## Disclosure Risks of Distance Preserving Data Transformations

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## Outline

## Motivation

The Attack

Conclusion

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## The Attack

Conclusion

## Data Analysis and Sharing

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- Outsourcing


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- Outsourcing - can the statistician be trusted?


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- Outsourcing - can the statistician be trusted?
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## Data Analysis and Sharing



- Outsourcing - can the statistician be trusted?
- Sharing - can they trust each other?


## Data Transformations

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Data Transformations - a way to get rid of trust.


Liu, Giannella, Kargupta: Attack on perturbed data.
Mutual distances:
Fact Are useful in many analytical techniques.
Claim Do not leak private information. Wrong!

## Attack Scenarios

Things an attacker might know:

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Data sample

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- Qualified guess


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Database $n$ objects with $d$ attributes

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Database $n$ objects with $d$ attributes
Published Distances between objects
Attacker Knows probability distribution
The attack:

1. Guess $d+1$ objects.
2. Use lateration to fix remaining objects.
3. Rotate and mirror to fit known distribution.

## Hyper-lateration

Known points $\bar{p}_{1}, \ldots, \bar{p}_{n} \in \mathbb{R}^{d}$

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Known points $\bar{p}_{1}, \ldots, \bar{p}_{n} \in \mathbb{R}^{d}$
Unknown point $\bar{x}$ at distance $\left\|\bar{x}-\bar{p}_{i}\right\|=\delta_{i}$

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Known points $\bar{p}_{1}, \ldots, \bar{p}_{n} \in \mathbb{R}^{d}$
Unknown point $\bar{x}$ at distance $\left\|\bar{x}-\bar{p}_{i}\right\|=\delta_{i}$
$n$ quadratic equations:

$$
\delta_{i}^{2}=\sum_{j=1}^{d}\left(x_{j}-p_{i j}\right)^{2}=\sum_{j=1}^{d} x_{j}^{2}-2 x_{j} p_{i j}+p_{i j}^{2}
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$n-1$ linear equations:

$$
\delta_{i}^{2}-\delta_{0}^{2}=\sum_{j=1}^{d} 2 x_{j}\left(p_{0 j}-p_{i j}\right)+p_{i j}^{2}-p_{0 j}^{2}
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If $n>d$ and $\operatorname{span}\left\{\bar{p}_{i}\right\}_{i}=\mathbb{R}^{d}$, solution is unique.

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Hyper-Iateration Unique up to orthogonal transform.

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Covariance matrix:

$$
\Sigma=\left[\begin{array}{ccc}
\operatorname{Cov}\left(A_{1}, A_{1}\right) & \cdots & \operatorname{Cov}\left(A_{1}, A_{d}\right) \\
\vdots & & \vdots \\
\operatorname{Cov}\left(A_{d}, A_{1}\right) & \cdots & \operatorname{Cov}\left(A_{d}, A_{d}\right)
\end{array}\right]
$$

$\operatorname{Cov}(A, B)=E[(A-\mu)(B-\nu)]$.


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$\operatorname{Cov}(A, B)=E[(A-\mu)(B-\nu)]$.
Eigenvectors, and values:
$\left(\bar{e}_{1}, \lambda_{1}\right), \ldots,\left(\bar{e}_{d}, \lambda_{d}\right)$.


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$\operatorname{Cov}(A, B)=E[(A-\mu)(B-\nu)]$.
Eigenvectors, and values:
$\left(\bar{e}_{1}, \lambda_{1}\right), \ldots,\left(\bar{e}_{d}, \lambda_{d}\right)$.
Do this for both hyper-laterated points and sample drawn from known distribution.

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4. Rotate to match principal components of known probability distribution

## The Attack

1. Guess first $d$ objects (unique up to rotation and mirroring)
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3. Find principal components
4. Rotate to match principal components of known probability distribution
5. Find best mirroring (optimized)

## Attack Accuracy (1)



Auto Miles per Gallon (using 5 attributes)

## Attack Accuracy (2)



US Adult Census (using 4 attributes)

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## Known <br> Leaked

Sample of $d+1$ objects Everything
Probability distribution Everything with high fidelity

## Conclusion

## Known Leaked <br> Sample of $d+1$ objects Everything <br> Probability distribution Everything with high fidelity

Never publish distances between data points!

## Conclusion

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## Thank You

