BEng(CompSc) Final Year Project
Computational meteorology: study of weather system evolution using radar data
Interim Report

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Abstract

Accurate measurements of the wind field are crucial to make a reliable prediction on the evolution of tropical cyclones. One of the most commonly used measurement apparatus for wind field is the Doppler radar. Algorithms for extracting the wind profile from radar data were intensively developed in the last century. However, these algorithms are have different limitations, thus fail to provide a robust estimation of the wind field in a high resolution manner. In this project, we attempt to develop a new algorithm that aims at providing uncertainty estimations of the measurements with unlimited spatial resolution. The algorithm will be tested against computer simulation, thus to compare with the current algorithms. In the current stage, we attempted to construct a radar signal generator for testing the new algorithm. We have formulated the simulation problem mathematically and implement it with Python. In the upcoming weeks, we will focus on testing the validity of our simulator and apply it to simulated weather systems.
Acknowledgements

I sincerely thank the continuous help from my supervisor, Dr CL Yip, who provide useful opinions and insights to the topic.
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Chapter 1

Introduction

1.1 Background

Tropical cyclones (TC) are storms developed in the lower atmosphere with a rapid spiraling wind field. Each year, more than 10 tropical cyclones impact the Eastern Asia, causing irrepairable damages.

In order to prevent the damages of TCs, it is crucial to have accurate real-time predictions of the evolution of TCs. The meteorology community has developed many strategies to predict the evolution of TCs (to name a few, see [1] [2] for a review). These predictions rely on accurate measurement of the physical parameters of TCs, as errors in measurements would propagate through the prediction algorithms. On the other hand, unlike other meteorological events which occur on the land side, TCs form over the ocean, at which it is difficult to place atmosphere monitoring instruments. Instead, it is more convenient to use remote sensing technologies to probe the nature of TCs on the ocean. For example, meteorological satellites are sent to the space to monitor the evolution of weather systems. However, it is difficult to measure the TC parameters precisely with meteorological satellites. One
major challenge is turbulence in the atmosphere, which limits the spatial
resolution of the satellite-based measurements. An alternative option is the
Doppler radar. When the TCs get closer to the land and enter the detection
range of doppler radar, it becomes possible to use doppler radars to probe
the nature of TCs.

1.2 Doppler Radar

Doppler radars are popular for monitoring the atmosphere in a remote
manner. The working principle of doppler radars relies on the doppler effect.
A detailed introduction was provided by, for example, Raghavan (2003) [7].
Based on his work, here we briefly outline the working principle of doppler
radars. The doppler radars emit radio wave, a kind of long wavelength elec-
tromagnetic wave. Upon hitting hydrometeors, part of the incidenting radio
wave would be reflected back (known as echos) from the hydrometeors. Dif-
ferent types of hydrometeors have different optical properties, like reflectance
and refractive index, therefore they respond differently to the radio wave.
The doppler radar would then listen to the echo. The echo as a function of
time would contain information about the remote hydrometeor. For exam-
ple, the velocity of the hydrometeors, along the line of sight of the radar (as
known as radial velocity), could be retrieved from the doppler spectrum of
the echos. As the hydrometeors recede away from (approach to) the radar,
the wavelength of the reflected radio wave would be elongated (shortened).
This effect is known as the doppler effect, and could be effectively used in
measuring the velocity of winds. An example illustrating the information
provided by a Doppler radar is provided in figure 1.1. The figure shows a
processed radar image. One goal of this project is to develop an algorithm
to generate this kind of image from the raw radar data.
Figure 1.1: This is a typical radar image showing the precipitation in a local region. Above shows Typhoon Imbudo approached Hong Kong on 24 July 2003. Note this image is not the raw data from the radar; instead, it is the estimated precipitation profile from the raw radar signal. Credits: Hong Kong Observatory.
1.3 Review of Literature

