Quantum Superposition of Communication Channels – Project Plan

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This is the project plan of Mao Wenxu's Final Year Project instructed by Chiribella Giulio on quantum superposition of communication channels. In this project plan, I will first briefly describe the project together with its objective and expected deliverable. Then I will go on to talk about the methodology applicable and a project schedule. Finally, *for those who have little experience with quantum information,* ***appendix A*** *is prepared to lay the background for understanding this project.*

I. INTRODUCTION

A. Project Overview

Seventy years ago, Shannon established the field of Information theory in his legendary paper "A Mathematical Theory of Communication"[1], in which classical physics is adopted as the framework for communication: classical bits act as the carriers of information and they can be transmitted along well-defined communication line. By using quantum systems as carriers of information, quantum information theory have shown great potentials in enhancing transmission of information including increasing transmission rates[2], enabling unconditional security[3], and introducing new means of information transmission[4][5]. Most existing researches have been focusing on the quantum superposition of the messages transmitted. However, the possibility of sending the same message along a superposition of different communication channel is not yet fully explored[6]. This project will try to define the notion of superposition of quantum channels and will explore how the superposition of quantum channels can be used as a resource of communication[6].

B. Motivation

One motivation of this project is the power demonstrated by the superposition of **order** of communication channels.

It is shown that when the relative order of two channel usage is controlled by quantum entanglement, the two channel as a whole demonstrates greater ability to transmit information than any definite order of usage[5](Figure1).

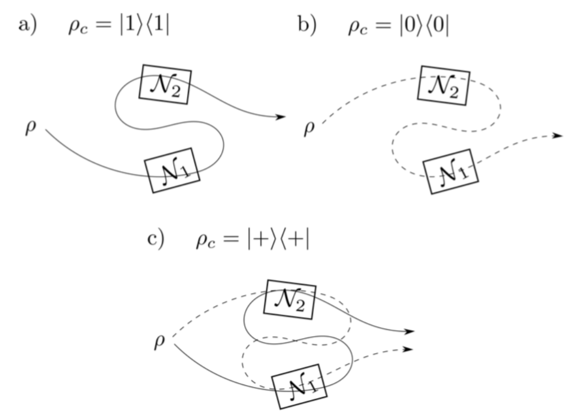

FIG. 1. Quantum Superposition of Order[5]

(a) shows a quntum system pass through N1 and then N2 (b) shows a quantum system pass through N2 and then N1 (c) shows a quantum system pass through N1 and N2 in superposition of order

The example given in [5] was that two depolarizing channel, which will fully randomized a quantum state if they are used in any definite order, could be used to transmit information when they are used in indefinite order controlled by an entangled state.

Such a result encourages one to wonder what kind of power entanglement of channel could bring? And this will be the core of this project.

II. PROBLEM DEFINITION

In this section, I will define the general outline of the setting that will describe the superposition of quantum channels (Figure 2). The rest of the project be based on this setup or its generalization.

We denote the Hilbert space of the message to be sent to be $\mathscr{H}^M$ and Hilbert space of a *control state* to be $\mathscr{H}^c$.

Suppose that there two quantum channels

$$\mathscr{N}_0 : \mathscr{H}^M \to \mathscr{H}^M$$

$$\mathscr{N}_1 : \mathscr{H}^M \to \mathscr{H}^M$$

$\mathscr{N}_0$ *and* $\mathscr{N}_1$ are composed into a larger channel $\mathscr{C}$ together with a *fixed control state* $\rho_c \in \mathscr{H}^c$ which is **independent** of the message. (In the case of this two channel senerio, $\mathscr{H}_c = \mathsf{Span}\{\langle 0|, \langle 1|\}$.) Later throughout this project report, $\mathscr{C}$ will be addressed as the **composite channel** while $\mathscr{N}_0$ *and* $\mathscr{N}_1$ will be called **sub-channels**.

$$\mathscr{C} : \mathscr{H}^c \otimes \mathscr{H}^M \to \mathscr{H}^c \otimes \mathscr{H}^M$$

The behavior of the channel is defined as following.

- When the control state is $|0\rangle\langle 0|$

  $\forall \rho \in \mathscr{H}^M, \mathscr{C}(|0\rangle\langle 0| \otimes \rho) = |0\rangle\langle 0| \otimes \mathscr{N}_0(\rho)$

- When the control state is $|1\rangle\langle 1|$

  $\forall \rho \in \mathscr{H}^M, \mathscr{C}(|1\rangle\langle 1| \otimes \rho) = |1\rangle\langle 1| \otimes \mathscr{N}_1(\rho)$

From this point above, the research questions of this project can be divided into following three:

- Mathematical characterization of superposition of quantum channels (developed from the above setting)
- Feasibility of realizing such a channel physically
- Calculation of channel capacity given different configuration (various sub-channels and control states)

