A Game-theoretic and Algorithmic Study of
the Toll Rates of
Hong Kong Road Tunnels

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1. Introduction

This section draws a brief image of the whole project.

1.1. Background Information

The transportation system of Hong Kong is highly congested. For instance, during the rush hour in the morning, the average driving speed of Hong Kong Island and Kowloon is about ten kilometers per hour and twenty kilometers per hour respectively (Legislative Council of Hong Kong, 2017). Meanwhile, due to the high building density of Hong Kong, the construction of new roads often faces resistance. The annual growth rate of total road length is estimated to be about 0.3% until 2020, while the growth rate of vehicles is predicted to be 3.4% (Legislative Council of Hong Kong, 2017), which indicates that the congestion problem will deteriorate without effective measures adopted.

Even though the congestion problem is severe now, not all roads in Hong Kong have reached the point of saturation. Therefore, the congestion problem can be alleviated by directing part of the traffics from the busy road to the relatively free road.

The cross-harbor tunnels system is an epitome of the whole Hong Kong transportation situation. Hong Kong has three cross-harbor tunnels. The Cross Harbor Tunnel (CHT) is connected to the most complete transportation network and charges the lowest toll rate. The Eastern Harbor Crossing (EHC) has the second lowest toll rate while the Western Harbor Crossing (WHC) has the highest rate. Recently, the transportation demand of the CHT and the EHC during the peak hours on weekdays has exceeded their design capacity of 77% and 38% respectively, while the WHC faces a demand of 90% of design capacity (Legislative Council of Hong Kong, 2017). Theoretically, the WHS can lighten the congestion of the CHT and the EHC by accepting part of their traffic flow.

Among different regulatory methods, road pricing is regarded as an effective one (Yan, 1996). By adjusting the toll rates of the three tunnels, it is possible that the traffic of the CHT and EHC will be diverted to WHC.

1.2. Previous Works

Some progress has been achieved in the field of selfish road balancing where every driver chooses the road maximizing his or her own benefits. Back to the 1920s, marginal cost pricing came out, suggesting that each road user should pay for the delay she or he causes for all the users using the same road (Pigou, 1932). Further research shows that the inefficiency caused by selfish actions can be counteracted by efficient pricing strategy (Beckmann, 1956). The theory was developed to the situation where both the road conditions and vehicles are heterogeneous (Cole, 2003). An efficient pricing strategy for different roads and different vehicles meanwhile came out (Cole, 2003).

For the specific congestion problem of the cross-harbor tunnels in Hong Kong, HAI YAN models the problem and proposed a procedure to figure out how to price each tunnel under the
equilibrium condition (Yan, 1996). He also figures out how the equilibrium condition reacts to changing of toll rates (Yan, 1996).

1.3. Scope of the Project

This project aims at alleviating the congestion problem of the three cross-harbor tunnels in Hong Kong and finally minimizing the congestion. I will first try to model this problem as a congestion game in the field of game theory. Different models will be proposed in this project, following the rule from simple to complex. As the model becomes more complex, the model becomes closer to the real situation. For each model, I will prove that an equilibrium of the game exists, that is the ideal toll rates exist. After that, a pricing strategy will be computed. Finally, the efficiency of the strategy will be evaluated.

1.4. Importance of the Project

The importance of the project is embodied in two aspects.

Firstly, it provides a detailed and up-to-date model close to the fact. With this model, the problem could be better understood. Hong Kong develops at a fast pace, so the models coming up twenty or ten years ago may not be suitable for the situation now. Besides that, since this project is based on fact, the work would not be feasible if it failed to accurately describe the fact.

Secondly, it could hopefully solve the congestion problem at a relatively lower price. In fact, Hong Kong government is seeking for different methods of alleviating the congestion problem of the three tunnels. Different suggestions are proposed. Some people suggests that electronic toll booths should take place of manual toll booths so that the waiting time for payment is reduced and the total time needed for passing the tunnel is less (Legislative Council of Hong Kong, 2017). Other people suggests that the road systems on the two endpoints of WHC need further improvement (Legislative Council of Hong Kong, 2017). Then more drivers would be willing to choose the convenient WHC. Both suggestions require huge money investment. Comparatively speaking, changing the pricing system requires nearly zero investment and is an economic method.

1.5. Deliverables

This project is mainly about theoretical analysis. A detailed report about the ideal pricing strategy under different congestion game models will be finished at the end of the project. If time allows, a program of calculating the price according to traffic volume might also be released.

1.6. Outline of the Following Report

In the following context, I will first demonstrate the methodology used by this project, inducing how to model the congestion problem and basic tools for analyzing the problem. Afterwards, the schedule of the project will be discussed. Finally, I will point out some limitations of the project.
2. Methodology

In this section, how to model the congestion problem as a congestion game will be introduced. Then the methods employed to figure out the answer will be discussed. Finally, how to evaluate the efficiency of the answer will be touched.