A number of algorithms have been developed to retrieve the wind velocity from the radar signal, among which the most popular algorithms are Velocity Azimuth Display (VAD) and Volume Velocity Processing (VVP). VAD was developed by Lhermitte and Atlas in 1961 [3]. This algorithm can be divided into two parts: the radar scanning strategy and the extraction of wind profile from the radar signal. The radar is operated, so that it first scans through the azimuth angles at a fixed elevation angle, then increases the elevation and perform another azimuthal scan. The process is repeated until the sequence of scannings cover the whole region of interest. Assuming that the wind field and the velocity of hydrometeors are homogeneous in the horizontal direction, it was shown that, the measured average radial velocity of the wind field is a sine function of the azimuthal angle. By performing a multi-variable fitting, the wind profile could thus be inferred. Later work by Browning and Wexler (1968) [4] further released the assumption of an uniform horizontal profile. It was shown that, upon expanding the wind velocity to the linear order, the parameters describing the wind are contained in the first few Fourier coefficients of the VAD scanned doppler spectrum. The VVP algorithm as proposed by Waldteufel and Corbin (1979) [5] also make use of the linear approximation to reconstruct the wind profile from radar signal. These algorithms are proven to be practical and reliable in the subsequent experiments. Based on these algorithms, recent research focus on extending and modifying these algorithms for achieving better accuracy. For instance, Gao et al (2004) [6] used the further information from the gradient of the radial velocity in the VAD analysis. On the other hand, machine learning, being the state of art in data processing, also emerge as a new tool for retrieving wind profile from doppler radar. For example, Kon and Tanaka et al (2011) [8] demonstrate the use of EM algorithm to extract the wind profile.
The aforementioned algorithms share a few common limitations, leading to their insufficiency to measure the wind velocity accurately. Firstly, these algorithms based on the linear approximation of wind field. The wind field of the TCs are far from an ideal fluid, and is compressible in nature. Results in fluid mechanics show that, turbulence could occur in such a wind field. This effect is far from linear, and it is expected that a linear approximation cannot handle this deviation from linearity. Secondly, these algorithms do not utilize the correlations between wind velocity of neighboring regions. The continuity equation of fluid demonstrate that, the mass flow should be conserved. With this elementary restriction, a constraint is imposed on the spatial variation of the wind field. This piece of extra information may be capable to improve the wind field measurement precision.

1.4 Scope and Deliverables

In this project, we aim to review the recently available algorithms for extracting the wind velocity from doppler radars’ data, and develop a new algorithm to recover the wind velocity field more accurately. With a focus on the drawbacks of the currently available algorithms, we aim to provide a more robust and reliable reconstruction of the wind profile. On the other hand, in order to facilitate a Bayesian treatment when using the reconstructed wind profile to forecast the evolution of TCs, we also demand our proposed algorithm being capable of providing the reconstruction uncertainty. As a side product of our goal, a radar signal simulator will be constructed. This radar signal simulator is responsible for providing test data for the evaluation of our algorithm’s performance. We expect that this simulator would be useful in future meteorology research.
CHAPTER 1. INTRODUCTION

1.5 Organisation of the Report

The remaining part of the report is organised as follow. In chapter 2 we outline the methodologies adopted in the project, including the generation of radar data for testing the algorithms, and the new framework for performing data analysis. The general data pipeline would also be illustrated. Next, for the purpose of monitoring the progress of the project, a draft timeline of the project would be provided in the chapter 3. The recent progress on the project would be presented in chapter 4. Finally, we summarize and discuss the limitations of our project, and thus propose corresponding improvements for future research in chapter 5.
Chapter 2

Methodology

The whole project consists of a number of parts. In order to evaluate the performance of our proposed algorithm, we need to run our algorithm on some test data. We can then compare the estimations from the current algorithm and our proposed algorithms on the same set of test data, so as to develop an insight on the strengths and inadequacies of different algorithms. Below we would illustrate the strategy to obtain test data. After getting sufficient test data, the radar signal inference algorithm can be developed. There are many ideas that are suitable for testing. In particular, we may extending the currently adopted algorithm, MM5. On the other hand, it is also possible to use machine learning for handling the task. Finally, the information field theory (IFT), being a relatively new technique, also demonstrated the possibility of handling this kind of signal inference problem. Below, we will outline these approaches.

2.1 Generation of Test Data

In order to evaluate whether the proposed algorithm is capable to estimate the wind profile accurately, a comparison between the true wind profile and the estimated wind profile is essential. However, it is impossible to get the underlying wind profile from observation and measurements, as these records
are subjected to systematic bias induced by the limitation of measurement instruments. On the other hand, stochastic fluctuations could exist in TCs, so deviations from the ideal signal is inevitable. Using these observed data to evaluate the performance of our algorithm would effectively make our algorithm bias towards existing algorithms. In the meantime, although the use of unsupervised machine learning algorithms is able to recover the wind profile without the complete knowledge of the true wind field, these algorithms usually require known and quantifiable measurement systematics (for example, the Wiener filtering algorithm). In general, unsupervised algorithms exploit the use of the expectation-maximization (EM) algorithm, and weak constraints from the underlying true signal are required to obtain a meaningful expectation (i.e.: the E-step of the EM algorithm). Therefore, it is preferable to compare the underlying true wind field with the radar-reconstructed wind field.

### 2.1.1 Generation of Mock Tropical Cyclones

As it is impossible to obtain the true wind field, the best approximation of the true wind field would be determined by computer simulation. There are numbers of open source weather simulation codes available online. For example, the Weather Research and Forecasting Model (WRF) as constructed by Skamarock et al (2008) [9] is able to provide a reliable and high resolution mesoscale simulation of TCs. However, the computational power required for a run of this model is high, and is therefore not suitable for the generation of a large set of simulated TCs dedicated for the training and evaluation of the wind field reconstruction algorithm. On the other hand, the Fifth-Generation NCAR / Penn State Mesoscale Model (MM5) [10], despite possessing a long history, is able to simulate TC systems with a relatively cheap computational resource. Therefore, in this work we would rely on MM5 simulation to generate mock TCs in the computer. To illustrate the information provided by a MM5 simulation trail, we present a simulation result provided by the MM5 official website [10] in figure 2.1. Instead of simulating a TC system, the
Figure 2.1: A typical MM5 run in the forecasting mode. This simulation predicts the precipitation in the United State. The more red the color it is, the heavier rainfall it implies.
simulation was run in the forecasting mode, and the precipitation information was presented. This illustrates the ability of MM5 to generate simulated weather system.

