Congestion Game with Agent and Resource Failures

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Abstract—Motivated by practical scenarios, we study congestion games with failures. We investigate two models. The first one is congestion games with both resource and agent failures (CG-RAF), where each agent chooses the same number of resources with the minimum expected cost. We prove that the game is potential and hence admits at least one pure-strategy Nash equilibrium (pure-NE). We also show that the Price of Anarchy and the Price of Stability are bounded (equal to 1 in some cases). The second model is congestion games with only resource failures (CG-CRF), where resources are provided in packages, and their failures can be correlated with each other. Each agent can choose multiple packages for reliability’s sake and utilize the survived one having the minimum cost. CG-CRF is shown to be not potential. We prove that it admits at least one pure-NE by constructing one efficiently. Finally, we discuss various applications of these two games in the networking field. To the best of our knowledge, this is the first work studying congestion games with the coexistence of resource and agent failures, and we give also the first proof of the existence of a pure-NE in congestion games with correlated package failures.

Index Terms—Congestion Game, Agent Failure, Resource Failure, Nash Equilibrium, Price of Anarchy.

I. INTRODUCTION

C ongestion games [1] are noncooperative games in which a collection of agents compete for a finite set of resources. Many practical problems, such as network routing [2], [3] and wireless spectrum sharing [4]–[6] can be modeled as congestion games. Congestion games have been proved to possess a Nash equilibrium in pure strategies, which is referred to as pure-strategy NE or pure-NE in short.

The vast majority of the existing studies on congestion games implicitly assume that there is no failure in the resources and agents (e.g., [2], [7]–[12]). However, in real applications, resources could fail to fulfill their assigned tasks [13], and agents, who are players in the game, may also fail to execute their chosen strategies [14]. Take for example two promising technologies, cognitive radio networks (CRNs) [15] and cloud computing [16]. In CRNs, each secondary user (SU) will choose one available channel that is not being used by primary users (PUs) in the network for communication. The availability of this channel may vary during communication due to the come-and-go of PUs. For an SU, the unavailability of a channel can be interpreted as resource failures. On the other hand, the SUs themselves (e.g., sensor nodes) could run out of power and quit the network, which can be regarded as agent failures. In cloud computing, remote resources are subject to a high degree of sharing. The tailor-made packages of CPU, RAM, and disk resources (usually in the form of virtual machines (VMs)) may become unavailable to some users because of jamming or outages (regarded as resource failures).

Taking failures into account brings great challenges to the study of congestion games. We list the state-of-the-art research outputs in Table I.

- Some existing works focused on resource failures [17]–[19], [27]. Penn et al. [17] were the first to incorporate the issue of failures in congestion games. They introduced a class of non-cooperative games and congestion games with failures (CGFs), where a selfish agent would choose multiple resources to execute its tasks for the sake of reliability, and resource failures were assumed to be independent from each other. In a CGF, the cost for an agent was the minimum of the costs of its successful attempts, and each agent aimed to minimize its expected cost. Penn, Polukarov, and Tennenholtz [18] considered a similar game model. Unlike the CGF model, they considered the total cost of the resources being utilized, where the aim of each agent was to maximize the difference between its benefit arising from a successful task completion and the sum of its costs over the resources it uses. Billand, Bravard, and Sarangi [19] introduced both independent link (resource) failures and player heterogeneity in a two-way flow version of the connections model, which was a non-cooperative model of network formation [28].

- Some studies focused on agent failures [20]–[25]. Porter et al. [20] extended the game theoretic framework of mechanism design to allow for uncertainty about agents’ execution of their assigned tasks. They considered a non-cooperative model and correlated failures, and took into...
We observe that there is a lack of studies considering both resource failures and agent failures together in congestion games, although these cases undoubtedly appear frequently in real applications (see Section VI). Moreover, even for the case in which only resources would fail, there is also a lack of work on congestion games where the failures are correlated.

In this paper, we first study congestion games with both resource and agent failures (CG-RAF), where each agent will choose exactly one resource to execute its tasks in order to minimize the expected cost or to maximize the expected payoff. Note that the cost minimization and the payoff maximization scenarios can be different, and we make no assumption that these failures are independent. After that we study congestion games with correlated resources failures (CG-CRF). In such a game, an agent may choose a set of subsets of the basic resources, and prove it is still a non-cooperative game as well, but the cost function for each agent significantly differs from the ones in CG-RAF, and there are only the correlated resource failures in this game. We investigate the properties of the above two types of congestion games, and our contributions can be summarized as follows.

- This is the first work that investigates congestion games with both resource and agent failures.
- We prove that CG-RAF always admits at least one pure-NE as it is potential. We show the uniqueness of the structure of the equilibria and provide tight bounds on the price of anarchy (PoA) and the price of stability (PoS) in symmetric games. Moreover, we extend CG-RAF to a more general case, called $l$-CG-RAF, where each agent can choose exactly $l \geq 1$ resources, and prove it is still potential.
- For CG-CRF, although it is not potential, we prove that it always admits at least one pure-NE. Two policies are also derived to guarantee a pure-NE, based on which we give an efficient algorithm that can achieve a pure-NE.
- Our study sheds light on how to design stable and efficient systems. We show by our analysis and numerical results that we can obtain a more stable and efficient system by controlling certain system parameters such as the number of agents competing for the public resources.

### Organization:
The rest of this paper is organized as follows. First, we give basic definitions and go over some preliminaries for our problems in Section II. Next, in Section III, we define CG-RAF, and prove that it always possesses a pure-NE. We also present the uniqueness of the properties of the best/worst NE, the PoA and the PoS. For CG-CRF, we prove the existence of pure-NE by constructing one in Section IV. We

### Table I: Comparison of Related Models.

<table>
<thead>
<tr>
<th>Models</th>
<th>Cooperative or Non-cooperative</th>
<th>Agent Failure</th>
<th>Resource Failure</th>
<th>Correlated or Uncorrelated Failure</th>
</tr>
</thead>
<tbody>
<tr>
<td>[17]–[19]</td>
<td>Non-cooperative</td>
<td>✓</td>
<td>✓</td>
<td>Uncorrelated</td>
</tr>
<tr>
<td>[20], [21]</td>
<td>Non-cooperative</td>
<td>✓</td>
<td>✓</td>
<td>Correlated</td>
</tr>
<tr>
<td>[22]</td>
<td>Non-cooperative</td>
<td>✓</td>
<td>✓</td>
<td>Uncorrelated</td>
</tr>
<tr>
<td>[23]–[25]</td>
<td>Cooperative</td>
<td>✓</td>
<td>✓</td>
<td>Uncorrelated</td>
</tr>
<tr>
<td>[26] *</td>
<td>Non-cooperative</td>
<td>–</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>CG-RAF [this work]</td>
<td>Non-cooperative</td>
<td>✓</td>
<td>✓</td>
<td>Correlated</td>
</tr>
<tr>
<td>CG-CRF [this work]</td>
<td>Non-cooperative</td>
<td>✓</td>
<td>✓</td>
<td>Correlated</td>
</tr>
</tbody>
</table>

*This is a different model, where a noise of the cost was introduced to capture the uncertainty (failure).
suggested some applications of congestion games in networking with resource and/or agent failures in Section VI. We present discussions in Section VII. Finally, we conclude the paper and suggest some future works in Section VIII.

II. PRELIMINARIES

A congestion game is defined by a set of $n$ agents $\mathcal{N}$, and a set of resources $V$. Each agent has some tasks that can be carried out by one or more resources. Each resource is coupled with a non-decreasing cost function or a non-increasing payoff function. Define $\mathcal{N}_v$ as the set of agents choosing resource $v$. The cost/payoff function can be resource-specific denoted as $c_{v,i}(\mathcal{N}_v) : 2^\mathcal{N} \rightarrow \mathbb{R}_+$, $\forall v \in V$, or agent-specific denoted as $c_i(\mathcal{N}_i) : 2^\mathcal{N} \rightarrow \mathbb{R}_+$, $\forall i \in \mathcal{N}, v \in V$, or the generalized resource-agent-specific denoted as $c_{v,i}(\mathcal{N}_i) : 2^\mathcal{N} \rightarrow \mathbb{R}_+$, $\forall i \in \mathcal{N}, v \in V$. In most scenarios (e.g., [1], [8]–[10], [17], [21]), the cost/payoff is congestion-sensitive rather than agent-sensitive.

That is to say, the cost/payoff of a resource will be affected only by the number of agents sharing it but oblivious to the agents’ identities. Therefore, the parameter in the cost/payoff function can be $|\mathcal{N}_v|$, denoted as $k$, instead of $\mathcal{N}_v$.

The pure strategy of one agent $i \in \mathcal{N}$, denoted as $\sigma_i \in \Sigma_i$, is defined by resources it chooses, where $\Sigma_i$ is the pure strategy set of $i$. Thus, $\sigma = (\sigma_1, \ldots, \sigma_n) \in \Sigma$ is a pure strategy profile over all agents and is the pure strategy set over $\mathcal{N}$.

