Interim Report for Final Year Project

A Game-theoretic and Algorithmic Study of the Toll Rates of Hong Kong Road Tunnels

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Abstract

Hong Kong is suffering from great traffic jams, the three crossing-harbor tunnels included. The good news is that not all three tunnels are congested. The western Harbor Tunnel is not fully used. Meanwhile, the Eastern Harbor Tunnel is facing less transportation than the Crossing Harbor Tunnel. By allocating traffic properly among the three tunnels, the congestion could be hopefully alleviated or even solved. This report explains the possibility and feasibility of re-allocating of the transportation by adjusting the toll rates of the tunnels. This report can hopefully offer a direction for solving the problem.

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Term	Meaning
POA	Price of Anarchy
CHT	Cross-Harbor Tunnel
EHC	Eastern Harbor Crossing
WHC	Western Harbor Crossing

1. Introduction

Hong Kong has been troubled from traffic jams, especially during peak hours. The cross-harbor tunnels system connecting Kowloon and Hong Kong Island is an epitome of the whole Hong Kong transportation situation.

Among the three tunnels, the Cross-Harbor Tunnel (CHT) carries the largest portion of traffic due to its convenient location and the lowest toll rate. The Eastern Harbor Crossing (EHC) charges the second lowest toll rate and attracts the second largest portion of traffic. Remaining traffic is carried by the Western Harbor Crossing (WHC).

Recently, both the CHT and the EHC are suffering from traffic congestion problem. According to the Legislative Council of Hong Kong, the transportation demand of the CHT and the EHC during the peak hours on weekdays has exceeded their design capacity by 77% and 38% respectively [1]. Given the fact that the WHC only faces a demand of 90% of design capacity, the congestion of the CHT and the EHC could be hopefully alleviated part of their traffic being redirected to the WHC.

The traffic could be redirected from the CHT and the EHC to the WHC by changing the toll rates of the three tunnels according to previous research. Back to the 1920s, marginal cost pricing came out, suggesting that each road user should pay for the delay she or he caused for all the users using the same road. [2] Further research shows that the inefficiency caused by the selfish actions of the drivers can be counteracted by efficient pricing strategy. [2] For the specific congestion problem of the cross-harbor tunnels in Hong Kong, HAI YAN models the problem and proposed a procedure to figure out how to price each tunnel under the equilibrium condition. [3] He also figures out how the equilibrium condition reacts to changing of toll rates. [3]

This project tries to model the problem of traffic congestion as a congestion game in the field of the Algorithmic Game Theory to lighten the traffic burden of the CHT and the EHC and optimize the performance of the cross-harbor tunnels system. In this model, the starting point and the ending point of each tunnel along with possible ways between the pair of points constitute a network. Each game player, known as the network user or driver, seeks to maximize their own profits and cares nothing about the performance of the whole network [4]. Given the traffic rate of all the possible edges in that network, the assignment of traffic to each edge will be found to minimize the sum of the travel time of each user (referred as total latency in the following part of the paper). This kind of assignment in this project is assumed to be mainly controlled by the toll rates charged by each tunnel. This project aims at finding a proper charging method to minimizing the congestion of the tunnel system.

The significance of the project is two-folded. On the one hand, it provides a detailed and up-to-date model close to the fact which can help understand the problem better. Hong Kong develops at a fast pace, so the models coming up twenty or ten years ago by Yan may not be suitable for the situation now. On the other hand, it could hopefully alleviate the congestion problem at a relatively lower price. In fact, Hong Kong government has been seeking for different methods of alleviating the congestion problem of the three tunnels. Different suggestions are proposed. Some people suggest that electronic toll booths should take place of manual toll booths so that the waiting time for payment is reduced and the total time needed for passing the tunnel is less [1]. Other people suggest that the road systems on the two endpoints of WHC need further improvement [1]. Then more drivers would be willing to choose the convenient WHC. Both suggestions require huge money investment. Comparatively speaking, changing the pricing system is an economic method.

In the following context, the report first introduces some technical concepts related to the project. Then the report elaborates the main methodology adopted by the project. After that, the work done is presented and evaluated. Next, the challenges and future work is discussed. Finally, a brief conclusion is offered.

2. Related Concepts

2.1. Non-cooperative Game

Non-cooperative game is competition between individual players caring about only their own benefits because of the absence of external authority. [5]

2.2. Nash Equilibrium

Nash equilibrium is a situation where no game player could benefit from changing only his or her own strategy when everything else stays the same in a no-cooperative game. [6]

2.3. Price of Anarchy (POA)

Price of anarchy refers to the ratio between the performance of the system when all game players choose their own most profitable strategy and the best possible performance of the system. In this paper, the POA refers the ration between the performance of the system at Nash equilibrium and the system's best possible performance.

2.4. Selfish Routing

Selfish routing in this report indicates the fact that all the drivers choose the best route for themselves.

3. Methodology

In this section, how to model the congestion problem as a congestion game will be introduced. Then the methods employed to figure out the answer will be discussed. Finally, how to evaluate the efficiency of the answer will be touched.

