, we adopt the algorithm introduced in [15]. More delicate key point extraction algorithms are available in [16~18], but they might incur heavier computational costs.

Assume that we have identified $v+1$ key columns from the matrix form parametric representation E_{k,l_s} of a synthesis reasoning source $\boldsymbol{P}_{k,l_s}$, which are the u_0-th, u_1-th, $\cdots$, u_v-th columns of the matrix. Here $1 = u_0 < u_1 < \cdots < u_v = col(E_{k,l_s})$, $s = 1, 2, \cdots, n$. We can then define the volume equalization operator for synthesis reasoning source ∇_t^e as:

$$(\nabla_t^e(E_{k,l_s}))\boldsymbol{e}_{t,i} \triangleq E_{k,l_s}\boldsymbol{e}_{col(E_{k,l_s}),\,\theta}, \quad i = 1, 2, \cdots, t$$

in which $\theta = \left[u_j + \dfrac{u_{j+1} - u_j}{\left[\dfrac{t \times (j+1)}{v}\right] - \left[\dfrac{t \times j}{v}\right]}\left(i - \left[\dfrac{t \times j}{v}\right]\right)\right]$; $\left[\dfrac{t \times j}{v}\right] < i \leqslant \left[\dfrac{t \times (j+1)}{v}\right]$; $j \in \{0, 1, \cdots, v-1\}$; $s = 1, 2, \cdots, n$ and [ ] is an integer truncation operator. Specifically, if each column of the matrix E_{k,l_s} is considered a key column, we can use a much simpler form of mapping to define the operator ∇_t^e as: $(\nabla_t^e(E_{k,l_s}))\boldsymbol{e}_{t,i} \triangleq E_{k,l_s}\boldsymbol{e}_{col(E_{k,l_s}),\left[\frac{i \times col(E_{k,l_s})}{t}\right]}$, in which $i = 1, 2, \cdots, t$ and [ ] is an integer truncation function.

Based on the definition of ∇_t^e, we can further define a synthesis reasoning source knowledge superimposing operator as ∇_n^f:

$$\nabla_n^f(M_1, M_2, \cdots, M_n) \triangleq \nabla_n^c(\nabla_h^e(M_1), \nabla_h^e(M_2), \cdots, \nabla_h^e(M_n)), \quad h = \max\{col(M_i)|i = 1, 2, \cdots, n\}$$

With the operator ∇_n^f, we can derive the matrix form representation of the superimposed synthesis reasoning sources as: $E_k \triangleq \nabla_n^f(E_{k,l_1}, E_{k,l_2}, \cdots, E_{k,l_n})$. With this operator, we can further calculate the feature matrix of all the synthesis reasoning source knowledge as:

$$S_k \triangleq E_k - \nabla_n^c(\nabla_h^e E_{k,nor}), \quad h = \max\{col(E_{k,l_i})|i = 1, 2, \cdots, n\}$$

In the above equation, $E_{k,nor}$ is the matrix form representation of the standard source knowledge $\boldsymbol{P}_{k,nor}$ at the k-th level of the knowledge representation. In our system, we assume that the shape of a constructive element written in the font style "Kai" (GB2312) as used in recent versions of Microsoft Word is the standard shape of an element $\boldsymbol{P}_{k,nor}$.

According to the user input intensities for different synthesis reasoning sources $\omega_{k,l_i,t}$, we can then compute the synthesis reasoning viewpoint matrix:

$$W_{k,t} \triangleq (\nabla_n^c(\omega_{k,l_1,t} \times I_{h\times h}, \omega_{k,l_2,t} \times I_{h\times h}, \cdots, \omega_{k,l_n,t} \times I_{h\times h}))^{\mathrm{T}}$$

in which $h = \max\{col(E_{k,l_i})|i = 1, 2, \cdots, n\}$ and $I_{h\times h}$ is an $h \times h$ dimensional unit matrix.