The cost (payoff) of each agent from a strategy profile $\sigma$ is $\Psi_i(\sigma) = \sum_{v \in \sigma_i} c_v(\mathcal{N}_v(\sigma))$. Each agent aims at minimizing (maximizing) its own cost (payoff) while executing its tasks with the resources. In addition, we denote $\sigma = (\sigma_i, \sigma_{-i})$ where $\sigma_{-i}$ is the strategy profile $\sigma$ excluding $\sigma_i$.

Definition 1 (Pure Strategic Nash equilibrium, pure-NE). $\sigma^* \in \Sigma$ is a pure strategy Nash equilibrium if and only if $\Psi_i(\sigma^*_i, \sigma_{-i}^*) \geq \Psi_i(\sigma_i, \sigma_{-i})$ in the case of cost minimization ($\Psi_i(\sigma^*_i, \sigma_{-i}^*) \geq \Psi_i(\sigma_i, \sigma_{-i}^*)$ in the case of payoff maximization) for any agent $i \in \mathcal{N}$.

When a pure-NE is reached, no agent can reduce (increase) its own cost (payoff) by unilaterally changing the strategy.

The classical exact congestion game framework has been well studied [8]. We present its properties which are related to our work here. Every finite exact potential game possesses a pure-strategy Nash equilibrium and the finite improvement property (FIP). The FIP provides a general method to achieve a pure-NE for every finite exact potential game.

The social cost (social welfare) in a game is a summation of the costs (payoffs) of each agent which we denote as SC. The social optimum is the optimal SC, which is the minimum social cost in the case of cost minimization, and is the maximum social welfare in the case of payoff maximization. The worst/best Nash equilibrium is the one with the maximum/minimum SC in cost minimization scenarios (the minimum/maximum SC in payoff maximization scenarios). The Price of Anarchy [29] is the ratio, $\frac{SC_{\text{in the worst NE}}}{SC_{\text{in the social optimum}}}$, in the case of cost minimization, and $\frac{SC_{\text{in the social optimum}}}{SC_{\text{in the worst NE}}}$ in the case of payoff maximization. The Price of Stability [30] is the ratio, $\frac{SC_{\text{in the worst NE}}}{SC_{\text{in the best NE}}}$, in the case of cost minimization, and $\frac{SC_{\text{in the best NE}}}{SC_{\text{in the social optimum}}}$ in the case of payoff maximization. PoA and PoS are used to evaluate the efficiency of the pure-NE, and by their definitions we have $PoA, PoS$ both equal to or greater than 1.

III. CONGESTION GAME WITH RESOURCE AND AGENT FAILURES (CG-RAF)

A. Model

In this section, we extend the basic congestion game and propose a model with both agent and resource failures (called CG-RAF for short), capturing the properties of the games in real applications (see Section VI).

1) Resource and Agent: Each agent in $\mathcal{N}$ is denoted as $i \in \{1, 2, \ldots, n\}$. Recall that $\sigma_i$ is the pure strategy of agent $i$. In CG-RAF, each agent will choose exactly one resource to perform its tasks, i.e., $|\sigma_i| = 1$. The chosen resource is denoted as $v_i \in \sigma_i$. We also discuss the scenario when $|\sigma_i| = l$, $\forall i \in \mathcal{N}$ in Section III-D.

Generally, agent failures may be independent but correlated, so we define an agent failure probability distribution as $p(\mathcal{S}) : 2^\mathcal{N} \rightarrow [0, 1]$, where $\mathcal{S}$ denotes that exactly the set $S$ of agents survives. For any subsets $\mathcal{N}' \subseteq \mathcal{N} \subseteq \mathcal{N}$, let $p(\mathcal{N}' \subseteq S) : \mathcal{S}^\mathcal{N} \rightarrow \mathbb{R}$, which is the probability of exactly the set of agents $\mathcal{N}'$ surviving from agents set $\mathcal{N}$. With this distribution, the failure probability of one agent can be calculated. We further assume that each agent $i \in \mathcal{N}$ survives with some constant probability $s_i (s_i \in [0, 1])$, and hence fails with the probability of $f_i = 1 - s_i$. Similar to agent failure, we set each resource $v \in V$ is associated with a constant (but not necessarily independent) failure probability, denoted as $f_v \in [0, 1]$. The survival probability of $v$ is denoted by $s_v$. We point out here that the constant failure probabilities, $f_i$ and $f_v$, can be easily achieved via statistics.

2) Cost/Payoff and Strategy: In this game, we consider the congestion-sensitive cost/payoff function which is more practical as stated in Preliminaries. Hence, resource cost/payoff for an agent to successfully execute its task on $v$ is denoted as $c_v(k)$ where $k = |\mathcal{N}_v|$. Here $\mathcal{N}_v$ is the set of survived agents who choose resource $v$. As both the resource and agent can fail with some probability, we have the following three cases for an agent $i$.

- If agent $i$ fails, it will receive an incompletion cost/payoff $\omega_i$.
- If agent $i$ survives, but its chosen resource $\sigma_i$ fails, it will receive another incompletion cost (payoff) $\omega_i$.
- W.l.o.g., we assume that $\omega_i > c_v(n)$ for the case of cost minimization (similarly, $\omega_i < c_v(n)$ for the case of payoff maximization), and otherwise agent $i$’s dominant strategy might be not to execute its tasks on any resource.

- If both of agent $i$ and its chosen resource $\sigma_i$ survives, its cost/payoff is only affected by other agents on $v$, which is given by $c_v^P(\mathcal{N}_v) = \sum_{R \subseteq \mathcal{N} \setminus \{i\}} p(R \cup \{i\} : \mathcal{N}_v) c_v(|R| + 1)$.

Therefore, the expected cost/payoff of agent $i$ from strategy profile $\sigma$ is:

$$
\Psi_i(\sigma) = \omega_i \cdot \left(1 - s_i\right) + s_i \cdot \left[\omega_i \cdot \left(1 - s_v\right) + s_v \cdot c_v^P(\mathcal{N}_v(\sigma))\right],
$$

where $\mathcal{N}_v(\sigma)$ is the set of agents choosing resource $v_i$ from the strategy profile $\sigma$. 

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B. Existence and Computation of Equilibria

We firstly show that the proposed CG-RAF possesses at least one pure-NE in the contexts of both cost minimization and payoff maximization. Then we show that one can always compute a pure-NE in CG-RAF by taking advantage of its finite improvement property.

1) Existence of Pure-NE in CG-RAF: We prove this by illustrating that the CG-RAF is a finite exact potential game.

**Theorem 1.** The CG-RAF is an exact potential game that has the following potential function:

\[
\phi(\sigma) = \sum_{R \subseteq N^v} p(R) \left[ \sum_{v \in V} \sum_{k=0}^{|R \cap N^v|} s_v c_v(k) \right] + \sum_{v \in V \setminus N^v} \sum_{i \in N_v} (1 - s_v) s_i \omega_i. \tag{2}
\]

**Proof.** We want to prove that for any agent \( i \) unilaterally deviating from \( \sigma = \{v_0\} \) to \( \sigma' = \{v_0\} \), the change in function (2) is the change in the cost of agent \( i \). Let \( \sigma = (\sigma_i, \sigma_{-i}) \) and \( \sigma' = (\sigma'_i, \sigma_{-i}) \) be two strategy profiles, where \( \sigma_i \neq \sigma'_i \). For simplicity, we denote in this proof that \( N_v (N^v) \) for any \( v \in V \) as the set of agents on resource \( v \) from strategy profile \( \sigma (\sigma') \). Therefore, we have \( N^v_i = N^v \setminus \{i\} \) and \( N^v_\emptyset = N^v \cup \{i\} \). Further denote \( N^v_i = N^v \setminus \{i\} \).

Let \( \Delta \phi = \phi(\sigma) - \phi(\sigma') = \Delta \phi(1) + \Delta \phi(2) \), where

\[
\Delta \phi(1) = \sum_{R \subseteq N^v} p(R) \cdot \left[ \sum_{v \in V} \sum_{k=0}^{|R \cap N^v|} s_v c_v(k) - \sum_{v \in V} \sum_{k=0}^{R \cap N^v} s_v c_v(k) \right],
\]

\[
\Delta \phi(2) = \sum_{v \in V \setminus i} (1 - s_v) s_i \omega_i - \sum_{v \in V \setminus i} \sum_{k=0}^{R \cap N^v} (1 - s_v) s_i \omega_i.
\]

We further analyze \( \Delta \phi(1) \):

\[
\Delta \phi(1) = \sum_{R \subseteq N^v} p(R) \cdot \left\{ s_v \left[ \sum_{k=0}^{R \cap N^v} c_v(k) - \sum_{k=0}^{R \cap N^v} c_v(k) \right] + s_v \left[ \sum_{k=0}^{R \cap N^v} c_v(k) - \sum_{k=0}^{R \cap N^v} c_v(k) \right] \right\}
\]

\[
= s_v \sum_{R \subseteq N^v} p(R : N^v | i) \times \left\{ s_v \left[ \sum_{k=0}^{R \cap N^v} c_v(k) - \sum_{k=0}^{R \cap N^v} c_v(k) \right] + s_v \left[ \sum_{k=0}^{R \cap N^v} c_v(k) - \sum_{k=0}^{R \cap N^v} c_v(k) \right] \right\}
\]

\[
+ (1 - s_v) \frac{p(R : N^v | \emptyset)}{N^v_i} \times \left\{ s_v \left[ \sum_{k=0}^{R \cap N^v} c_v(k) - \sum_{k=0}^{R \cap N^v} c_v(k) \right] + s_v \left[ \sum_{k=0}^{R \cap N^v} c_v(k) - \sum_{k=0}^{R \cap N^v} c_v(k) \right] \right\}
\]

