Quantum Machine Learning with Superposition of Causal Order

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1 Introduction

Since first introduced by Richard P. Feynman in 1981 in his epochal-making speech on quantum computation [1], tremendous progress has already been made in both theory and physical realization. Along the line of research, many algorithms was proposed and gained public attention although they were only of pure theoretical interests in 90s when created. Examples include the well-known Shor's algorithm, a quantum algorithm which factorizes a large integer into its prime factors [2], and the Grovers search algorithm [2] which was later extended to a general scheme called amplitude amplification [3]. The Shor's algorithm surprised the public for it not only demonstrated the potential of quantum computer to provide exponential speedup against their classical counterpart, but it overthrew some most widely used public-key cryptography schemes such as RSA scheme [2]. The latter one, though offers only quadratic speedup, has been proven extremely useful, since searching problem are often important building blocks of other much more complicated algorithms [2].

What followed the advancement of the study of quantum algorithms was the attempt of physical realization of quantum computer [2]. Leading forces includes giants such as IBM [4], Google [5] and the competition have become increasingly heated as these giants started to race to announce their new achievements. The most advanced quantum chip at the moment is Googles new Bristlecone quantum chip which contains 72 qubits and, according to many scientists, such device overshadows any existing classical supercomputer [6].

In addition to its astonishing application in cryptography, quantum computation is also considered advantageous in solving optimization problems. In particular, much efforts have been devoted into a quantum computation model called adiabatic quantum computation [7]. This model was first introduced to attack a long-standing problem called 3SAT problem which was the first known to be NP-complete [8]. Later on. It was then found useful in other tasks such as clustering algorithms [9].

The introduction of Adiabatic quantum computation into unsupervised machine learning is in fact only a small portion of the wide application of quantum computation techniques in tasks arising from the study of machine learning [10] [11]. Many classic classical machine learning algorithms witnessed the construction of their quantum counterpart, where in most cases, the quantum versions are asymptotically much faster, although the quantum algorithms usually come with restrains that classical version do not suffer. Typical example is the HHL algorithm which can solve a linear system of equations exponentially faster than their classical counterparts, under the constrain that the matrix can be simulated efficiently (for instance, when the matrix is sparse, row computable and well-conditioned) to ensure that the phase estimation subroutine can be implemented efficiently [12]. When a machine learning task can be formulated as a search problem, Grovers search algorithm (and its variants) can usually offer speedup (mostly quadratically), concrete examples are quantum clustering algorithms in which tasks such as searching for minimal are repeated [13]. Other techniques are also popular in quantizing classical machine learning algorithm, for instance, combining the power of density operator exponentiation and phase estimation, one can perform principle component analysis efficiently [14].

On the other hand, one can also consider the possibility of utilizing classical machine learning techniques to accomplish quantum learning tasks. Existing machine learning algorithms provide general frameworks to understand, to extract and to define certain information of data [15]. This suggests we may consider making use of machine learning techniques to understand quantum resources, the first idea of such kind would probably be to cluster a set of quantum states. Given the well developed clustering algorithm, the core of the task in fact lies in the estimation of the "difference" between quantum states (we shall elaborate more in section 2). Naturally, one could expect to conduct the same tasks on other objects, for instance, one could imagine a task such as, with the access to a set of quantum gates for n times each, cluster the gates in terms of some predefined notion of discrepancy. This leads to the first possible direction of the project and one can even go further to generalize such task to other quantum resources such as quantum operations.

As mentioned, one of the central problem in this task is to define sensible and computable notions of differences between 2 quantum resources. The obvious classical counterpart of this task is to define and compute the differences between 2 data vectors in $\mathbb{R}^n$. This is often considered an easy task, primarily due to the fact that the metrics and inner products on such spaces ($\mathbb{R}^n$ or $\mathbb{C}^n$) are well studied, and the computations are straightforward because data vectors stored in classical computers can be accessed repeatedly and directly). However, such computational simplicity does not sustain in quantum cases, forcing people to consider indirect methods. In the quantum state case, simple methods such as swap test and others alike exist [16] [9], and offers insight into the possible solution to the same problem on quantum gates and quantum operations. It is expected, such task would require a simple but powerful quantum device: quantum switch.

This brings out second part of the project title: superposition of causal order. However, to concretely elaborate on the relation between the quantum device quantum switch and indefinite causal order, we first briefly go through the basic notations in section 2.