2.1.2 Computation of Radar Response

Following the simulations of TC systems in computers, the corresponding radar signal obtained from these simulated systems could be computed. During this process, instrumental effects and random fluctuations could be added in the signals. To achieve this, we can adopt the ray-tracing algorithm, a commonly used algorithm in computer graphics. In computer graphics, this algorithm is used to simulate the propagation of light rays and thus generate shadows with a given light source. Note the radar emits radio wave, and the optical interaction between the atmosphere and the radio wave can be used to infer the signal. Furthermore, similar to visible light, radio wave is also an electromagnetic wave. Therefore, it becomes possible to use the ray-tracing algorithm to simulate realistic radar signals from the TC systems. There are existing software for performing such tasks, for example, Cheong et al (2008) [11] demonstrated a three-dimensional time series radar signal simulation; Lischi et al (2014) [12] illustrated an example for simulating mock radar signal for polarimetric doppler radar. However, the source code for such simulations is often unavailable, and is offered as a commercial product (e.g.: Rockwell Collins [13]). Therefore, instead of paying for commercial product, it is possible for us to follow previous published result, and make the simulation by ourselves.

2.2 Analysis of Radar Signal

Signal processing is a classical task in information engineering. Many algorithms have been developed for recovering the underlying signals from the noisy data. These algorithms are often based on the assumption of a Gaussian
uncorrelated noise (i.e.: white noise). Upon careful treatment to marginalize the effect of noise, there are different approaches to process the filtered signals. In particular, there are two major groups of algorithm, namely a supervised approach and an unsupervised approach. The former requires fitting the filtered signal with a theoretically motivated model, and minimize the loss function between the signal and the model. Typical applications in the meteorology community fall into this category. For example, the Velocity Azimuth Display (VAD) algorithm [3] fits the radar signal to a sine curve, where the sine curve model is determined from the geometry of radar and the wind field.

2.2.1 Machine Learning

Influenced by the recent advances in computation technologies, a large class of machine learning algorithms is now possible to run on computers. As mentioned in the introduction, the current traditional algorithms is not able to utilize the information of correlating the spatial distribution of the wind profile. On the other hand, the convolution neural network (CNN) proposed by Krizhevsky et al in 2012 [14] demonstrated the power of a convolution filter to capture the local spatial correlations. Inspired by his work, we would also try a convolution based algorithm to utilize the spatial correlations information in the estimation of wind field. By considering the conservation laws and symmetry of TC systems, weak constraints could be put on the estimated wind field, and thus enabling a unsupervised strategy (the EM algorithm) to diminish the size of the parameter space spanned by the wind field parameters. Experiments on this kind of algorithms will be carried out in this project.

2.2.2 Information Field Theory

Despite the possibility to apply machine learning to tackle the radar-wind field reconstruction problem, a new data analysis framework, the Informa-
CHAPTER 2. METHODOLOGY

Information Field Theory (IFT) is promising to address this kind of signal influence problem. The IFT was originated in the astronomy community by Ensslin (2009) [15]. This new technique is based on Shannon’s famous work on the information theory, and allows a Bayesian inference of continuous varying field under significant uncertainty. Originally, this algorithm was used to infer the nonlinear growth of structures in the universe from telescope signals. Following his work, his students Selig et al (2013) [16] presented an attempt to apply this algorithm to astronomical image processing, where the photons received by the telescope are considered to be sampled from a scalar field (the sky). Instead of treating the corresponding digital images on discretized pixels, they attempt to use a continuous parameterization. By doing this, the spatial correlations between neighboring points are utilized. In Ensslin’s unpublished lecture notes (2018) [17], he also outlined an extension of the IFT on analysing vector field data (or general, tensor field of any rank). The wind velocity profile is intrinsically a continuous vector field. By considering this nature, the IFT would be a promising tool in the reconstruction of wind field from radar signals. As the computational aspect of IFT relies heavily on the quantum field theory, a framework for doing particle physics, the computer science community is not familiar with this theory, so that its ultimate applications in digital image processing is hidden. One aim of this project is to experiment with the possibility of applying this useful framework in digital image processing and other data analysis task, and bring this technology into the CS community.
Chapter 3