Let \( \Delta \Psi_i = \Psi_i(\sigma) - \Psi_i(\sigma') = \Delta \Psi_i^{(1)} + \Delta \Psi_i^{(2)} \), where

\[
\Delta \Psi_i^{(1)} = s_v \sum_{R \subseteq N^v} p(T : N^v | i) \times \left\{ s_v \left[ \sum_{k=0}^{R \cap N^v} c_v(k) - \sum_{k=0}^{R \cap N^v} c_v(k) \right] + s_v \left[ \sum_{k=0}^{R \cap N^v} c_v(k) - \sum_{k=0}^{R \cap N^v} c_v(k) \right] \right\}
\]

\[
- s_v v(a) \sum_{Q \subseteq N^v \setminus i} p(Q : N^v | i) c_v(|Q| + 1).
\]

Then we have \( \Delta \phi(1) = \Delta \Psi_i^{(1)}, \Delta \phi(2) = \Delta \Psi_i^{(2)} \Rightarrow \Delta \phi = \Delta \Psi_i \), and Theorem 1 is proven.

We point out here that Theorem 1 is valid in both cost minimization and payoff maximization games. Obviously, CG-RAF is a finite game, so we have the following.

**Theorem 2.** CG-RAF possesses at least one pure Nash equilibrium.

2) Computing Pure-NE: As CG-RAF is a finite exact potential game, its FIP provides an algorithm, which we call the FIP algorithm, to achieve a pure-NE [8]. There are also distributed methods (e.g., [31]) that can efficiently learn the pure-NE in exact potential games.

C. Nash Equilibria and Social Optimum

We present some important characteristics of pure-NE in symmetric CG-RAF, a special case of CG-RAF where agents and resources are symmetric. That is, \( s_i = \hat{s}, \forall i \in N^v \), and \( s_v = \hat{s}, \forall v \in V \), where \( \hat{s} \) and \( \hat{s} \) are constants. The resource cost/payoff function \( c_v(k) = k, \forall v \in V, \forall k \in \mathbb{N} \) is nonnegative, strictly increasing (strictly decreasing) and convex. The incompleteness costs/payoffs \( \omega_i = \hat{\omega}_i = \omega, \forall i \in N^v \). Recall that \( |N^v| = n \). Denote \( |V| = m \) and \( r = n \mod m \).

Based on the above setting, we study the best and worst Nash equilibria as well as the social optima in the CG-RAF. Our analysis considers both the scenarios of the cost minimization (with strictly increasing resource cost functions) and the payoff maximization (with strictly decreasing resource payoff functions). Interestingly, we find the best and the worst possible social costs incurred by agents in equilibrium are the same in both scenarios. We then analyze the social optima in the two scenarios respectively. Based on these findings, we derive the PoA and PoS.

1) The Best and Worst Equilibria:

**Lemma 1.** In symmetric CG-RAF, if there exists two resources \( v_a \) and \( v_b \) such that \( |N^v_a| - |N^v_b| \geq 2 \), then there always exists an agent \( i^* \) who can benefit from deviating from \( v_a \) to \( v_b \) and will receive a lower cost (or a higher payoff).

**Proof.** Under the symmetric setting, the cost of an agent \( i \) for choosing resource \( v \) is \( \omega_i[(1 - \hat{s}) + \hat{s}(1 - \hat{s})] + \hat{s} \cdot \hat{s} \).
c(|N_v|). |N_v| − |N_v'| ≥ 2 implies |N_v| ≥ (|N_v| + 1) + 1 and c(|N_v|) > c(|N_v'| + 1) as the cost function is strictly increasing. Hence by deviating from v_a to v_b, agent i receives a lower cost. Similarly, for the case of payoff maximization, we can prove agent i can benefit from deviating from v_a to v_b, and will receive a higher payoff.

From Lemma 1 we know that ||N_v|−|N_v'|| ≤ 1, ∀v, v' ∈ V. Therefore, we have

**Theorem 3** (Unique Structure of pure-NE). In each pure-NE of symmetric CG-RAF, there are exactly r resources chosen by \( \frac{n}{m} + 1 \) agents, and (m − r) resources chosen by \( \frac{n}{m} \) agents, where \( r = n \mod m \).

We recall that the social cost (social welfare) is a summation of costs (payoffs) of agents. From Theorem 3 we know the social cost/ social welfare in every pure-NE of symmetric CG-RAF is \( \text{SC}_{NE} = nw_1(1−s) + \hat{s} \bar{s}r(\frac{n}{m} + 1)c(\frac{n}{m} + 1) + (m − r)(\frac{n}{m})c(\frac{n}{m}) \), which depends only on n and m. Therefore the following corollary holds.

**Corollary 1.** In the symmetric CG-RAF, the SC in the worst pure-NE is equivalent to the SC in the best pure-NE.

We point out here that Theorem 3 and Corollary 1 are valid in both the case of the cost minimization and that of the payoff maximization.

2) PoA and PoS: In this part, we first characterize the unique structure of social optima and then derive the PoA and PoS. Note that the PoA and PoS is different between the case of cost minimization and that of payoff maximization.

**Theorem 4** (Unique Structure of Social Optimum in Cost Minimization). In each social optimum of symmetric CG-RAF in the case of cost minimization, there are exactly r resources each chosen by \( \frac{n}{m} + 1 \) agents, and (m − r) resources each chosen by \( \frac{n}{m} \) agents, where \( r = n \mod m \).

**Proof.** Firstly, we show the following property of convex functions.

**Lemma 2.** \( c(x) \) is convex, so \( c(b)−c(a) \leq c(z−a)−c(z−b) \), where \( a \leq b \leq z/2 \).

**Proof.** By Lagrange’s mean value theorem, we have: \( c(b)−c(a) = c′(\delta_1)(b−a) \), and \( c(z−a)−c(z−b) = c′(\delta_2)(b−a) \), where \( a \leq \delta_1 \leq b \) and \( z−b \leq \delta_2 \leq z−a \). Since \( a \leq b \leq z/2 \), we have \( b−a \geq 0 \), \( \delta_1 \leq \delta_2 \), and \( c′(\delta_1) \leq c′(\delta_2) \) (by convexity). Hence, \( c(b)−c(a) \leq c(z−a)−c(z−b) \).

The social cost is a summation of the costs of agents, which is given by \( SC = nw_1[1−s+\hat{s}(1−s)] + \hat{s} \bar{s} \sum_{v \in V} |N_v|c(|N_v|) \). Suppose an agent deviating from \( v_a \) to \( v_b \) causes the social cost changing from \( SC \) to \( \hat{SC} \). The set of agents on \( v_a \) and \( v_b \) changes from \( N_{v_a} \) and \( N_{v_b} \) to \( N′_{v_a} \) and \( N′_{v_b} \), where \( |N′_{v_a}| = |N_{v_a}| + 1 \) and \( |N′_{v_b}| = |N_{v_b}| + 1 \). Further supposing \( |N_{v_a}| \geq 2 \). Recall that \( c(x) \) is strictly increasing over \( \mathbb{R}_+ \). We have

\[
\hat{SC} = SC + |N_{v_b}| \cdot c(|N′_{v_b}|) − c(|N_{v_b}|) + |N_{v_a}| \cdot c(|N′_{v_a}|) − c(|N_{v_a}|) ≥ 2.
\]

This theorem implies that it is optimal to approximately evenly distribute agents to each resources, and this unique structure leads to the unique social optimum value. As PoA and PoS are the ratios to the social cost, the following theorem can be derived from Theorem 3 and 4.

**Theorem 5** (PoA and PoS in Cost Minimization). In symmetric CG-RAF in the case of cost minimization, PoA = PoS = 1.

**Theorem 6** (Unique Structure of Social Optimum in Payoff Maximization). In each social optimum of symmetric CG-RAF in the case of payoff maximization, 1) there are exactly n resources each chosen by one agent, if \( n \leq m \), 2) there are \( m − r \) resources each chosen by exactly one agent, and the left one resource chosen by \( n − m + 1 \) agents, if \( n > m \).

**Proof.** It is trivial to verify the social optimum for the cases when \( n \leq m \). We then focus on the cases when \( n > m \). Recall that the payoff function \( c(x) \) is strictly decreasing and convex over \( \mathbb{R}_+ \). By Lemma 2 we have \( SC − \hat{SC} \geq 0 \), where the definition of \( SC \) and \( \hat{SC} \) is the same as those in the proof of Theorem 4. This means we can achieve the social optimum when there are \( m − r \) resources each of which is chosen by exactly one agent, and the left one resource chosen by \( n − m + 1 \) agents.