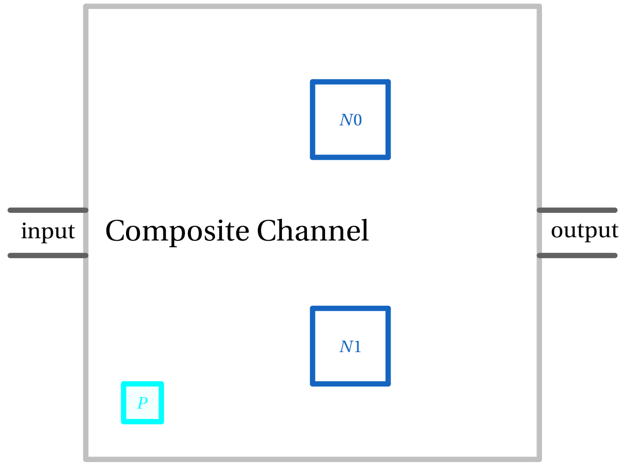

FIG. 2. Composite channel of $\mathscr{N}_0$ *and* $\mathscr{N}_1$

III. PROJECT PHASES

The aim of this project is to define the notion of quantum superposition of communication channel and explore how it could be used a resource for communication. Such an objective naturally divide the project into following phases.

A. Mathematical Model Formulation

The first task is to discover the general mathematical model corresponds to the superposition of two quantum channels. It is necessary to work out the mathematical representation of the composite quantum channel ($\mathscr{C}$ in Figure 2) consists of two sub-quantum channels ($\mathscr{N}_0$ *and* $\mathscr{N}_1$ in Figure 2). Only after that could I answer some critical questions like: how does the composite quantum channel look like mathematically; what properties characterize it and how can it enhance communication etc.

This phase should end after a clear mathematical representation of the above case is defined.

B. Investigate physical realization

Next problem that need to be solved is whether such a model can be realized physically and if so, how could it be constructed. The completion of this phase is marked by composing a feasible recipe for constructing such a channel.

C. Compute channel capacity in different configurations

After the mathematical model is well defined, I can move on to calculate the channel capacity of the superposition of quantum channels with some fixed configuration. Further more, it would be ideal to find some pattern on which configuration can result in best capacity given the set of quantum channels. Finally, some concrete examples are needed to demonstrate the advantage of superposition of quantum channels (eg: higher capacity, more robust to noise etc.)

The length and contents of this phase is rather flexible. The baseline consists the calculation of channel capacity under some fixed control states and sub-channels and some examples showing the advantage superposition could bring while further consideration of how to optimize such a configuration could also be included if time allows.

IV. METHODOLOGY

A. Literature Review

As this project is mainly theoretical, literature review will consist a significant part of the project. Currently known books and paper that might be helpful to this project is listed below, and the list is expected to grow as the project proceeds

- Quantum information theory[7]
- Quantum communication in a superposition of causal orders[8]
- Enhanced Communication with the Assistance of Indefinite Causal Order[5]
- A theoretical framework for quantum networks[9]
- Quantum Circuit Architecture[10]
- to be continued ......

B. Calculations and proofs

The other important components of this project are proofs and calculations. The key here is to learn the techniques and forms of proofs of others while conducting literature review in order to make my own proof rigorous and clear.

V. SCHEDULE AND MILESTONES

The project started at the beginning of September and will end in April next year. Approximately 7 months (28 weeks) are available for the project. Current plan of schedule is listed as following (open to change in future)

Time Period	Task
1st Sep - 20th Sep	1. Review previous course material 2. Project inception 3. Literature review
20th Sep - 30th Sep	1. Problem definition 2. Mathematical formulation 3. Write project plan 3.Build project website
1st Oct - 31th Oct	1. Refinement of mathematical formulation 2. Literature review 3. Start survey on physical realization
1st Nov - 20th Nov	1.Finish surveys on physical realization 2. Enrich knowledge base for computation part
20th Nov - 5th Dec	1. Halt for quiz/assignments/finals
6th Dec - 31st Dec	1. Calculate channel capacities 2. Find suitable examples 3. Wrap things up
1st Jan - 20th Jan	1. Prepare for first presentation 2. Write interim report
12th Jan - April	1. Time for anything unfished or new problems encountered 2. Write final report

Appendix A: A brief on quantum information

This section will brief you on some background knowledge of quantum information and computation and assume no previous experience except basic linear algebra. [11] However, this is neither a rigorous nor thorough view over quantum information. For more information, [7] or Wikipedia are recommended.

To understand quantum information, there are three major components: quantum states, quantum evolution and quantum measurement.

1. Dirac Notation and Hilbert Space

Dirac notation can make lives easier when comes to the representation of vectors and matrices. In Dirac notation, $|k\rangle$ can be regarded as a column vector and $\langle k|$ as a row vector and $\langle k|$ is the complex conjugate of $|k\rangle$. Then, $\langle k|k\rangle$ naturally represents the inner product between the two vector. $|k\rangle\langle k|$ is a rank one matrix.

$\mathscr{H}^n$ is a Hilbert space of dimension n where Hilbert space can be thought of as a vector space with inner product defined.

$\{|0\rangle, |1\rangle\}$ is a simple way to represent a *orthonormal basis* for $\mathscr{H}^2$.