Schedule

The project consists of numerous elements, and the tasks are divided and planned as follows. First of all, the development of algorithm relies on the cross validation between a realistic radar signal and the underlying wind field which generates such a radar signal. Therefore, the first few months will be dedicated to the construction of a radar signal simulator. As guided by previous studies, we expect the simulator could be finished at the end of 2018. The remaining time will be devoted to exploring different algorithms. As a start, experiments on understanding the recently used VAD algorithm will be carried out, which will thereafter used as a baseline of other algorithms. After gaining enough experience with VAD, we will start the integration of machine learning algorithms, and test the performance of algorithms which relies purely on statistics and without involving physical knowledge. Last but not the least, we will attempt to integrate IFT in the wind profile estimation. To reserve buffer time for organising the data and the writing the report, the last month would be devoted to wrapping up the project and the writing of the FYP thesis. The timeline of the project is summarized in table 3.1.
3.1 Delayed work

According to the planned timeline, we should have completed the radar simulator and performed the required testing by the end of December. Unfortunately, due to unexpected difficulties encountered in the implementation of the radar simulator, we are lagged behind the schedule. By the submission time of this report, the radar simulator has been partially finished: the core function and simulation has been implemented on time, yet we still need testing the simulator against simulated weather system. To adapt for the delay, we have to postpone the schedule accordingly. On the other hand, after the discussion of the project supervisor on 18th Jan 2019, we have decided to use the real data from the Hong Kong Observatory (HKO) to design and test our algorithm. The implemented radar simulator will mainly used to double check if the conclusion drawn by the real data and the algorithm are consistent. In the period where we are waiting for the administration work required to get the HKO data, we will keep improving our radar simulator. In this way, here we proposed to compress the work on testing and experimenting with traditional algorithms (VAD) in February. The original schedule can therefore be retained after February.
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<th>Deliverables</th>
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<td>30 Sept 2018</td>
<td>Confirmation of project scope and direction. First round literature review.</td>
<td>Project plan and website</td>
<td>completed</td>
</tr>
<tr>
<td>30 Dec 2018</td>
<td>Implementation of radar signal simulator</td>
<td>Radar signal simulator</td>
<td>in progress with delay</td>
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<tr>
<td>5 Jan 2019</td>
<td>Organization of previous result</td>
<td>First presentation</td>
<td>completed</td>
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<tr>
<td>20 Jan 2019</td>
<td>Testing VAD algorithm and its variants</td>
<td>Interim report; Presentation</td>
<td>pending</td>
</tr>
<tr>
<td>28 Feb 2019</td>
<td>Experiment with machine learning based algorithms</td>
<td>Test data</td>
<td>pending</td>
</tr>
<tr>
<td>31 Mar 2019</td>
<td>Experiment with IFT</td>
<td>Algorithm for wind profile retrieval</td>
<td>pending</td>
</tr>
<tr>
<td>14 Apr 2019</td>
<td>Organization of results and project wrap up</td>
<td>Final report; New algorithm</td>
<td>pending</td>
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Table 3.1: The timeline of the project.
Chapter 4

Radar Simulator

To validate the algorithms that we are going to develop in this project, it is essential to compare the algorithm-reconstructed parameters with the ground truth (the correct answer). In machine learning, this is done by running the machine learning algorithm on the test data, where the ground truth is contained in the testing set (for supervised learning). However, in meteorology, it is difficult to determine the ground truth, as the true atmospheric conditions could only be extracted via measurements, yet different kind of measurements would suffer from different kind of systematic bias. In contrast, we can generate a simulated weather system in the computer, in which we have complete knowledge on the underlying parameters that are responsible for generating such atmospheric conditions. We can thereafter simulate the radar response on the simulated weather system. Finally, we can compare the algorithm-reconstruction of the simulated radar response with the simulated weather system for validating the robustness of the algorithm.

In this chapter, we will first discuss the underlying assumptions of our proposed approach of using radar simulator. Arguments on the validity of our assumptions, and the following resolutions, would be provided in section ??.
4.1 Assumptions

The most critical assumption to make this approach make sense is that, we have assumed the simulation is consistent with the real world scenario, up to a parameterizable noise. This relies on the accurate modelling of the underlying physical processes that governs the evolution of the atmosphere and the propagation of radar signals in such an atmosphere. Unluckily, it is not practically feasible to incorporate all the known physical processes in the simulation, because simulating low-level details, despite boosting the accuracy of the simulation, require unrealistic computational resources. On the other hand, the research community is still uncertain on whether the current understanding of those physical processes are correct. To make our assumptions valid, in this work we formulate these uncertainties by a probabilistic noise distribution. From an information-theoretic point of view, noise could be considered a description of lack of knowledge about a system. Of course, it is yet another critical issue to parameterize the noise distribution. According to the principle of (information) entropy maximization, Gaussian distribution is the most probable noise distribution if we only model the mean and the variance of the noise. Moreover, gaussian distribution has the nice property that its Fourier transformed counterpart also follows the gaussian distribution. Due to these reasons, they are intensively used to model noise in the modern day data analysis and signal processing tasks, and there are hardware-level acceleration for reducing the computation cost with gaussians. Therefore, in our project, we will use a zero-mean gaussian to model the noise, and thus the insufficiency of the physical processes modelling.