This unique structure tells us that when the agent number is no greater than the resource number, each agent selecting a different resource achieves a social optimum. When there are more agents than the resources, it can reach a social optimum by assigning the extra \( n − m \) agents to one resource (i.e., a total of \( n − m + 1 \) agents on it). The following theorem can be derived by Theorem 3 and 6.

**Theorem 7** (PoA and PoS in Payoff Maximization). In symmetric CG-RAF in the case of payoff maximization, 1) PoA
= PoS = 1 when \( n \leq m, 2 \)

\[
P_oA = PoS
\]

\[
= n\omega(1 - \tilde{s}) + \tilde{s}((m - 1) + (n - m + 1)c(n - m + 1)] \]

\[
= n\omega(1 - \tilde{s}) + \tilde{s}[r(\frac{n}{m} + 1)c(\frac{n}{m} + 1) + (m - r)(\frac{n}{m} + 1)]
\]

when \( n > m \).

**Theorem 8** (Bounded PoA and PoS in Payoff Maximization).

In symmetric CG-RAF,

\[
P_oA = PoS < \frac{n\omega(1)}{(n - 1)c(\frac{n}{m} + 1)} .
\]

**Proof.** It is trivial to prove the result when \( n < m \). As for \( n > m \), we have

\[
P_oA = PoS
\]

\[
= n\omega(1 - \tilde{s}) + \tilde{s}((m - 1) + (n - m + 1)c(n - m + 1)] \]

\[
= n\omega(1 - \tilde{s}) + \tilde{s}[r(\frac{n}{m} + 1)c(\frac{n}{m} + 1) + (m - r)(\frac{n}{m} + 1)]
\]

\[
\leq \frac{(m - 1)c(1) + (n - m + 1)c(n - m + 1)}{r(\frac{n}{m} + 1)c(\frac{n}{m} + 1) + (m - r)(\frac{n}{m} + 1)} \]

\[
\leq \frac{n\omega(1)}{(n - 1)c(\frac{n}{m} + 1)} .
\]

From Theorem 5, we can see in the case of cost minimization, each NE can achieve the social optimum for symmetric CG-RAF. From Theorem 7 and 8, we can see in the case of payoff maximization, the social welfare in any NE of symmetric CG-RAF can be bounded. We will show how these theoretical results sheds light on a better system design in Section VI.

**D. Extension to l-CG-RAF**

We can extend the results of CG-RAF to a more general case called l-CG-RAF, where each agent chooses a fixed number \( l(1 \leq l \leq |V|) \) resources. That is to say, let \( L_i \subseteq V \) be the set of resources that agent \( i \) will choose. We have \( |L_i| = l, \forall i \in \mathcal{N} \). Similar to the CG-RAF, we can prove that l-CG-RAF is still potential. Furthermore, by the uniqueness of the structure of the equilibria, we can prove that, for symmetric l-CG-RAF, PoA = PoS = 1 in cost minimization and PoA = PoS < \( \frac{n\omega(1)}{(n - 1)c(\frac{n}{m} + 1)} \) in payoff maximization. Due to the limited space, we omit the detailed proof here.

**IV. CONGESTION GAME WITH CORRELATED RESOURCE FAILURE (CG-CRF)**

**A. Model**

In this section, in order to model practical congestion games (e.g., in cloud computing as discussed in Section I), we propose a kind of congestion games with correlated resource failures, which extends the result in [17] where they assumed an independent and constant failure probability for each resource.

1) Resource and Agent: As resource failures can be correlated, we assume a resource failure probability distribution as \( p'(R) : 2^\mathcal{N} \rightarrow [0, 1] \), where \( R \) denotes exactly the set \( \mathcal{R} \) of resources that survive. Let \( \mathcal{J} \) be the package set 1, a combination of all available packages. We set \( j \in \mathcal{J} \) as a resource package, and \( V_j \subseteq V \) as the set of resources contained in \( j \). We denote \( V_j = \{ V_j : j \in \mathcal{J} \} \).

A package that survives (i.e., it is not a package failure) can be utilized if and only if all resources it contains survive. This setting is reasonable in practice. For example, the whole communication path cannot be utilized if one node on the path fails, and a VM cannot be created if some of its needed resources cannot be provisioned. More applications can be found in Section VI. Each agent needs one survived package to carry out its tasks. As stated above, packages are not reliable, and to play safe, an agent might choose multiple packages and utilize one of the survived packages.

2) Cost/Payoff and Strategy: We consider a generalized resource-agent-specific cost/payoff for agent \( i \) to utilize \( v \), denoted as \( c_{i,v}(k_v) \), where \( k_v = |N_i| \). Here \( N_i \) is the set of survived agents who choose the packages containing resource \( v \). To simplify the notations, we use \( c_{i,v} \) and \( c_{i,v}(k_v) \) interchangeably. Similar to the existing works, we assume a symmetric model for agents’ strategy set, which means all given packages are available to every agent. The cost/payoff for agent \( i \) to choose a package \( j \) is defined as \( c_{i,j} = \sum_{v \in V_j} c_{i,v} \), called the package cost/payoff. As stated above, an agent might choose multiple packages for reliability, and then select the survived package having the minimum cost (the maximum payoff) to utilize. Agent \( i \)’s cost (payoff) when successfully completing its tasks is determined by the cost (payoff) of the survived package the agent utilizes, while its cost with an uncompleted task is evaluated by its nonnegative incompletion cost (payoff) \( \omega_i \), which is similar to those in CG-RAF. Generally, \( \omega_i \) might be smaller than \( c_{i,j} \). That is to say, completing a task by utilizing an extremely congested package might cost more (gain less) than leaving the task unfinished. Define \( c_{i,j}(k) = \sum_{v \in V_j} c_{i,v}(k) \) as the cost/payoff for agent \( i \) to choose a package \( j \), each of whose resources have been chosen by \( k \) agents. We assume that \( c_{i,j}(1) = \sum_{v \in V_j} c_{i,v}(1) < \omega_i, \forall i,j \) for cost minimization \( (c_{i,j}(1) = \sum_{v \in V_j} c_{i,v}(1) > \omega_i \) for payoff maximization). Otherwise, the obvious dominant strategy of agent \( i \) is to avoid assigning its task to any package.

Different from the model of CG-RAF, the set of pure strategies \( \Sigma_i \) for agent \( i \in \mathcal{N} \) is the power set of the package set: \( \Sigma_i = 2^\mathcal{J} \). Then \( \Sigma = \times_{i \in \mathcal{N}} \Sigma_i \) is the set of pure strategy profiles of all agents, and \( \sigma = (\sigma_1, \ldots, \sigma_n) \in \Sigma \) is a combination of pure strategies. The \( |V| \)-dimensional congestion vector that corresponds to \( \sigma \) is \( h^\sigma = (h^\sigma_v)_{v \in V} \), where \( h^\sigma_v = |\{ i \in \mathcal{N} : v \in V_j, j \in \sigma_i \}| \). The outcome from \( \sigma \) is a survived subset \( T_{\mathcal{J}} \subseteq \mathcal{J} \) and a corresponding subset \( T_V \subseteq V \) of the resources that have successfully executed their assigned tasks. The cost/payoff of agent \( i \) from a strategy profile \( \sigma \) and the outcome \( T_{\mathcal{J}} \) is denoted as \( \psi_i(\sigma, T_{\mathcal{J}}) \).

1Recall that each package is given initially, which can be a communication path, a combination of a VM’s resources, and etc.
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ψ_i(σ, T_J) is either ω_i (if σ_i ∩ T_J = ∅) or min_j∈σ_i \ T_J \sum_{v_j ∈ V_j} c_{i,j}(h^{ω_i}_{v_j}) (if σ_i ∩ T_J ≠ ∅). Given a strategy profile σ, let T_J and T_V denote the random variables representing the subsets of successful packages and resources respectively. T_J is distributed over 2^{J \cup \sigma_i}, and its distribution is determined by (f_j)_{j ∈ J \cup \sigma_i}, while T_V is distributed over 2^{U_j ∈ (T_j) \cup \sigma_i}, and its distribution is determined by (f_j)_{j ∈ J \cup \sigma_i, U_j ∈ V_j}.

We define a subset Π^σ_i(i) ⊆ σ_i \ \forall i ∈ Σ, \ \forall i ∈ Σ \ \forall j \in σ_i such that by unilaterally switching to any package from j, agent i will receive a lower cost (a higher payoff). We further consider two events, A_j, the event that package j survives eventually, and B(X) the event that the packaging sleeve belongs to the set X all fail. Note that (\bigcap_{j∈J}A_j) ∩ (\bigcap_{j∈J \setminus T_J A_j}) ≠ ∅ will not hold for some T_J’s (\bigcap_{j∈J}A_j) ∩ (\bigcap_{j∈J \setminus T_J A_j}) ≠ ∅. For those infeasible T_J ∈ 2^J, we have Pr[(\bigcap_{j∈J}A_j) ∩ (\bigcap_{j∈J \setminus T_J A_j})] = 0.