3.1. Model the Problem

Different models of the tunnel system according to the complexity of the related factors will be built. For each driver, the cost function is assumed to be mainly determined by the time used by the driver and the toll rates of each tunnel. That is,

$$\text{cost}(x) = f\,(\text{time, toll rates}).$$

There are two cases of the toll rates. On the one hand, I investigate the case where the toll rates are fixed. On the other hand, the case where the toll rates are dynamically adjusted to the number of drivers using the tunnels is investigated.

3.2. Calculation of Ideal Price

To figure out the ideal price, I should first try to prove the existence of the Nash equilibrium of each model, which means that the solution exists. Then I will try to figure out the price minimizing the total waiting time caused by the congestion by figuring out the minimum value of the functions in the section 2.1.

3.3. Evaluation of the Calculated Price

To evaluate the efficiency, I make comparisons between the equilibrium case and the optimal case.

For a fixed price x, I will first calculate the total latency of tunnels if everyone chooses the tunnel which is most profitable for himself or herself. Then I calculate the shortest latency of the traffic flow. Finally, price of anarchy (POA) of is employed to indicate the efficiency of the toll rates. If the computed POA is smaller than the present value, the pricing strategy can be considered as efficient.

4. Project Progress

4.1.Construction of the Project Model

For the model construction, this project uses the model put forward by Tim Rougharden in 2003 for reference. [8]

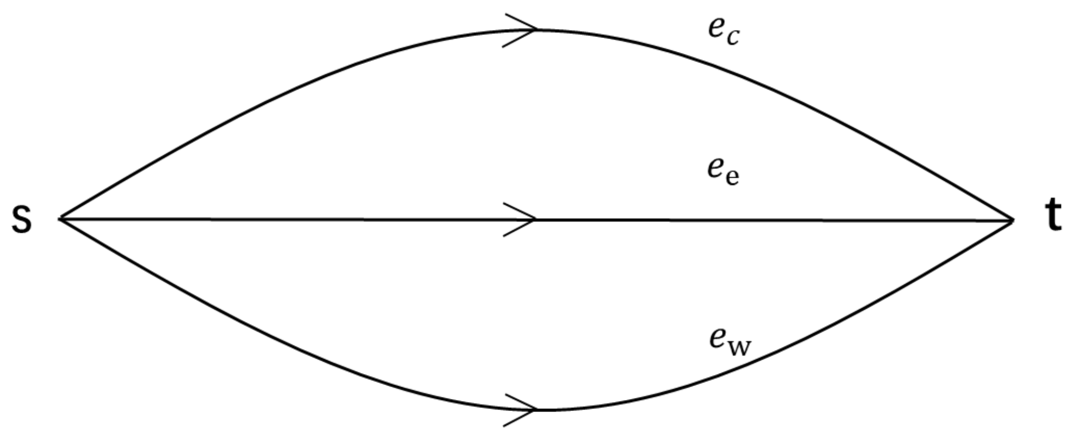

Figure 1: the model of the tunnel system

The tunnel system is modeled as a directed graph $G = (V, E)$ with the starting point *s* and the ending point *t* (see Figure 1). The three directed edge (e_c, e_e, e_h) in the figure represent the three tunnels (CHT, EHT, WHT) respectively.

I define the amount of traffic using each tunnel as *a flow on each edge*, namely, f_e. Then, I use a positive, non-decreasing, and continuous latency function l_e describing the delay caused by the flow of the edge. Thus, the total delay of the graph $\mathrm{L(G)} = \sum_{e \in E} l_e(f_e)$. The minimal-latency is represent as $\mathrm{L}(\cdot)$. [8]

Besides that, I represent the toll rate as t_e for each tunnel. The toll rates can either be fixed or change with the flow on the edge. [8]

I assume that all the drivers are selfish. They care only about their own benefits and care nothing about the whole performance of the tunnel system. Meanwhile, all drivers prefer shorter latency to longer latency as well as prefer lower toll rate to higher toll rate. Each driver has their own evaluation of the importance of the latency and the toll rates. For a driver d, an edge's score equals to $a_d(l_e(f_e)) + b_d(t_e(f_e))$, where the positive, non-decreasing, and continuous functions *a* and *b* indicate the importance of the two factors for the users respectively. The driver chooses the edge with the lowest score.

The project aims at minimizing $\mathrm{POA} = \mathrm{L}\big(\mathrm{G(E)}\big)/\mathrm{L}(\cdot)$.

4.2.Rationality of the Model

The tunnel system is modeled as a directed graph because the traffic from Kowloon to Hong Kong Island is independent of the traffic from Hong Kong Island to the Kowloon. Meanwhile, the traffic capacity of the three tunnels are evenly distributed between the two directions. Therefore, I can reveal the full information of the tunnel system by evaluating the model twice with different parameters of the two directions.'