4.2 Radar Modelling

4.2.1 Instrumental Properties of Radar

The instrumental configurations of a radar play an important role in radar signal processing. First of all, the power of a radar and the detection thresh-
old of the radar would jointly impose constraints on the maximum detection range of the radar. On the other hand, the emitted radar signal, in the form of radio wave, is not isotropic. Ideally, the radar emits a pencil-beam like radio wave to probe the state of the atmosphere. However, due to engineering difficulties, the radar wave, when propagate in the atmosphere, diffuse and span a 3-dimensional volume of the atmosphere. The size of this volume would depends on the angular size of the radar wave, which is technically called the beam width. Therefore, the received radar signal would be an integrated signal along the whole volume spanned by its beam width. Lastly, the wavelength of the emitted radar signal is also restricted in the construction of the radar. Therefore our radar signal simulator should be capable of adapting to different inputted radar configuration, and simulating the operation of the radar using these input parameters.

4.2.2 Energy Decay of Radar Signal

The energy emitted from a radar should be conserved, which leads to the area law of energy: the energy flux should decay with the surface area. In the same spirit, the energy flux, and thus the energy flux per time - power flux of the radar would also decay as radar wave propagate. By measuring the decay of energy imprinted on the amplitude of the received radar signal, we can extract information on the distance between radar wave scatters and the radar.

To mathematically quantify this, we consider the emitted power $P_e$ and the received power by the scatter, $P_s$. The surface area spanned by the wave front that has travelled a distance $r$ is $4\pi r^2$, therefore we have,

$$P_s = \frac{P_e}{4\pi r^2}$$  \hspace{1cm} (4.1)

However there are still other effects to be modelled. First of all, the geometry of the radar antenna would determine the gain $G_r$ and the cross-section $\sigma_r$ of the radar, which are both responsible for quantifying the collection of more reflected signal by the radar. Up to this point, we have only considered
the radar emitted signal. In the following, we examine the effect of the reflected signal by the scatters in the atmosphere. The intensity of the reflected signal would also follows the area law of energy. Therefore, two effects jointly contribute and gives a stronger decay in energy density with distance, the composite effects gives:

\[ P_r = \frac{P_e G^2 \lambda^2 \sigma}{(4\pi)^3 r^4} \]  

This is known as the radar equation. Clearly we can compare the intensity ratio between the emitted and the received signal, \( P_r / P_e \), and determine the distance \( r \) between the radar and the scatter. Intuitively, we also see the intensity dependence on the scattering cross section of the scatter, \( \sigma \), because the properties of the scatter would affect the ability the scatter to reflect radar signal.

### 4.2.3 Scattering Cross-section of Hydrometeors

In the above sub-section, we have introduce an important concept, the scattering cross-section of the scatter. In reality, there is no perfect scatter. Incidenting radar signal can be partly be absorbed (and possibly re-emitted in random direction) by the scatters, or even transmitted with refraction. This property is related to the temperature and pressure of the atmosphere. On the other side, the size of the scatter also dominate the reflection process, as larger area will reflect more signal. To quantify this effect, we introduce the scattering cross section \( \sigma \), which is a parameter assigned to each signal scatters.

The scattering effect of electromagnetic wave is well studied in theoretical physics. In radar meteorology, we use long wavelength (sub-mm scale) electromagnetic wave (radio wave) as the carrier of radar signal. In this regime, the wavelength of the radar signal is comparable to the size of the scatter. This process is well described by the Mie scattering process. In this regime,
the scattering cross section of a scatter can be modelled as:

\[ \sigma = D^6 \frac{\pi^2}{\lambda^4} \left| \frac{m^2 - 1}{m^2 + 2} \right|^2 \]  

(4.3)

where \( D \) is the diameter of the scatter, \( \lambda \) is the wavelength of the radar wave, and \( m \) is the complex refractive index of the scatter, which is defined as:

\[ m = n - i\kappa \]  

(4.4)

, with \( n \) the real refractive index and \( \kappa \) the attenuation parameter, and \( i^2 \equiv -1 \). It is clear that the scattering cross section is tightly related to the size of the scatter by \( D^6 \), meaning larger scatter reflects more signal. The absorption and reflection behaviour of the scatter are contained in the \( \left| \frac{(m^2 - 1)}{(m^2 + 2)} \right|^2 \) term.