The expected cost/payoff of agent i from strategy profile σ is therefore:

Ψ_i(σ) = \sum_{T_J ∈ 2^J} \psi_i(σ, T_J)Pr[(\bigcap_{j∈J}A_j) ∩ (\bigcap_{j∈J \setminus T_J A_j})] = ω_i Pr(\bigcap_{j∈J}A_j) + \sum_{j ∈ J} c_{i,j}(σ) Pr[(\bigcap_{j∈J}A_j) ∩ (\bigcap_{j∈J \setminus T_J A_j})] = ω_i Pr(\bigcap_{j∈J}A_j) + \sum_{j ∈ J} c_{i,j}(σ) Pr[B(\Gamma^σ_i)(A_j)],

and each agent aims at minimizing (maximizing) its own expected cost (payoff).

**B. Existence of Equilibria**

It has been shown in [17] that even congestion games with independent constant resource failures are not ordinal potential and have no finite improvement property. Our generalized game model has the same properties. In this section, we constructively prove the existence of equilibria. We first show the conditions that guarantee the Nash equilibrium, according to which we design an efficient algorithm to construct a Nash equilibrium. For simplicity reasons, we illustrate the results for cost minimization scenario only. Based on this, one can easily derive similar results for the payoff maximization scenario.

1) Policies that Guarantee the Existence of Equilibria: Let (σ^ω, ..., σ^ω), σ^ω ∈ Σ be a pure strategy profile of a given CG-CRF. We recall that h^ω = (h^ω_v)_{v ∈ V} is the corresponding congestion vector, and V_j is the resource set of package j.

We further let V^ω = \bigcup_{j ∈ σ_j^ω}V_j for any agent i from a pure strategy profile σ. We first give two policies that act as the foundations of our proofs of the existence of Nash equilibria.

**Policy 1.** c_{i,j}(σ^ω) = \sum_{v_j ∈ V_j} c_{i,j}(h^ω_{v_j}) ≤ ω_i, \ \forall j ∈ σ^ω_i.

**Policy 2.** c_{i,j}(σ^ω) = \sum_{v_j ∈ V_j ∩ V^ω} c_{i,j}(h^ω_{v_j}) + \sum_{v_j ∈ V_j \setminus V^ω} c_{i,j}(h^ω_{v_j} + 1) ≥ ω_i, \ \forall j \notin σ^ω_i.

For any agent i ∈ Σ and any strategy profile σ^ω = (σ^ω_i, σ^ω_j) that satisfies Policies 1 and 2, we have the following lemmas.

**Lemma 3.** For any j ∈ σ^ω_j, if j \notin σ^ω_i : V_j = V_j exists, we have c_{i,j}(σ^ω) = c_{i,j}(σ^ω^ω_i).\n
**Proof.** From V_j = V_j, we have c_{i,j}(σ^ω) = \sum_{v_j ∈ V_j} c_{i,j}(h^ω_{v_j}) = \sum_{v_j ∈ V_j} c_{i,j}(h^ω_{v_j}). Meanwhile, since j \notin σ^ω_i, by Policy 2, we have c_{i,j}(σ^ω) ≥ ω_i. Thus, c_{i,j}(σ^ω) = c_{i,j}(σ^ω^ω_i) = \omega_i.

**Lemma 4.** For any feasible T_J ∈ 2^J, we have ψ_i(σ^ω, T_J) ≤ ω_i.

**Proof.** Consider the following two cases. Case I: If σ^ω_i ∩ T_J = \∅, we have ψ_i(σ^ω, T_J) = ω_i; Case II: If σ^ω_i ∩ T_J ≠ \∅, by Policy 1, we have ψ_i(σ^ω, T_J) = min_{j ∈ σ^ω_i \cap T_J} c_{i,j}(σ^ω) ≤ ω_i. Lemma 4 is reached by the above.

Recall that σ^ω is a pure-NE if and only if ψ_i(σ^ω, σ^ω^ω) = ψ_i(σ^ω, σ^ω^ω), \ \forall i ∈ Σ, \ \forall i ∈ Σ. Next, we want to show under all circumstances, each agent has no incentive to deviate to any other strategy. For any σ^ω_i = (σ^ω_i, σ^ω^ω_i) : σ^ω_i ≠ σ^ω^ω_i, we have the following results.

**Lemma 5.** For any feasible T_J ∈ 2^J, if σ^ω_i ∩ σ^ω_i ∩ T_J = \∅, we have ψ_i(σ^ω, T_J) ≥ ψ_i(σ^ω, T_J).

**Proof.** Consider the following two cases.

Case I: If σ^ω_i ∩ T_J = \∅, by Lemma 4 we have ψ_i(σ^ω, T_J) = ω_i ≥ ψ_i(σ^ω, T_J);

Case II: If σ^ω_i ∩ T_J ≠ \∅, for j ∈ σ^ω_i ∩ T_J we have the following two subcases:

Subcase 1: If j ∈ \{ j : j = J \in σ^ω_i ∩ T_J, V_j = V_j \}, by Lemma 3 we have c_{i,j}(σ^ω) = c_{i,j}(σ^ω^ω_i) = ω_i;

Subcase 2: If j \notin \{ j : j = J \in σ^ω_i ∩ T_J, V_j = V_j \}, by Policy 2 we have c_{i,j}(σ^ω) = \sum_{v_j ∈ V_j ∩ V^ω} c_{i,j}(h^ω_{v_j}) + \sum_{v_j ∈ V_j \setminus V^ω} c_{i,j}(h^ω_{v_j} + 1) = c_{i,j}(σ^ω) ≥ ω_i.

Then we have ψ_i(σ^ω, T_J) = min_{j ∈ σ^ω_i ∩ T_J} c_{i,j}(σ^ω) ≥ ω_i ≥ ψ_i(σ^ω, T_J). Lemma 5 is reached.

**Lemma 6.** For any feasible T_J ∈ 2^J, if σ^ω_i ∩ σ^ω_i ∩ T_J ≠ \∅, we have ψ_i(σ^ω, T_J) ≥ ψ_i(σ^ω, T_J).

**Proof.** For all j ∈ σ^ω_i ∩ T_J \ σ^ω_i, by the result of case II in the proof of Lemma 5, we have c_{i,j}(σ^ω) ≥ ω_i. All cases are considered as follows:

Case I: If arg min c_{i,j}(σ^ω) ∈ σ^ω_i ∩ σ^ω_i ∩ T_J, with c_{i,j}(σ^ω) ≥ ω_i we have ψ_i(σ^ω, T_J) = min_{j ∈ σ^ω_i ∩ σ^ω_i ∩ T_J} c_{i,j}(σ^ω) = ψ_i(σ^ω, T_J).

Case II: If arg min c_{i,j}(σ^ω) ∈ σ^ω_i ∩ T_J \ σ^ω_i, we have min_{j ∈ σ^ω_i ∩ T_J} c_{i,j}(σ^ω) = min_{j ∈ σ^ω_i ∩ T_J} c_{i,j}(σ^ω) = min_{j ∈ σ^ω_i ∩ T_J} c_{i,j}(σ^ω) ≥ min_{j ∈ σ^ω_i ∩ T_J} c_{i,j}(σ^ω).

Then we have Theorem 9 that shows the two policies sufficiently guarantee a Nash equilibrium.

**Theorem 9.** The strategy profile σ^ω is a Nash equilibrium in CG-CRF if Policies 1 and 2 are satisfied for all i ∈ Σ.
Lemma 7. \( \sigma^* \) satisfies Policy 1.

Proof. Suppose there exists an agent \( i \in \mathcal{N} \) such that the subset \( Q_i = \{ j \in \sigma_i^* : c_{i,j}(\sigma^*) > \omega_i \} \subseteq \sigma^* \) is not empty. Let \( \chi \in Q_i \) be such a package such that \( c_{i,\chi}(\sigma^*) = \max_{j \in Q_i} c_{i,j} > \omega_i \).

Similarly, let \( \psi_i(\sigma^*, T_J) \) be agent \( i \)'s cost from \( \sigma \) and \( T_J \).

To consider the following three failure realizations.

- Case I: \( T_J = \emptyset \). We have \( \psi_i(\sigma^*, \emptyset) = \psi_i((\sigma_i^* \setminus \{ \chi \}, \sigma_i^*), \emptyset) \).
- Case II: \( T_J = \{ \chi \} \). Agent \( i \) benefits from removing \( \chi \) from its strategy set, that is \( \psi_i((\sigma_i^* \setminus \{ \chi \}, \sigma_i^*), \{ \chi \}) = \omega_i < \psi_i(\sigma^*, \{ \chi \}) \).
- Case III: Otherwise, \( T_J \neq \emptyset \) and \( \{ \chi \} \). Since \( c_{i,\chi} \geq c_{i,j} \) for any \( j \in \sigma^* \), we have \( \psi_i((\sigma_i^* \setminus \{ \chi \}, \sigma_i^*), T_J) = \psi_i(\sigma^*, T_J) \).

Note that \( \psi_i((\sigma_i^* \setminus \{ \chi \}, \sigma_i^*), \{ \chi \}) < \psi_i(\sigma^*, \{ \chi \}) \) for Case II. Thus, we can conclude agent \( i \) can strictly improve its expected cost by removing package \( \chi \) from its strategy set, that is \( \Psi_i(\sigma^*) > \Psi_i((\sigma_i^* \setminus \{ \chi \}, \sigma_i^*), \{ \chi \}) \), which contradicts \( \sigma^* \) is a NE pure strategy profile. \( \square \)

Lemma 8. \( \sigma^* \) satisfies Policy 2.