2 Definitions and Quantum Algorithms[1]

2.1 Basic Notions in Quantum Computation

Definition:

Associated to any isolated physical system is a complex vector space with inner product (a Hilbert space) known as ***state space*** of the system. The system is completely described by its ***density operator***, which is a positive operator ρ with trace one, acting on the state space of the system. If a quantum system is in the state ρ_i with probability p_i, then the density operator for the system is $\sum_i p_i \rho_i$

Definition:

A ***quantum*** operation $\mathcal{E}$ from the set of density operator of the input space Q_1 to the set of density operators of the output space Q_2, satisfies the three axioms:
A1: $\mathcal{E}(\rho)$ is the probability that the process represented by $\mathcal{E}$ occurs, when the initial state is ρ:

$$0 \leq tr[\mathcal{E}(\rho)] \leq 1,$$

A2: $\mathcal{E}$ is convex-linear on the set of density operators, that is, for probabilities p_i:

$$\mathcal{E}(\sum_i p_i \rho_i) = \sum_i p_i \mathcal{E}(\rho_i)$$

A3: $\mathcal{E}$ is a **completely positive** map. That is $\mathcal{E}(A)$ must be positive for all positive A. Further more, if we introduce an extra system R of arbitrary dimensionality, it must be true that $\mathcal{I} \bigotimes \mathcal{E}(A)$

[1]unless specified, all definitions are taken from [2]

is positive for any positive operator A on the combined system RQ_1, where $\mathcal{I}$ is the identity map on R.
The map $\mathcal{E}$ satisfies axioms **A1**, **A2** and **A3** if and only if:

$$\mathcal{E}(\rho) = \sum_i E_i \rho E_i^\dagger$$

for some set of operators E_i which map the input Hilbert space to the output space, and $\sum_i E_i^\dagger E_i \leq I$.

Note that, if one consider only the evolution of a closed quantum system, the general representation of the quantum operation reduces to a unitary transformation, U, acting on the density operators ρ or state vector $|\psi\rangle$ of the system, by $U\rho U^\dagger$ or $U|\psi\rangle$. This is useful when we are working on the circuit model of quantum computation.

Definition (***quantum SWITCH***) [17]:

Denote the Kraus operators of the channel $\aleph_1$ as K_i^1 and the channel $\aleph_2$ as K_i^2. The overall Kraus operators of the resulting channel from the switching is:

$$W_{i,j} = K_j^1 K_i^2 \bigotimes |0\rangle\langle 0|_C + K_i^2 K_j^1 \bigotimes |1\rangle\langle 1|_C$$

Then the action of the quantum SWITCH is given by:

$$\mathcal{S}(\aleph_1, \aleph_2)(\rho \otimes \rho_c) = \sum_{i,j} W_{ij} \rho \otimes \rho_c W_{ij}^\dagger$$

Note that the system C works as a control qubit, that determines the order in which $\aleph_1$ and $\aleph_2$ are applied. In addition to that, observe that the quantum switch are interesting only when the Kraus operators of $\aleph_1$ and $\aleph_2$ are non-commutative, for otherwise, $\aleph_1$ and $\aleph_2$ will be applied in a definite order whatever the control qubit is.

2.2 Brief Introduction to Important Quantum Algorithms

we present here several useful (certainly not exhaustive) quantum algorithms that might be useful in the project. In particular, we first introduce 2 classical methods for estimating the "difference" between quantum states, and hopefully we can generalize such notions in the future. Following that, we introduce briefly the Grover's search algorithm. This algorithm together with other classical quantum algorithm such as quantum Fourier transform, phase estimation and density operator exponentiation, may serve as building blocks for more complicated algorithms.

2.2.1 *Swap Test and Euclidean Distance Estimation*

Using a swap gate S, one can estimate the "overlapping" between two pure states in terms of the standard inner product $\langle \cdot, \cdot \rangle$ on the state space (the Hilbert space $\mathbb{C}^n$). Swap gate does the following: $S|\psi\rangle|\phi\rangle = |\phi\rangle|\psi\rangle$ where $|\psi\rangle, |\phi\rangle$ are arbitrary ***state vectors*** in the state space. Swap test does the following: first adjoin an ancillary system C in the state $|0\rangle$, apply Hadamard gate to the system C, then apply controlled swap on $|\psi\rangle, |\phi\rangle$, controlled on system C. Then measure the system C on Fourier basis. The probability distribution of the result encodes the desired information.
Before the final measurement on the Fourier basis, the state of the system is given by:

$$\frac{1}{\sqrt{2}}(|0\rangle_C|\psi\rangle|\phi\rangle + |1\rangle_C|\phi\rangle|\psi\rangle)$$

Measuring the control system on the Fourier basis, the probability of getting output $+$ is thus:

$$\frac{1}{2}(1 + |\langle\phi|\psi\rangle|^2)$$

Repeatedly doing the swap test, one can obtain an estimate of the probability up to any desired precision, and thus able to estimate the overlapping between the two states.