2. Quantum States

a. *From classical to quantum bits*

Unlike classical quantum bit which can only be in state 0 or 1, a quantum bit (qubit), which has two perfectly distinguishable state $|0\rangle, |1\rangle (\||0\rangle\| = 1, \||1\rangle\| = 1)$, can have infinitely many states that can be characterized by the linear combination of the form

$$|\phi\rangle = \alpha|0\rangle + \beta|1\rangle,\ \ \alpha, \beta \in \mathbb{C}, |\alpha|^2 + |\beta|^2 = 1$$

In fact, the word "superpostion" describes just this phenomenon that quantum state can not only be in the perfectly distinguishable state $|0\rangle, |1\rangle$, but also be in the "combination" of these states.

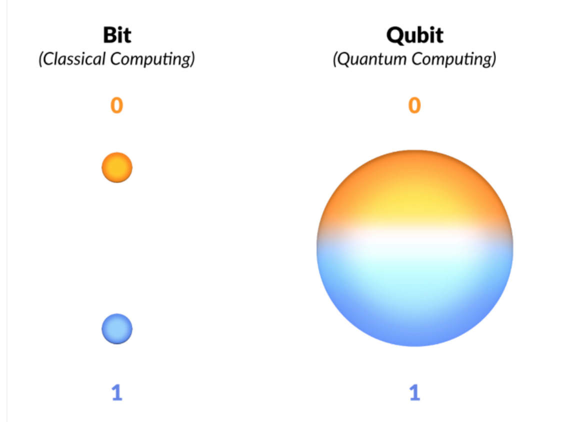

FIG. 3. qubit vs bit[12]

b. *General Quantum States*

All quantum states ρ can be put into the following form

$$\rho = \sum_i p_i |\phi_i\rangle\langle\phi_i|$$

where

$$\sum_i p_i = 1$$

and

$$\langle\phi|\rho|\phi\rangle, \forall\phi$$

c. *Tensor Product*

Tensor product of Hilbert space A of dimension dimA and Hilbert space B of dimension dimB results in a Hilbert space $\mathscr{H}^A \otimes \mathscr{H}^B$ of dimension $dimA \times dimB$.

Tensor product of two operators of Hilbert space A and Hilbert space B give a operator in Hilbert space AB.

$$\forall A \in \mathscr{H}^A, B \in \mathscr{H}^B,\ \ A \otimes B \in \mathscr{H}^A \otimes \mathscr{H}^B$$

.

Some handy formulas for tensor product:

$$A \otimes kB = kA \otimes B = k(A \otimes B)$$
$$(A \otimes B)(C \otimes D) = AC \otimes BD$$

3. Quantum Channel

All changes to a quantum state can be described in terms of quantum channels.
All quantum channels (C) can be put in Kraus representation

$$C(\rho) = \sum_i C_i \rho C_i^\dagger,\ where\ \sum_i C_i^\dagger C_i = I$$

4. Quantum Measurement

To obtain information of an unknown quantum state, quantum measurement has to be performed.

A general quantum measurement is characterized by POVM (positive-operator valued measure).

POVM is a set of matrices $\{P_0, P_1 \dots P_n\}$ satisfies

1. $P_i \geq 0$
2. $\sum_i P_i = I$

For a measurement carried out with a POVM on state ρ, the possibility of each outcome i is

$$p_i = \mathrm{Tr}[P_i \rho]$$

[1] C. E. SHANNON, "A mathematical theory of communication," *The Bell System Technical Journal*, 1948.

[2] J. A. S. C. H. Bennett, P. W. Shor and A. V. Thapliyal, "Entanglement-assisted capacity of a quantum channel and the reverse shannon theorem," *Physics Review Letter*, 1999.

[3] C. H. Bennett, C. o. C. S. G. Brassard, and S. Processing, "Quantum cryptography: Public key distribution and coin tossing," pp. 175–9, 1984.

[4] C. C. Ì. R. J. A. P. C. H. Bennett, G. Brassard and W. K. Wootters, "Teleporting an unknown quantum state via dual classical and einstein-podolsky-rosen channels," *Physics Review Letter*, 1993.

[5] S. S. Daniel Ebler and G. Chiribella, "Enhanced communication with the assistance of indefinite causal order," *Physics Review Letter*, 2018.

[6] C. Giulio, "Final year project description." `http://example.cohttps://community.cs.hku.hk/web/csis0801/projectlist`, Jul 2018.

[7] M. M. Wilde, *Quantum information theory.* Cambridge University Press, 2013.

[8] . Sina Salek, 1 Daniel Ebler and G. Chiribella, "Quantum communication in a superposition of causal orders," 2018.

[9] â. Giulio Chiribella, âĹŮ Giacomo Mauro DâĂŹAriano and P. PerinottiâĂą, "A theoretical framework for quantum networks," *Physics Review Letter*, 2009.

[10] G. M. D. G. Chiribella and P. Perinotti, "Quantum circuit architecture," *Physics Review Letter*, 2008.

[11] Unless specified, all contents in this section have reference [7] and COMP3316 lecture notes.

[12] T. Bassan, "A brief introduction to quantum computing." `https://hackernoon.com/a-brief-introduction-to-quantum-computing-d21e578cb7ed`.