Proof. Suppose there exists an agent \( i \in \mathcal{N} \) and a package \( \chi \notin \sigma_i^* \) such that \( c_{i,\chi}(\sigma^*) < \omega_i \). Then we consider the following three failure realizations.

- Case I: \( T_J = \emptyset \). We have \( \psi_i(\sigma^*, \emptyset) = \psi_i((\sigma_i^* \cup \{ \chi \}, \sigma_i^*), \emptyset) \).
- Case II: \( T_J = \{ \chi \} \). Agent \( i \) benefits from including \( \chi \) into its strategy set, that is \( \psi_i((\sigma_i^* \cup \{ \chi \}, \sigma_i^*), \{ \chi \}) < \omega_i = \psi_i(\sigma^*, \{ \chi \}) \).
- Case III: \( T_J \neq \emptyset \) and \( \{ \chi \} \) (otherwise). Since \( c_{i,\chi} \geq c_{i,j} \) for any \( j \in \sigma^* \), we have \( \psi_i((\sigma_i^* \cup \{ \chi \}, \sigma_i^*), T_J) \leq \psi_i(\sigma^*, T_J) \).
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locally; and 2) disseminate its decision to all the other agents;

- Step 4: after receiving the decisions of all the other agents on this package, each agent updates the congestion on the resources of this package locally, i.e., updating $\Delta h_v$ as in Line 13 in Algorithm 1.

The above sketch contains two basic distributed operations, sorting and information dissemination, which have been well-studied in the area of distributed computing (e.g., [32]–[34] and [35]–[37]). Due to the limited space, we omit the details here.

### Algorithm 1: Efficient Construction of a Nash equilibrium.

1. $\sigma_i = \emptyset, \forall i \in \mathcal{N}; \Delta h_v = 0, \forall v \in \mathcal{V}$
2. Let $c(i,j)(h) = \sum_{v \in \mathcal{V}} c_{i,v}(h + \Delta h_v)$ and $H_{i,j} = \{h' : c(i,j)(h' + \Delta h_v) < \omega_i, h' = 1, \ldots, n\}, \forall i \in \mathcal{N}, \forall j \in \mathcal{J}$
3. For each agent $j$, do
   4. For each $v \in \mathcal{V}$ do
      5. If $H_{i,j} = \emptyset$ then
         6. $h'_{i,j} = \max_{h \in H_{i,j}} h$
      7. Else
         8. $h'_{i,j} = 0$
      9. Reorder the agents of $h'_{i,j}$ in a non-increasing way: $\pi_j : i \rightarrow \pi_{i,j}$
     10. For $\pi_{i,j} = 1$ to $n$ do
         11. If $\pi_{i,j} \leq h'_{i,j}$ then
             12. $\sigma_i = \sigma_i \cup \{j\}$
         13. Update $\Delta h_v$
     14. Return $\sigma_v, \forall v \in \mathcal{V}$

### V. NUMERICAL ANALYSIS

We conduct extensive simulations to study the equilibria properties of the proposed congestion games with failures, CG-RAF and CG-CRF. We examine the existence of the pure-NE first, and then move on to the impact of key parameters on the equilibria properties, e.g., PoA.

#### A. Setup

The number of agents $n$ ranges from 6 to 200, and that of resources $m$ from 3 to 50. In CG-CRF, we randomly choose $|\mathcal{J}| = \lceil 2m/3 \rceil$ packages, each of which contains $\lceil 2m/3 \rceil$ resources. The survival probability of resource $s_v, \forall v \in \mathcal{V}$ and of agent $s_i, \forall i \in \mathcal{N}$ is uniformly randomly chosen in $[0,1]$. Each resource $v \in \mathcal{V}$ is coupled with a strictly increasing convex function $c_{v}(\cdot)$. We consider the case of cost minimization in the experiments. The experiments on payoff maximization work in a similar way. We then focus on analyzing the equilibria properties. To better understand the features of such properties of our proposed congestion game with failures, in CG-RAF, we let the resource cost function be $c_v(k) = \alpha k, \forall v \in \mathcal{V}$, where $k$ is the number of agents using it and $\alpha$ ranges from 1 to 5. The incompletion costs, $\omega_i$ and $\hat{\omega}_i(v)$, are set randomly following the uniform distribution in $[100, 200]$ for CG-RAF so that each agent would be incentivized to choose a resource. In CG-CRF, we set the resource cost function as $c_{i,v}(k_v) = \alpha v k_v + i, \forall i \in \{1, \ldots, n\}$, $\forall i \in \{1, \ldots, |\mathcal{V}|\}$, where $k_v$ and $v$ are set the same as those in CG-RAF. The incompletion cost in CG-CRF is set to $\omega_i = \max_{j} c_{i,j}(1) + u_i, \forall i \in \mathcal{N}$, where $u_i$ is uniformly randomly chosen in $(0,10]$. For each setting in CG-RAF, we repeat the FIP algorithm 150 times from different starting points, and calculate the average value of the metrics under consideration. For CG-CRF, Algorithm 1 is utilized to efficiently reach an NE, and we further use exhaustive search to find its PoA.

#### B. Convergence to the pure-NE

We define $\Delta N_V = \max_{x,y \in \mathcal{V}} (N_x - N_y)$ for CG-RAF, i.e., the maximum difference in the number of agents on each resource. We first simulate the FIP of CG-CRF. Figure 1(a) and Figure 1(b) illustrate the evolution of the $\Delta N_V$ and the potential function value $\phi$ under two settings respectively, where Figure 1(a) includes 10 resources with failure probability of 0.5, 10 agents with failure probability of 0.3, and Figure 1(b) includes 15 resources with failure probability of 0.5, 50 agents with failure probability of 0.7. We can see in these two figures that the $\Delta N_V$ and $\phi$ are gradually reduced and remain unchanged eventually, which verifies our theoretical results, i.e., the existence of pure-NE. For CG-CRF, we have proved the existence of the pure-NE by constructing one using Algorithm 1 in Section IV-B. We have also shown in Section IV-B the distributed implementation of Algorithm 1. Next, we test whether the output strategy profile of Algorithm 1 is an NE according to the definition of pure-NE (Definition 1). In our experiments, we consider a CG-CRF with 100 agents, 20 resources, and 50 packages each containing 1 to 5 resources. We repeat Algorithm 1 150 times. In each time, we pick uniformly at random the order of the packages that the agents traverse in the algorithm. The results show that the output strategy profiles of Algorithm 1 are all NEs, which validate Theorem 11.

#### C. Impact of Parameters on Equilibria Properties

In this part, we evaluate the impact of key parameters on three equilibria properties, 1) the convergence time of computing a pure-NE (for CG-RAF only), 2) the social cost of the pure-NEs, and 3) the PoAs (for both games). We first analyze the impacts in CG-RAF (Section V-C1, V-C2, and V-C3). Then we focus on CG-CRF, and analyze the impacts on the social optimum and the PoA (Section V-C4).

1) **Number of Resources and Agents:** For CG-RAF, we evaluate the impact of the number of resources $m$ and the number of agents $n$ by executing the FIP algorithm, where $m$ ranges from 10 to 50 and $n$ ranges from 40 to 200. During these comparison tests, we fix the probabilities of agent

2) It is better to efficiently compute the PoA or its nontrivial bounds, which we leave to our future work.
Fig. 1: (a),(b): Convergence to the pure-NE; (c),(d): the impact of the number of resources and agents; (e): the impact of survival probabilities of resources and agents; (f),(g): the impact of sensitivity of the cost function to congestions; (h): the impact of survival probabilities of resources and agents and the sensitivity of the cost function to congestions in CG-CRF.

Failure and resource failure to be 0.5 and 0.7, respectively. Simulations with $5 \times 5$ different values of $m$ and $n$ are conducted to evaluate the impacts of these two parameters.

In Figure 1(c) and Figure 1(d) we show that as the number of agent increases, both of the convergence time and the value of social cost increase. As we have proved PoA=$\text{PoS}=1$ in CG-RAF, the social cost is equal to the social optimum. We can also observe that the smaller the resource number, the lower convergence time and the higher the social cost would be obtained. This is because fewer resources will lead to a greater congestion on each resource.

2) Failure Probabilities: For CG-RAF, we evaluate the impact of the resource survival probability, $s_r$, and the agent survival probability $s_i$. To reveal the trends more clearly, we set $s_i = s_r, \forall v \in V$ and $s_i = s_a, \forall i \in N$ where $s_r$ and $s_a$ both range from 0.2 to 1. During these comparison tests, we fix the number of agent and resource to be 100 and 10 respectively. Simulations with $5 \times 5$ different values of $s_r$ and $s_a$ are conducted.

Since we consider constant survival probabilities, from the potential function, we know that the failure probability will not affect the convergence to pure-NE through FIP. So we only evaluate the impact of failure probabilities on the social cost here. The results are shown in Figure 1(e), where the social cost decreases as the resource survival probability or the agent survival probability increases. This is expected as the higher failure probability leads to a higher chance to receive the relatively higher incompletion cost.