2.1. How to Model the Problem

Different models of the tunnel system according to the complexity of the related factors will be built. For each driver, I assume he or she have the same cost functions for each tunnel \( C(x) \) and the driver will choose the tunnel with the least cost.

The cost function is assumed to be mainly determined by the time used by the driver and the toll rates of each tunnel. That is,

\[
\text{cost}(x) = f(\text{time}, \text{toll rates})
\]

For the time, two situations are taken into consideration. In the first case, I only consider the time of passing the tunnel. In the second case, I will consider the total time a driver spends from the departure point to the destination.

There are also two situations of the toll rates. On the one hand, I investigate the case where the toll rates are fixed. On the other hand, the case where the toll rates are dynamically adjusted to the number of drivers using the tunnels is investigated.

Both toll rates and passing time are related to the number of drivers using the tunnels. Tow cases are considered. The situation where the total number of users is fixed and the situation where the total number of users fluctuates have different impact on the time and price.

2.2. How to Figure out the Ideal Price

To figure out the ideal price, I should first try to prove the existence of the Nash equilibrium of each model, which means that the solution exists. Then I will try to figure out the price minimizing the total waiting time caused by the congestion by figuring out the minimum value of the functions in the section 2.1.

2.3. How to Evaluate the Efficiency of the Price

To evaluate the efficiency, I make comparisons between the equilibrium case and the optimal case. For a fixed price \( x \), I will first calculate the total time \( \text{equ}(x) \) required for passing the tunnels if everyone chooses the tunnel which is most profitable for himself or herself. Then I calculate the shortest total time of the traffic flow, recorded as \( \text{opt}(x) \). Finally, price of anarchy (POA) of is employed to indicate the efficiency of the toll rates. Price of anarchy refers to the ratio between the performance of the system with rational players and the best possible performance of the system. In this problem, \( \text{POA} = \frac{\text{equ}(x)}{\text{opt}(x)} \). If the computed POA is smaller than the preset value, the pricing strategy can be considered as efficient.
3. Schedule

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<thead>
<tr>
<th>Date</th>
<th>Tentative schedule</th>
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<tbody>
<tr>
<td>September 1, 2018</td>
<td>Finish the detailed project plan.</td>
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<tr>
<td></td>
<td>Launch project website.</td>
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<tr>
<td>October 30, 2018</td>
<td>Finish the model with fixed number of drivers, fixed toll rates and the time passing the tunnels only.</td>
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<tr>
<td>November 30, 2018</td>
<td>Finish the model with fixed number of drivers, fixed toll rates and the total travelling time.</td>
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<tr>
<td>December 30, 2018</td>
<td>Finish the model with fixed number of drivers, the total travelling time and the dynamic pricing strategy.</td>
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<td>January 7, 2019</td>
<td>Deliver interim presentation.</td>
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<td>January 20, 2019</td>
<td>Finish the interim report (basic model and evaluation included).</td>
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<tr>
<td>February 28, 2019</td>
<td>Finish the model with dynamic number of drivers, the total travelling time and the dynamic pricing strategy.</td>
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<tr>
<td>March 30, 2019</td>
<td>Explore the possible improvement of the project.</td>
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<tr>
<td>April 4, 2019</td>
<td>Deliver the final report.</td>
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<tr>
<td>April 15, 2019</td>
<td>Deliver the final presentation.</td>
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4. Limitations of the Project

The project mainly has three limitations.

Firstly, I assume that the government can freely change the toll rates of the three tunnels. However, changing the toll rate of the WHS is not easy because the right to operate WHS belongs to private companies rather than the Hong Kong Government. In other words, if the pricing strategy suggests the toll rates of WHS decreases, the companies owning the operation rights might refuse to accept the suggestion.

Secondly, this project assumes the number of drivers is fixed. However, the total number of drivers might vary as the toll rates fluctuates. For example, if the price of the three tunnels decreases, the people using other methods of cross the harbor might choose to drive instead. Then the whole congestion problem will be worse than the project’s expectation. Similarly, if the price of the three tolls is too high, even though fewer people choose to drive and the congestion reduces, other modes of transportations will inevitably face greater pressure.

Finally, the drivers in the model have the same cost functions. Whereas, drivers driving different vehicles might care about different factors and have different cost functions.
5. Conclusion

This project aims at finding out a suitable pricing system of the three cross-harbor tunnels of Hong Kong, so that the congestion problem of the tunnels could be alleviated and minimized. To achieve this, I model this problem as a congestion game in the field of game theory. Then I explore the existence of the ideal pricing system and figure out what it exactly is. Even though this project might not be able to provide a very accurate model because of the constraints of real factors, this project could hopefully provide some constructive suggestions for alleviating the congestion problem.
6. References