We can now generate the synthesis reasoning feature result $S_{k,r}$ through the synthesis reasoning process as described by the following form:

$$S_{k,r} = (\nabla_m^c((S_k \otimes W_{k,1})^{\mathrm{T}}\boldsymbol{e}_{m,1}, (S_k \otimes W_{k,2})^{\mathrm{T}}\boldsymbol{e}_{m,2}, \cdots, (S_k \otimes W_{k,m})^{\mathrm{T}}\boldsymbol{e}_{m,m}))^{\mathrm{T}}$$

In the above equation, $\otimes$ is a synthesis reasoning simulation operator. If the synthesis reasoning process models human beings′ creative thinking as a linear process, then $E_k \otimes W_k \triangleq E_k \bullet W_k$, where $\bullet$ is an ordinary matrix multiplication operator. If the synthesis reasoning process models human beings′ creative thinking as a z-order polygonal, then $\otimes$ is defined as:

$$A_{p\times q} \otimes B_{q\times r} = C_{p\times r} \Rightarrow c_{i,j} = \sqrt[z]{\sum_{k=1}^{q}(a_{i,k}\ b_{k,j})^z}, \quad i = 1, 2, \cdots, p;\ j = 1, 2, \cdots, r$$

We can also model creative thinking as a geometrical averaging process, as indicated by the following equation:

$$c_{i,j} = \sqrt[q]{\prod_{k=1}^{q} (a_{i,k}\ b_{k,j})}, \quad i = 1, 2, \cdots, p;\ j = 1, 2, \cdots, r$$

Generally speaking, we can overload the synthesis reasoning simulator $\otimes$ to realize the simulation over different kinds of creative thinking using the synthesis reasoning model.

Finally, by adding back the shape of $E_{k,nor}$, the standard constructive element associated with the reasoning result $\boldsymbol{P}_{k,nor}$ in the synthesis reasoning process, we obtain the parametric representation $E_{k,r}$ of $\boldsymbol{P}_{k,r}$: $E_{k,r} = \boldsymbol{S}_{k,r} \oplus \nabla_h^e E_{k,nor}$, where $E_{k,nor}$ is the matrix-form parametric representation of the shape of $\boldsymbol{P}_{k,nor}$.

If all the reasoning intensities in the current synthesis reasoning are within the range of [0,1], that is, $0 \leqslant \omega_{k,l_i,t} \leqslant 1$ ($t = 1, 2, \cdots, m;\ i = 1, 2, \cdots, n$), the synthesis reasoning process is essentially an interpolation process. Otherwise, it is an extrapolation process. From a psychological point of view, if $\exists \omega_{k,p,q} < 0$, the current synthesis reasoning process employs backward thinking of the brain. If $\exists \omega_{k,p,q} > 1$, the current synthesis reasoning process employs forward exaggerative thinking of the brain. Meanwhile, if $n \geqslant 3$, then the current synthesis reasoning based creative thinking simulation mimics combined thinking activity.

Note that the synthesis reasoning process can be applied not only to the matrix representations of all the reasoning sources by evaluating a series of matrix operations simulating the reasoning, but also to the topological constructors of all the reasoning sources. Let $\boldsymbol{T}_{k,l_1}, \boldsymbol{T}_{k,l_2}, \cdots, \boldsymbol{T}_{k,l_n}$ correspond to the field intensities $\omega_{k,l_1}, \omega_{k,l_2}, \cdots, \omega_{k,l_n}$, respectively, where $\sum_{i=1}^{n} \omega_{k,l_i} = 1$. Then the synthesis reasoning result is: $\boldsymbol{T}_{k,r} = \bigoplus_{i=1}^{n} (\boldsymbol{T}_{k,l_i} \times \omega_{k,l_i})$, in which $\oplus$ is a synthesis reasoning simulation operator. Similarly, we can overload the definition of operator $\oplus$ to realize different innovative thinking activities simulated by our system. Some simple synthesis reasoning simulation operators for topological constructors are: arithmetic mean, $\boldsymbol{T}_{k,r} = \frac{1}{n}\sum_{i=1}^{n} (\boldsymbol{T}_{k,l_i} \times \omega_{k,l_i})$; geometric mean, $\boldsymbol{T}_{k,r} = \sqrt[n]{\prod_{i=1}^{n} (\boldsymbol{T}_{k,l_i} \times \omega_{k,l_i})}$; harmonic mean, $\frac{1}{\boldsymbol{T}_{k,r}} = \frac{1}{n}\sum_{i=1}^{n} \left(\frac{1}{\boldsymbol{T}_{k,l_i} \times \omega_{k,l_i}}\right)$, *etc.*

The suggested synthesis reasoning process is essentially either an interpolation or an extrapolation process. That is, ω^i is the interpolation or extrapolation weight for $\boldsymbol{P}_{k,l}^i$ with the constraint that $\sum_{i=1}^{n} \omega^i = 1$. In our intelligent calligraphy generation system, all the analogous reasoning intensities can be manually adjusted by the user through a graphical interface.