3) Sensitivity of Cost Function to Congestions: The sensitivity of the cost function on congestions may have a great effect on the equilibria properties. We use the parameter $\alpha$ to characterize this kind of impact, where $\alpha$ ranges from 1 to 5 with different settings of the numbers of agents and resources.

We observe in Figure 1(f) that the sensitivity of the cost function on congestion does not influence the convergence time through FIP. This can be also explained by the mechanism of FIP [8]. However, the sensitivity affects the value of the social cost which is shown in Figure 1(g). As the value of $\alpha$ increases, i.e., the sensitivity on congestion increases, the value of social cost increases. We can know from Theorem 3 that the equilibria are with the same unique structure on the congestion distribution over all resources. The social cost of one equilibrium therefore increases with the sensitivity on the congestion of each resource.

4) Impacts on the Social Optimum and the PoA in CG-CRF: The situation becomes much more complicated in CG-CRF, as it is not even an ordinal potential game. Here, we analyze the impacts of the resource failure probability and the sensitivity to congestions have on the social optimum and the PoA respectively in CG-CRF. We denote $\mu_v$ and $\sigma_v$ the mean value and the standard deviation of the resource failure probabilities, respectively. Recall that the cost function we consider is $c_{i,v}(k_v) = \alpha v k_v + i$. We use a 3-tuple $(\mu_v, \sigma_v, \alpha)$ to denote a case in the game. We consider 7 different cases (each with 10 agents, 10 resources, and 5 packages each containing 1 to 5 resources) as shown in Table II. In each case, we generate the failure probabilities that meet $\mu_v$ and $\sigma_v$, and the cost functions of the basic resources with parameter $\alpha$.

The results are shown in Figure 1(h). We can observe that 1) the PoAs in different cases are with similar values; 2) the resource failure probability does not influence the social optimum or the PoA; 3) the sensitivity to congestions does not affect the PoA but it does affect the social optimum. As the sensitivity on congestion increases, the value of social optimum increases. Our conjecture of the reason behind is that the structure of the pure-NE is similar to that of the social optimum. We also analyze the general cases when the failures are correlated, and the results are similar to the above. We omit them here due to the lack of space.
D. Extension

In this section, we analyze two extensions of the CG-RAF.

1) The PoA in Asymmetric CG-RAF: We derive in Section III-C PoA=PoS=1 theoretically for the symmetric CG-RAF. In this section, we analyze the PoA of the more general game model, asymmetric CG-RAF. That is, the (agent and resource) failure probabilities are not necessarily the same, so are the cost functions. We consider 6 agents and 3 resources in a CG-RAF. Agent failure probabilities are generated to meet the given parameters ($\mu_i, \sigma_i$), where $\mu_i$ and $\sigma_i$ are the mean and standard deviation of the probabilities. For the resource failure probability, the parameters are denoted as ($\mu_v, \sigma_v$). W.l.o.g., we set $\mu_i = \mu_v = \mu$. Recall that $\alpha$ denotes the sensitivity to congestions. In the following, we analyze the impacts of $\mu$, $\sigma_v$, and $\alpha$ on the social optimum and the PoA. Our starting setting is ($\mu, \sigma_i, \sigma_v, \alpha$) = (0.5, 0.1, 0.1, 1).

- The mean value of the failure probabilities, $\mu$ (see Figure 2(a)). As $\mu$ increases, the social optimum increases while the PoA decreases. When $\mu$ reaches 1, the PoA tends to be 1. This is because the incompletion costs of the agents increase as the failure probabilities increase; and when the agents and resources are easy to fail in the game, even the social optimum cannot perform well.

- The standard deviation of the resource (agent) failure probabilities, $\sigma_v$ ($\sigma_i$) (see Figure 2(b) and 2(c)). In general, when the heterogeneity of the (agent or resource) probabilities increases, neither the social optimum nor the PoA is much affected. If we consider the failure probabilities as the “weights” on the agents or resources, then the total weighted resources are evenly chosen by the weighted agents. In this case, the total weights of either the resources or the agents remain unchanged, which might account for the results.

- The sensitivity of the cost function to congestions, $\alpha$ (see Figure 2(d)). As the sensitivity to congestions increases, the social optimum increases naturally while the PoA remains stable. The reason might be as the sensitivity to congestions increases, both the social costs in the pure-NE and the social optimum increase.

2) Inaccuracy of the Failure Probabilities: As mentioned in the previous sections, the failure probabilities can be achieved via statistics [38], [39]. However, it is probable that agents will have inaccurate values of those probabilities. In this section, we first discuss the impacts of such inaccuracies on the existence of the NE. Then, we move on to the analysis of their impacts on the social optimum and the PoA. In the following, we focus on the CG-RAF, and the analysis is similar in the case of CG-CRF. We denote, for agent $i$, the (possibly inaccurate) estimations of resource $v$’s and agent $j$’s failure probabilities as $f'_{i,v} = f_v + \epsilon_{i,v}$ and $f'_{i,j} = f_j + \epsilon_{i,j} (j \neq i)$ respectively, where $f_v$ and $f_j$ are the actual values of resource and agent failure probabilities, and $-f_v \leq \epsilon_{i,v} \leq 1 - f_v$ and $-f_j \leq \epsilon_{i,j} \leq 1 - f_j$ are the estimation errors. To better analyze $\epsilon_{i,v}$’s and $\epsilon_{i,j}$’s impacts, we suppose there is an adversary who is able to control the values of $\epsilon_{i,v}$’s and $\epsilon_{i,j}$’s. There are two cases, both of which can happen in practical networks.

- If the adversary sets $\epsilon_{i,v} = \epsilon_i, \epsilon_{i,j} = \epsilon_j, \forall i \in N$, i.e., $f'_{i,v} = f'_v, f'_{i,j} = f'_j, \forall i \in N$ (this happens in situations where the estimations are common knowledge, e.g., a transportation network where a main road is jammed with some probability), then the existence of pure-NE is definitely not influenced. The main reason is that agents are unaware of the inaccuracies (even if they are aware of them, they do not know whether the values are greater or less than the actual ones). This implies that we end up with a CG-RAF with $f'_v$’s and $f'_j$’s being the resource and agent failure probabilities.

- If the adversary sets the differences among $\epsilon_{i,v}$’s and $\epsilon_{i,j}$’s to be large enough, agents will have estimations of the failures largely different from each other (this happens where the estimations are private knowledge, e.g., in a wireless network, transmitters may have different estimations of the channel conditions). In this case, the game becomes complicated, and the potential function in our previous presentation does not hold any more. Our conjecture is that the new game still possesses a pure-NE if the errors meet certain conditions. This is an important branch of work which we will certainly consider in the future.

Next, we focus on the case where $f'_{i,v} = f'_v$ and $f'_{i,j} = f'_j$, and further analyze the social optimum and the PoA. We consider 6 agents and 3 resources. W.l.o.g., we set $\epsilon_{v} = \epsilon_{i} = \epsilon, \forall i, v$. We denote Social OPT 1 and Social OPT 2 as the real social optimum and the virtual social optimum when considering $f'_v, f'_j$ as the failure probabilities, respectively. We analyze three cases, as follows, and the results are shown in Figure 3.

- Case 1 (Figure 3(a)): the real failure probabilities are set to $f_v = 0.3, \forall i \in N$ and $f_j = 0.5, \forall j \in V$. The error $\epsilon$ is picked from the set $\{-0.2, -0.1, 0, 0.1, 0.2\}$.

- Case 2 (Figure 3(b)): the real failure probabilities are set to $f_v = 0.3, \forall i \in N$ and $f_j = 0.5, \forall j \in V$. $\epsilon$ is uniformly randomly chosen in $[0, 0.5]$. We denote Social OPT 1 and Social OPT 2 as the real social optimum and the virtual social optimum when considering $f'_v, f'_j$ as the failure probabilities, respectively. We analyze three cases, as follows, and the results are shown in Figure 3.

- Case 3 (Figure 3(c)): different from the above, we consider the asymmetric failure probability in this case, where $f'_v$’s are set to 0.4, 0.5, 0.6, 0.7, 0.8 respectively and $f'_j$’s are set to 0.4, 0.5, 0.6 respectively. $\epsilon$ is uniformly randomly chosen in $[0, 0.5]$. We denote Social OPT 1 and Social OPT 2 as the real social optimum and the virtual social optimum when considering $f'_v, f'_j$ as the failure probabilities, respectively. We analyze three cases, as follows, and the results are shown in Figure 3.

We can observe that although the virtual social optimum’s fluctuation is greater or less than that of the real social optimum as the value of the error increases, the PoA increases as the deviation of the error increases, which implies the strategy profile that the agents believe to be the “pure-NE” does not perform well in practice.
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VI. APPLICATIONS IN NETWORKING

In this section, we give some examples of CG-RAF as well as CG-CRF in real applications. We also show how our theoretical results can shed light on a better system design.