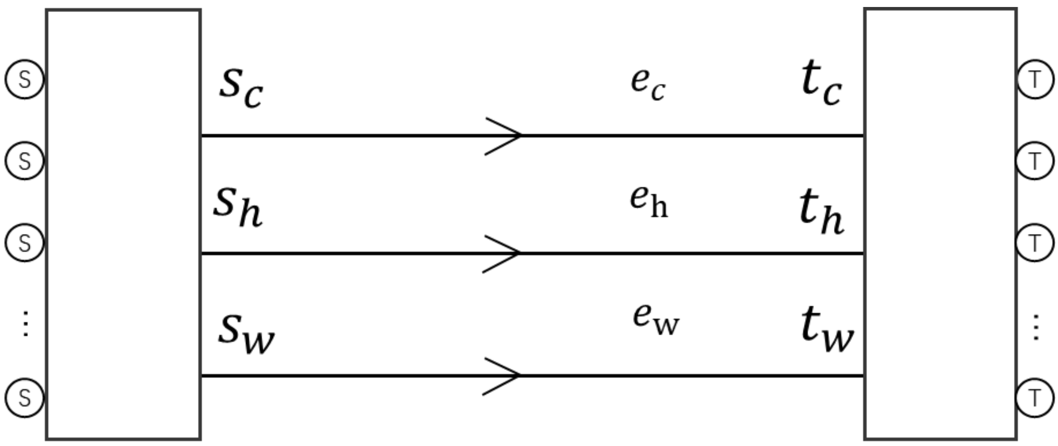

Figure 2: intuitive model of tunnel system

That the three edges share the same starting point and ending point is also reasonable. Intuitively, the model of the tunnel system should be Figure 2. The circles with S represent the starting points of the drivers and the circles with T represents the destinations of the drivers. They will choose any of the three tunnels, as indicating by the rectangle. However, the starting point and destination are fixed for each driver. Meanwhile, the condition of roads connecting these locations and the three tunnels do not change with the change of allocation of traffic over the tunnels. In other words, the latency of these roads does not change when the flows of the tunnels change. Therefore, the starting points, the destinations and the ways to the three tunnels can all be viewed as parameters in function $b(t_e)$. For example, for two drivers using EHC, the driver with longer way to the WHC is probably less willing to change to the WHC when WHC lowers its toll rate.

It is also reasonable to assume that the drivers are selfish game players because a driver usually knows little about other drivers. Meanwhile, we cannot expect there are always some people willing to sacrifice their own benefits for a better overall performance. Therefore, that the drivers care only about time and money makes sense.

The network reaches a Nash equilibrium when for each user and edge $e \in \mathrm{E}$:

$$a_d(l_{e^*}(f_{e^*})) + b_d(t_{e^*}(f_{e^*})) \leq a_d(l_e(f_e)) + b_d(t_e(f_e)),$$

where e^* is the edge the user chooses. Since such a Nash flow exists in every such network according to Schmeidler, [7] calculating and evaluating the POA of the graph is possible and meaningful.

4.3.Implementation of the Functions

A general graph of how to construct and evaluate the functions have also been carried out.

This model introduces two important functions. The first is the latency function $l_e(f_e)$, connecting the time required for passing an edge *e* with the flow on the edge *e*. The second is the function $a_d(l_e(f_e)) + b_d(t_e)$, reflecting a typical game player's degree of care to the latency and the toll rate of an edge.

The project first investigates homogeneous users, which is in progress. The homogeneous users have the same a_d and b_d functions while the functions vary for heterogeneous users. In the simplest homogenous hypothesis, the users only care about the fixed toll rates of the edge and all the users choose the cheapest tunnel. In a more complex one, the users care about the changing toll rates. In the most complex one, the user cares about both the latency and the changing toll rates. Then the project investigates the heterogeneous users in a similar order.

Each of the edge has their own latency function. These latency function holds for all the hypotheses. For each hypothesis, the important role of directing traffic played by toll rate is discussed. Then, I try to seek a set of toll rates minimizing $\mathrm{POA} = \mathrm{L}\big(\mathrm{G}(\mathrm{E})\big)/\mathrm{L}(\cdot)$.

5. Challenges

To be closer to the real situation, more data about the three tunnels are required. Up to now, the functions are mainly based on the assumption. However, the Hong Kong government does not offer much information in this aspect. In the future work, I plan to collect data for the latency function by field trip. By collecting the number of vehicles passing the entrance of the three tunnels and the waiting time of the vehicles, a more accurate latency function can be discovered. For the score function of the users, questionnaire survey can help a lot.

6. Conclusion

This project draws a thorough picture of the Hong Kong cross-harbor system and seeks how to solve the congestion problem of the tunnels by changing the toll rates. By modeling the system as a congestion game, the project proves that toll rate is a meaningful tool in redirecting part of the traffic of one tunnel to other tunnels. Besides that, by adjusting the toll rates, the whole congestion problem can be hopefully alleviated.

At this stage, the model of the harbor system has been constructed. The basic methods to further analyze the model have been finished too. Future work will focus on collecting data about the real situation and evaluating the congestion model.

This project assumes that the government can freely change the toll rates of the three tunnels. However, changing the toll rate of the WHS is not easy because the right to operate WHS belongs to private companies rather than the Hong Kong Government. In other words, if the pricing strategy suggests the toll rates of WHS decreases, the companies owning the operation rights might refuse to accept the suggestion. Even though the final result of the project might not be applied to the reality due to the limitations mentioned, it can at least offer a direction for solving the problem.

7. References

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