In our situation, the scatters in the atmosphere are often raindrops, hails and (less probably) snow. It has been shown in various researches that, the refractive index of raindrops (and thus water) have a slight dependence on temperature and the wavelength of the incidenting radar signal. The dependence could be calibrated via laboratory experiments. In principle, detailed information of the temperature dependence of refractive index can provide extra information on atmospheric temperature estimation. However, as shown in the radar equation, the received radar power only depends on the temperature via the weak dependence on refractive index. And the sensitivity of temperature \( T \), on the scattering cross-section \( \frac{d\sigma}{dT} \) is much smaller than the uncertainty in distance estimation \( r^4 \). As a result, in our following implementation and testing, we will fix the refractive index of raindrop to be constant, so as to simplify the simulation. This should not be a constrain, and therefore in the code design of the simulator, the refractive index is a variable that can be changed by the user, enabling more flexibility in future research.
4.2.4 Radial Velocity of Hydrometeors

In the above subsections, we focused on discussing the radar effects that is related to estimate the radar-scatter distance. Yet, the information provided by a radar measurement is not limited to the distance, but also the velocity of the scatter. As shown in classical mechanics, with complete information about both the position (distance) and the momentum (velocity), the evolution of the scatter (in a closed system) can be completely determined.

To determine the velocity of the scatter, we will utilize the Doppler effect. The Doppler effect is the elongation (shortening) of emitted wavelength by a moving away (approaching) wave emitter. In our application, as the local wind field blows the hydrometeors in the atmosphere, the hydrometeors will move approximately along the wind and thus gaining a non-zero velocity. The doppler shifted radar signal can therefore used to determine the wind velocity.

Mathematically, the Doppler effect is quantified by:

$$\lambda' = (1 + v/c)\lambda$$

(4.5)

, where $\lambda'$ is the doppler shifted wavelength and the unprimed $\lambda$ is the original wavelength. The constant $c$ is the propagation speed of the signal, which is simply the speed of light in our application.

In practice, instead of estimating the change in wavelength, we measure the phase shift of the signal contributed to the doppler effect. The doppler phase shift, as a function of the radial velocity $v_r$, $\phi_z(v_r)$ is therefore:

$$\phi_z(v_r) = 2\pi f' t_r = 2\pi \frac{c}{(1 + v/c)\lambda} \frac{2r}{c} = \frac{4\pi r}{(1 + v/c)\lambda}$$

(4.6)

4.2.5 Intrinsic Dispersion of Hydrometeors

So far in the above discussion we consider scattering with single scatter. However, in reality there are millions of millions of scatters in each radar scanning
volume. Moreover, although the velocity of the hydrometeors approximately follows from the wind velocity, the wind velocity is not constant through the whole scanning volume. We can release the assumption of hydrometeors-follow-wind, as heavier but smaller (i.e.: more dense) hydrometeors are more difficult to be driven by the wind. To model these uncertainties, in the following, we will break these systematics into parts and at the end join them together with basic rules in probability theory.

First of all, the size of the hydrometeors is clearly not uniform in the atmosphere. Multiple raindrops can merge to form larger raindrop. We therefore assign a prior probability distribution of the hydrometeors size $P(D)$. In principle, the distribution $P(D)$ should depends on the atmospheric parameters which control the time scales for merging. In our application, we simply model $P(D)$ as a Poisson distribution, as the merging of raindrops to a good approximation satisfied the assumption of Poisson process.

The next component is the dispersion of wind velocity over the scanning volume. In meteorology simulation, the wind velocity is usually calculated at the boundary at each grids, and the exact wind velocity inside the grid is interpolated. This model clearly ignore the uncertainty of in-grid wind velocity generated by the turbulence. We will model this uncertainty by a zero-mean Gaussian distribution. Therefore, given the interpolated wind velocity $\psi_{\text{approx}}(x)$ at position $x$, the true wind velocity follows a conditional probability distribution $P(\psi_{\text{true}}(x)|\psi_{\text{approx}}(x)) \sim \mathcal{G}(0, \Sigma)$.

Finally, we can combine the two effect and derive the conditional probability for a diameter $D$ hydrometeor at position $x$, moving at velocity $v$ under wind velocity $\psi_{\text{true}}(x)$:

$$
P(v|D, \psi_{\text{true}}(x)) \sim \mathcal{G}(\psi_{\text{true}}(x), \Sigma_v)
$$

(4.7)

, and the marginalized velocity distribution of the hydrometeors is therefore

$$
P(v, D|\psi_{\text{approx}}) = \int d\psi_{\text{true}} P(v|D, \psi_{\text{true}}(x)) P(\psi_{\text{true}}(x)|\psi_{\text{approx}}(x)) P(D)
$$

(4.8)
It is unrealistic to calculate all the scattering of radar signal of the millions of millions hydrometeors. To cut down the computational cost, we will approximate the total signal by a Monte-Carlo integration. If the radar signal reflected by the \( i \)-th hydrometeor of velocity \( v_i \), diameter \( D_i \) is \( s_i(v_i, D_i) \), when there are \( N \) hydrometeors, the received signal \( S \) is:

\[
S = \sum_{i}^{N} s_i(v_i, D_i)
\]

(4.9)

However, we have workouted the distribution of hydrometeors velocity, therefore the above summation could be approximated by:

\[
S = N \int dv \int dD \, s_i(v, D) P(v, D | \psi_{approx})
\]

(4.10)

This could be further simplified by using monte carlo method to draw \( n \ll N \) samples from \( P(v, D | \psi_{approx}) \), and do the integration as:

\[
S \approx N \sum_{j}^{n} s_i(v_j, D_j)
\]

(4.11)

where \( v_j, D_j \) are random number drawn from \( P(v, D | \psi_{approx}) \). If we restrict \( n \sim 10^3 \), we can practically evaluate the sum and therefore obtain the signal generated by the large amount of hydrometeors.