A. Channel Access in Networks

In wireless networks deployed in a limited area, at any one time each wireless channel can be occupied by one wireless node to avoid interference. Therefore, the nodes have to compete with each other to access the channels. Clearly, the chance to access a channel is affected by its congestion level, i.e., the number of nodes competing on it. Some channels might be not available (e.g., occupied by a primary user in cognitive radio networks). Some nodes may fail and quit the competition due to running out of power. Since the network area is limited, it is reasonable to assume that the channel availability for all the nodes is the same. So the chance to successfully occupy a channel is symmetric to each node.

In the case that all the nodes are selfish, the above scenario can be modeled as a congestion game with failures, where resources and agents are the channels and nodes, respectively. The payoff for each agent to choose a resource $v$ can be defined as the chance to get access to the corresponding channel, which can be captured by a non-increasing function $c_i(k)$ of the congestion level $k$. By taking into account the possible failure events, our proposed CG-RAF can model the game properly.

B. Heterogeneous Cellular Networks

A cellular network is a heterogeneous wireless network consisting of $|V|$ base stations (BSs) and $N$ users (agents) [40], [41]. Here a BS can be an AP in a Wi-Fi network, a femtoBS in a femto-cells network, an NB in a 3G network, and etc. We denote the set of users that share base station $v \in V$ by $N_v$. At any time, each user uses a single radio access technology (RAT) to access a subset of BSs. The throughput achieved by user $i$ via BS $v$ is denoted as $c_{i,v}$, which depends on the the congestion level on the BS, i.e., the number of users sharing it. The congestion level on $v$ is given as $c_{i,v} = R_v b_v(|N_v|), \forall v \in N_v$, where $R_v$ is the data rate when there is only one user on BS $v$, and $b_v$ is a non-increasing function. Each user will transmit on the BS with the lowest congestion level and therefore achieve the highest throughput. Consider that a BS may fail to serve a user in practice, e.g., the unavailability of the channel at a particular moment. The game that each user chooses a BS matches a light-version of CG-CRF (Section IV) where each package contains only a single BS $v$. When the communication is multi-hop, a package can be a communication path, and the failure of any BS in it leads to the failure of this package.

C. Other Applications

Because of limited space, we mention some other applications only briefly below.

Wireless Sensor Networks (WSNs). In a wireless sensor network (WSN) [42], task initiators (agents) need to choose a specific number of sensors to process their tasks. In selecting a subset of sensors, an agent’s aim is to receive the highest possible quality of service, i.e., finishing the task on hand as quickly as possible, which is determined by the congestion level on each sensor as caused by other agents also engaging the sensor. Agents (they themselves are sensors) may “fail”, i.e., running out of power, and therefore may quit the competition for the resources (sensors). This game can be modeled as our proposed $l$-CG-RAF as presented in Section III.

Cloud Computing. In a cloud computing environment, computing resources, e.g., CPU, memory, are assigned to virtual machine instances for job processing [43]. The quality of
service for a job (an agent) depends on its completion time, which is affected by the system’s conditions such as the status of agents competing for the same resources. In most scenarios, jamming and outages might happen to the resources, and such “resource failures” are not independent among the resources concerned [16]. With the existence of selfish agents which persistently aim at maximizing their own quality of service, the game can be modeled as a CG-CRF.

In addition to the above, CG-RAF and CG-CRF can model games in many other networking scenarios such as energy harvesting networks [44] and business logistics [45] as well.

D. Insights for better system designs

In our analysis of CG-RAF, we can identify specific failure parameters, i.e., symmetric failure probabilities and costs/payoffs, that lead to the unique structure of the Nash equilibria. NE in such a congestion game can achieve the social optimum (by Theorem 5). This sheds light on how to design stable system with small efficiency loss. While the ideal symmetric failure parameter values are hard to determine in practice, it is always beneficial to reduce the variance of the parameters among the agents and the resources. By controlling the level of heterogeneity, we can achieve a better system efficiency. For CG-CRF, we prove the existence of Nash equilibrium which can help us to understand better a system with correlated failures of resources.

Numerical results in Appendix V suggest that 1) for CG-RAF, we can obtain a more stable and efficient system by controlling the number of agents competing for the public resources and by tuning down the parameter of congestion sensitivity by admission control; and 2) for CG-CRF, we show how to make the practical networks more stable and efficient (Figure 1(g)).

VII. DISCUSSION

A. Dependency on Failure Events

Unlike some existing works, we do not have a strong assumption about the independence of failure events. In CG-RAF, agent failures can be correlated, and so can the resource failures. We only set a constant failure probability for each resource or agent in this game. In CG-CRF, resource failures can be correlated. The dependency among failures changes the game fundamentally. For example, CG-CRF with the resource-specific cost/payoff function is so complicated that it is not even an ordinal potential game. Although the pure-NE for CG-CRF is proven to exist with the resource-agent cost/payoff function, the PoA and PoS are hard to obtain.

B. The Game Model

Here, we discuss the models, i.e., settings in some other context, of the proposed CG-RAF and CG-CRF. We compare the closest existing works, paper [21] and paper [17], and ours and present their differences.

Meir et al. [21] might be the first to introduce (correlated) agent failures to congestion games where each selfish agent, with the knowledge of the correlated failure probabilities, would choose one resource to execute its tasks to minimize the expected cost. They proved that this class of games possess Nash equilibria in pure strategies, while not being isomorphic to potential games [8]. The model in [21] significantly differs from CG-RAF. In CG-RAF, not only the agents can fail but also the resources can fail to execute the assigned tasks, which is the first time in the literature. And similar to the agent failures in CG-RAF, the resource failures can be correlated as well (see Table I). Based on the new game model, we prove the CG-RAF is an exact potential game, and therefore possesses pure Nash equilibria. Moreover, we extend the CG-RAF to the I-CGRF, which captures a more general case and also possesses pure Nash equilibria. It will be interesting to study the general case that the agents can choose different numbers of resources.

The closest work to CG-CRF is Penn et al. [17] where each agent chooses independent basic resources. This is different from the proposed CG-CRF. In CG-CRF, each agent is allowed to include multiple packages at first instead of the basic resources, but process its task on the survived one with the minimum cost. The above difference implies that our proposed generalized model has a wider range of practical applications. Intuitively, the correlation of package failure itself makes the analysis more challenging and somehow technically different. The difference in the two models can be illustrated with a routing game in graphs, where each node might fail with some probability, and a path fails when any of the nodes on it fails. The agents arrive at one source-destination pair. Penn et al. [17] considers a special graph where the paths do not share common nodes except for the source and destination nodes, while CG-CRF can be interpreted as the routing game in a general graph such that paths might share nodes. Furthermore, we remark that Theorem 9 can be extended to a more generalized asymmetric congestion game, where agents can arrive at different source-destination pairs. It would be interesting to analyze the PoA and PoS under this setting, which can be a meaningful extension of this work.

C. Cost Function

In CG-CRF, we consider the resource- and agent-specific cost function $c_{i,v}(k_v)$ which is general enough to model the practical scenarios. In CG-RAF, we first consider a basic cost function $c_{i}(k_v)$ in the original congestion game without any failure (see Section III-A2). Then, after introducing failures into the game, the agents are not completely symmetric. The probabilities of agent failures are correlated, and agents could fail with different probabilities. It leads to the (expected) cost for a survived agent $i$ on a survived resource $v$ to be $c_{i,v}^e(N_v)$ as shown in Section III-A instead of $c_{i}(N_v)$ in some existing works on congestion games (e.g., [1], [8], [10], [29]). We can observe that $c_{i,v}^e(N_v)$ is a resource- and agent-specific cost function, based on which we have derived the expected cost of agent $i$ (not necessarily being survived), $\Psi_i(\sigma)$ in Equation 1. Our main results on the existence of pure-NE in CG-RAF are based on this expected cost (Section III-B). The problem would be over-complicated when the basic cost function is resource- and agent-specific, i.e., $c_{i,v}(k_v)$. The
congestion game might not posses even an ordinal potential function, which would require totally different techniques in the analysis. We believe our work here can open up additional interesting directions in the study of congestion games with failures.

VIII. CONCLUSION

In this paper, we proposed two models, CG-RAF and CG-CRF, for congestion games with failures. In CG-RAF, resources and agents might fail with some probability. Each agent chose exactly one resource to perform its tasks. We considered both the cost minimization and the payoff maximization scenarios. We proved the game is potential and the existence of pure-strategy Nash equilibrium is therefore guaranteed. We derived the bounds for the PoA and PoS which captured the property of the social cost (or social welfare) in the equilibrium. We also showed the extension of the results in CG-RAF to l-CG-RAF where each agent could select a constant number of resources. In CG-CRF with correlated resource failures, each agent was allowed to choose multiple packages (each containing a specific number of resources) and then perform its task on the one with the minimum cost (or the maximum payoff). We proved the existence of a pure-NE in CG-CRF by constructing one efficiently. To the best of our knowledge, this work is the first studying congestion games with the coexistence of resource and agent failures, and it is also the first time that the existence of a pure-NE in congestion games with correlated package failures is proven. We further showed that our theoretical study sheds light on how to design stable and efficient systems. For the future work, besides the extensions of this work mentioned in the discussions, one important direction is to consider CG-CRF together with agent failures, and analyze the existence of pure-NE as well as PoA and PoS.

IX. ACKNOWLEDGEMENTS

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