6 Experiment result

Given limited space, we can only show a few samples of the results here. Fig. 3(b) shows the results from the synthesis reasoning model being applied to a single character using six training examples, Fig. 3(a), as the reasoning source. The results demonstrate that our approach can yield different new styles after having digested some existing styles.

Fig. 3 (a) Six input sources for synthesis reasoning at the single character level

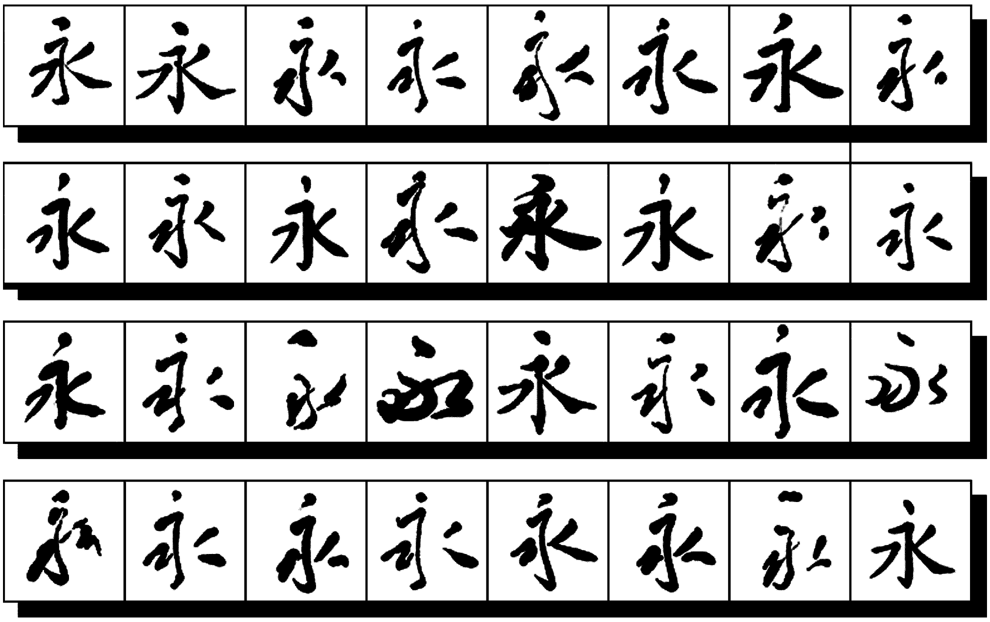

Fig. 3 (b) The results generated by the synthesis reasoning process at the single character level

Similarly, for couples of Chinese calligraphic artwork, we can also apply synthesis reasoning process to achieve novel calligraphy. Fig. 4(a) gives the reasoning source and Fig. 4(b) lists some selected reasoning results. Note the consistency in style among characters within the same newly generated calligraphic piece. The experiment presented by Fig. 4 shows that not only can synthesis reasoning process generate calligraphic artworks in different new styles, but also can it generate results with consistently styled characters within the same new style when necessary. In our case, the characters in one Chinese calligraphic couple are all in one style.

To verify that the system was indeed able to generate quality outputs, we invited a group of judges consisting of six amateur calligraphists with at least more than two years′ writing experience and four professional calligraphists including a professor in calligraphy in an art school. They cast votes on the calligraphic artwork generated by the system. If an artwork received more than seven votes, it was considered a new acceptable calligraphic artwork. The result is that they considered most of our generated calligraphy to be acceptable.

Fig. 4 (a) The learned samples

Fig. 4 (b) Selected computer generated results of Chinese calligraphic couples

7 Discussions

After designing and developing the intelligent Chinese calligraphy generation system, we find there are constraint satisfaction requirements arising from the synthesis reasoning model. We also generalize the experiences gained from the system development to propose a generic methodology for designing synthesis reasoning based intelligent CAD systems for solving real world problems.

7.1 Constraints of synthesis reasoning

The rigid constraint of synthesis reasoning is that all the activated reasoning sources in one synthesis reasoning process must be homogeneous, *i.e.*, their matrix form knowledge representations must have the same number of rows. This means that each of the reasoning source knowledge components has the same number of properties to be reasoned about. Such a constraint must be satisfied during any synthesis reasoning process.