### 4.2.6 Abnormous Propagation

In the above discussion we described the effects that we have modelled in our radar simulator. However, there are higher order effects that we have ignored in our simulator. To characterise these effects, we treat it as noise. In the following we will have a brief discussion on the higher order effects, known as abnormous propagation.

The first contribution of abnormous propagation comes from the fact that, although in general, hydrometeors reflects the most portion of incidenting signal, some of the signal can still pass through the hydrometeors by refraction. The refracted radar wave will continue propagating, and eventually hit another hydrometeors and repeated the refraction mechanism; with
some chance they can bounce back to the radar antenna and enriching the complexity of the received signal. We argue that this secondary reflection effect is very weak, as the intensity of the reflected radar signal, after $n$ refraction, discounted as $\gamma^n$, with $\gamma$ the ratio of refracted energy to the reflected energy in each scatter. As $\gamma \ll 1$, this eventually drops to zero at a power-law rate.

Another contribution is the curvature of the Earth. In the above calculations in this section, we rely on the (global) geometrical quantities, namely, the position of the hydrometeors and distance between the hydrometeors and the radar. In these calculation, we have implicitly assumed a flat, Euclidean geometry, where the distance calculation follows the simple Pythagorean law, described by an identity metric tensor. However, for application of radar where the range of the radar can spans hundreds kilometers, the effect of Earth curvature starts to emerge. This makes the actually (geodesic) distance between the radar and the hydrometeors larger than the flat Euclidean space by a factor of $\int \Gamma_{jk} dx^j dx^k$, with the Christoffel connection coefficient $\Gamma_{jk}$ depends on the (derivative of) Earth curvature. We argue, in our application, this effect will not dominate the signal. It is because our project aims mainly on estimating the wind velocity where we rely on the doppler shift of radar signal, which is independent on the distance.

### 4.3 Testing of the Radar Simulator

#### 4.3.1 Specification of Radar

To test the radar simulator, we first specify a set of parameters related to the radar instrument we are to simulate. To make these parameters realistic and with more practical use, we adopt the set of radar parameter of Tate’s Carin Doppler radar owned by the Hong Kong Observatory. This radar is intensively used by the Hong Kong Observatory to make measurement on the atmosphere and thus generating weather prediction. The set of parameters
is summarized in table 4.1.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Antenna beamwidth</td>
<td>1.8</td>
<td>degree</td>
</tr>
<tr>
<td>Transmitting frequency</td>
<td>3</td>
<td>GHz</td>
</tr>
<tr>
<td>Detection range</td>
<td>500</td>
<td>km</td>
</tr>
<tr>
<td>Pulse duration</td>
<td>$5 \times 10^{-7}$</td>
<td>s</td>
</tr>
<tr>
<td>Pulse repetition frequency</td>
<td>1</td>
<td>kHz</td>
</tr>
</tbody>
</table>

Table 4.1: The radar parameters adopted in the simulation. Most of the parameters are directly adopted from the actual value of the Tate’s Carin Doppler radar owned by the Hong Kong Observatory.

### 4.3.2 Radar Pulse

Another radar property that have to be specified is the waveform of the radar signal pulses. The key parameters of a pulse are the pulse repetition time, pulse duration, and the pulse carrier wavelength. We show the meaning of each term schematically in figure 4.1. A longer pulse duration can provide more time to sample the received signal, which improve the accuracy of the doppler shift, thus wind field measurements. On the other hand, the remaining two parameters will jointly determine the range of detectable wind velocity. This is set by the Nyquist-Shannon sampling theorem, which states that, the sampling rate should be at least twice the period of the signal. As Doppler effect will change the period of the received signal, this imposes constraints on the detectable range of velocity as:

$$v_{\text{max}} = (\text{p.r.f.})ct_c/4$$  \hspace{1cm} (4.12)

where the (p.r.f.) stands for pulse repetition frequency, which is just inverse of the pulse repetition time, and $t_c$ is the period of the signal carrier.