A similar procedure can be used to estimate the Euclidean distance between $|\psi\rangle, |\phi\rangle$ [9]:

One assumes the existence of a ***quantum Random Access Memory*** [18], and two data vectors are stored in quantum form $|\psi\rangle, |\phi\rangle$ in the qRAM, by querying the qRAM, one obtains $\frac{1}{\sqrt{2}}(|\psi\rangle|0\rangle_C + |\phi\rangle|0\rangle_C)$. Then repeatedly measuring the control system C on Fourier basis, we can obtain an estimate of the Euclidean distance, since simple calculations shows that the probability of obtaining output $-$ is:

$$\frac{1}{2}||\psi\rangle - |\phi\rangle|^2$$

Generalizing these methods to extract information between a broader class of quantum resources will be one of the focus of this final year project.

2.2.2 Grover's Search and Search Based Quantum Clustering Algorithms [2]

Grover's search algorithm assumes the existence of an Oracle C, we simply assume that the oracle add a phase $-1^{f(x)}$ to the *index register* $|x\rangle$, where $f(x)$ is 0 when x is a hit, 1 otherwise.
Write $|\psi\rangle = \frac{1}{N^{\frac{1}{2}}}\sum_i |i\rangle$, where we assume $N = 2^n$ is the size of the search space. In addition, let the size of target space to be $M < \frac{N}{2}$. Then an Grover iteration is defined as:

$$G = (2|\psi\rangle\langle\psi| - \mathcal{I})O$$

Starting with $|\psi\rangle$, repeatedly apply the Grover Iteration G for $R = \lceil \frac{\pi}{4}\sqrt{\frac{N}{M}} \rceil = O(N)$ times, then measure on the computational basis, obtaining resulting state $|x_a\rangle$. Then x_a, with high probability, is in the target space.

The importance of this algorithm in this project is that a big class of clustering algorithms have a subroutine on searching for minimal or maximal, where by making use of Grover's search algorithm, one can obtain at least a quadratic speedup in expectation against the classical counterpart [13]. We shall also study these algorithms in better depth in this project.

3 Objective

Summing up the working directions mentioned in section 2. The objective of this final year project is to explore how a quantum learning machine can take advantage of a new quantum resource, namely the ability to query different processes in a indefinite order.

In better details, the contemporary goal of this project is to solve the following problem: Given accesses to a set of black boxed quantum resources, which can be unitary gates or certain quantum operations, making use of the device quantum switch, define and extract certain notion of differences between individual quantum resources.Then, with the aid of such notion of difference, cluster the black boxed quantum resources into clusters where in each cluster, the quantum resources are similar to each others.

One could imagine scenarios such as, someone, say, Alice, has accesses to other laboratories' quantum measurement devices where she can only send quantum states to be measured but cannot examine the detailed constructions of these devices. In this case, these measurement can be seen as block boxed quantum resources, then the question to ask is can we cluster/classify these quantum measurements?

Note that, this is not necessarily the only focus of the final year project. Upon completion of this task, the student may explore other areas in quantum computation and information, given the permission of the supervisor.

4 Methodology and Schedule

The objective will be studied mainly theoretically, i.e. mathematical arguments shall be the main form of research. This includes concrete mathematical definitions, mathematical properties of the definitions and proofs of correctness. However, actual implementation of the algorithms proposed are also feasible. This could be achieved by applying for an access to IBM's 5 qubit quantum computer [4].

A precise schedule is not applicable in this case, primarily because the project is not software engineering oriented. A tentative plan is the following:

October - November: Literature review and study on necessary knowledge.

November - January: Study on the task defined in section 3.

January - March: Implementation of codes and submission to IBM Q.

March - April: Preparation of final deliverable.

April - May: Preparation of presentation.

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