There are also several soft constraints in synthesis reasoning process. 1) All the source knowledge components taking part in the synthesis reasoning process must be of the same syntax. That is, if we are reasoning about two primitive strokes $\boldsymbol{P}_{1,s}$ and $\boldsymbol{P}_{1,t}$, we should make sure that they are both horizontal strokes (or vertical strokes, or left slating strokes, *etc.*). 2) All the source knowledge components must have the same capacity. That is, if we are reasoning about $\boldsymbol{P}_{k,s}$ and $\boldsymbol{P}_{k,t}$, it should be guaranteed that their respective matrix representations $(E_{k,s})_{m_1\times n_1}$ and $(E_{k,t})_{m_2\times n_2}$ satisfy the relationship $n_1 = n_2$. This can be interpreted as that the precondition of composing a synthesis reasoning space is that all the reasoning sources must have the same number of components. 3) All the source knowledge components for a synthesis reasoning process must be at the same level. That is, if we are reasoning about $\boldsymbol{P}_{m,s}$ and $\boldsymbol{P}_{n,t}$, $m = n$ must be true. Otherwise, the semantics would be different for different contributing reasoning sources.

To solve real world problems, we can relax the soft constraints of synthesis reasoning to make the reasoning process more widely applicable. For source knowledge components that are of different dimensions, we can define and then apply a source knowledge equalization operator ∇_t^e to convert them to be of the same dimension. For source knowledge components that are not homogeneous or at different levels in the hierarchy, we can also carry out synthesis reasoning by ignoring all the soft constraints. This corresponds to the simulation of the thinking process of a calligraphist while creating an artwork of running style, a style which is the freest of all styles. With all these relaxations on the synthesis reasoning process, the number of novel calligraphic artwork thus generated is vastly increased. But note that at the same time, the ratio of generating aesthetically attractive novel calligraphy decreases. It is obvious that too strict a set of constraints can limit the system′s overall creativity, while too loose a set of constraints could harm the overall acceptability of the results.

The constraints of the synthesis reasoning process can be relaxed in order to allow for results with new styles that cannot be easily imagined. From a computational psychology′s perspective, relaxing or ignoring the constraints in our analogous reasoning process corresponds to the creative brain activity of a calligraphist performing cursive and running style writing.

7.2 Generic methodology to develop synthesis reasoning based intelligent systems

The synthesis reasoning model we have proposed is capable of solving a class of shape reasoning problems. We have successfully applied the model to resolve many real world design problems. The automatic Chinese calligraphy generation system introduced in previous sections is one of these systems. Other successful problem solving systems developed based on synthesis reasoning model include an intelligent advertisement design system[5], an intelligent decoration design system[6], and an intelligent chair design system[8]. Although each of these systems has its own problem solving semantics and background knowledge, the synthesis reasoning approaches they employed are very similar. Based on the practices and experiences we accumulated through designing and developing those systems, we propose a generic methodology for developing a synthesis reasoning based intelligent system for solving any particular type of design problem in real world, as follows.

1) According to the specific application domain, choose the synthesis reasoning sources and its properties. By superimposing all the synthesis reasoning sources together, we can construct the synthesis reasoning space.

2) Introduce a hierarchy of knowledge representation by picking a suitable granularity of knowledge for the different levels in the hierarchy so that the reusability of the reasoning sources and the reasoning results, as well as the reasoning efficiency, can all be improved.

3) Derive the operators to convert between knowledge representations at different levels in the hierarchy.

4) Based on the work done in steps (1)$\sim$(3), a full parametric and hierarchical knowledge representation of the complete synthesis reasoning space can be derived.

5) According to the semantics of the specific application problem to be solved, construct operators to instantiate the synthesis reasoning model as a concrete computable algorithmic framework, based on which the overall system architecture can be deduced. These operators include the synthesis reasoning source superimposing operator, the reasoning source equalization operator, the synthesis reasoning evaluation operator, *etc.*

8 Conclusions

Our thesis has been that with the formal method of reasoning based on knowledge synthesis, one can design intelligent systems having machine creativity that can solve real-world shape design problems. A Chinese calligraphy generation system is presented as a case study to show the system construction procedures. Our paper also proposes a generic methodology to design and develop synthesis reasoning based intelligent CAD systems for specific applications in different domains.

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