Again, we will adopt practical pulse parameters. Note the pulse repetition time is set to 1 ms while the pulse carrier frequency is determined by the
Figure 4.1: This is a typical waveform generated by a practical doppler radar. Here we illustrate the meaning of each radar pulse parameters. Note, in the plot, we exaggerated the pulse repetition time by shortening it by a factor of $10^6$ to make the plot clearer. Our simulator will send this signal to the atmosphere and measure the echos.
frequency of the radio wave, namely $\sim 3$ GHz, and period is therefore $\sim 3$ ns, which is $10^6$ shorter than the pulse repetition time. And the phase shift of each pulse would be used to estimate the doppler shifted phase and therefore the (radial) wind velocity.

### 4.3.3 Scattering with raindrop

After all the setting being fixed, we can run our radar simulator against simulated weather systems. For a validation purpose, we study the case of scattering with two hydrometeors in the form of raindrops. We place two raindrops at distance 50km and 52km respectively, with radial velocities of 10 m/s and 20 m/s. These raindrops are in the same sampling volume of one radar pointing. Using the above radar parameters, we emit radar signal to these to raindrops and observe the resulting signal generated by these two raindrops. The results is plotted in figure 4.2. We showed in the plot, our simulator successfully capture the reflected radar signal. After zooming in the time axis, we obtained the different time delays of radar signal contributed from the two raindrops. Using the wavelength of the reflected pulses, we can use the doppler relation to extract the velocity of the raindrops. This therefore illustrate a successful simulation on two scatters.

### 4.4 Conclusion

In this chapter, we first illustrated and derived the related mathematical formalisms of the propagation of radar signal. Then we implemented it on computer, and tested against simple situation using practical radar parameters. In the subsequent stage of the project, we will integrate the simulator with simulated atmospheric condition provided by MM5 simulation, so as to collect simulated data of radar measurements. These would provide a rich dataset for us to test the new wind field measurement algorithm that we are going to develop in this project.
Figure 4.2: A testing of the radar simulator with two raindrops. The dots in the plot marks the receiving time of each radar pulses. To make the time delay of signals due to different scatters distance, we enlarge the plot in the left panel, and show the comparison of one single pulse. Note we cannot see the shape of the wave carrier as in the waveform illustration in 4.1 as the frequency of the carrier wave is too high to be plotted here.
Chapter 5

Conclusion

In this project, we attempted to design and implement a new algorithm for extracting the wind field from the raw signal of Doppler radar. Upon testing the algorithm on simulated radar signal, we demonstrated the possibility and the effectiveness of applying the algorithm to real world scenario. By comparing the new algorithm with the currently adopted traditional algorithm, vertical azimuth display (VAD), a more objective evaluation of the algorithm could be performed. Recently, the tropical cyclone simulation code, MM5 has been reviewed and experimented. We aim to finish the implementation of the radar simulator by the end of December.

5.1 Limitations

However, the implementation of the new algorithm suffers from several limitations. Firstly, we only have limited computational resource. When testing the algorithm, we rely on computer simulation to generate test data. The insufficient computational power restrict us to perform simulation in restricted resolution. Without high enough resolution, we could not effectively compare the new algorithm, which model the wind profile in a continuous manner, with the traditional algorithm, which model the wind profile on discretized grid points. This elementary restriction would also make our al-
gorithm unable to capture small scale features of the TCs systems. Besides, the simulated radar signal was generated by ray tracing algorithm. However, ray tracing algorithm is computationally expensive, therefore a maximum recursion depth was imposed in our implementation. In this sense, complicated scattering of the mock radar signal by hydrometeors was not fully captured by our method; only the radar signal which interacted with the hydrometeors and bounced back in a few recursions was captured. Because the testing and evaluation of our algorithm relied on these simulated radar signal, our design of algorithm also ignored this kind of fine grained detail. Or in other words, the simulated signal by our algorithm was too ideal, which may leads to a deviation from real world scenario.

5.2 Future Improvements

Further refinement of our method can be performed in the future work. In particular, the measured wind field is being used intensively in weather forecast. However, in our work, the inference of wind field from radar signal, and the process of forecasting based on the inferred wind field, were decoupled. The recovery of true wind field from radar signal is not perfect; on the other hand, even with perfectly information of the wind field, weather forecast could also suffered from uncertainty, due to the lack of theoretical knowledge about the evolution of weather system. In other words, the error in reconstructing the wind field would be coupled to the forecast of the weather system. In order to design a reliable weather forecast algorithm, we may need to consider the inference of wind field and the subsequent forecasting altogether in one shot. By doing this, the uncertainties introduced in each step could be considered on an equal footing, so that the total uncertainties of all the steps could be minimized and handled together.
Chapter 6

References


10. MM5 Community Model Homepage. Available at [http://www2.mmm.ucar.edu/mm5/mm5-home.html](http://www2.mmm.ucar.edu/mm5/mm5-home.html)


17. MM5 Online tutorial. Available at [http://www2.mmm.ucar.edu/mm5/On-Line-Tutorial/mm5/mm5.html](http://www2.mmm.ucar.edu/mm5/On-Line-Tutorial/mm5/mm5.html)