Epidemic Forwarding in Mobile Social Networks

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Abstract—Recent years have witnessed the prosperity of mobile social networks, where various information is shared among mobile users through their opportunistic contacts. To investigate efficiency of information dissemination in wireless networks, epidemic models have been employed to study message forwarding delays, presuming message delivery whenever an opportunistic contact occurs. A practical concern is typically neglected, that one mobile user may only be willing to pass information onto others with social ties, rather than anyone upon contact. Under such a constraint, information dissemination may behave differently, according to the pattern of social ties that exist in the network.

In this paper, we model social-aware epidemic forwarding in mobile social networks using mean-field equations, and carefully study the end-to-end unicast message propagation delays under different levels of social ties among users. Both cases of limited and unlimited message validity are considered in our models, i.e., whether relay nodes may delete a message after carrying it for some finite time $T$ or never. Through careful theoretical analysis and empirical studies, we made a number of intriguing observations: First, the topology of social relation graphs significantly influences message forwarding delays, i.e., the more skewed the social relationship distribution is, the larger delay it results in. Second, the average delivery delay remains fairly stable with the growth of system scale, presenting a sharp contrast with the case without social awareness. Third, we observe that with a moderate choice of $T$, message delivery can achieve a successful ratio of almost 100% with an expected delay very close to the case of unlimited validity, signifying that a good tradeoff can be achieved between end-to-end message delivery efficiency and energy/storage overhead at the relay nodes in a network. All these provide useful guidance for efficient information dissemination protocol design in practical mobile social networks.

I. INTRODUCTION

With the advent of mobile technologies, various mobile social networks have emerged, which enable direct exchange of data among mobile users when they are in physical proximity, including pictures and news [1]. It is an interesting topic to investigate the delay of message propagation in a mobile social network, as an in-depth understanding would give useful insights to guide efficient information dissemination protocol design in practice.

Various epidemic models have been employed to study message forwarding delays in opportunistic wireless networks [2], [3], [4], where message delivery occurs whenever there is an opportunistic contact between two users. Groenevelt et al. [2] and Zhang et al. [3] show that the end-to-end unicast delay scales as $\frac{\ln N}{\ln \ln N}$ using Markovian models, where $N$ is the size of the network. Under the assumption that a message can flood over a connected component of a network instantaneously, Kong et al. [4] study with arguments from percolation theory the asymptotic behavior of unicast delay with respect to the initial Euclidian distances between the source and the destination in a large-scale mobile wireless network. Their conclusions are: (1) if the network is supercritical (i.e., the density of nodes is high such that a giant component of a size proportional to the network size is guaranteed to exist with high probability), the delay scales sublinearly with the distance; (2) when the network is subcritical (i.e., no giant component exists with high probability), the delay scales linearly with the distance.

These existing work have largely ignored a practical concern in real-world mobile social networks, that one mobile user may only be willing to pass information onto others with social ties, rather than anyone upon contact. Under such a constraint, epidemic information dissemination may behave differently, according to the pattern of social ties that exists in the network. In particular, when no social awareness is present, all possible transmissions among nodes constitute a complete graph, while when social awareness is present, a scale-free topology is formed. There is a limited amount of work concerning the temporal behavior of dynamics in scale-free networks. Barthélemy et al. [5] demonstrate that epidemics pervade in scale-free networks in a precise hierarchical fashion, from higher-degree nodes to lower-degree ones. However, to the best of our knowledge, no results have been obtained in terms of the end-to-end message delivery delay in such a topology.

In this paper, we model unicast epidemic message forwarding in a mobile social network, where information is only shared among socially connected users upon their contacts. Such a social-aware epidemic forwarding process is modeled using mean-field equations, and the end-to-end message propagation delays are carefully studied under different levels of social ties among the users. We consider both cases of limited and unlimited message validity in our models, i.e., whether nodes may delete a message after carrying it for some finite time $T$ or never.

Through careful theoretical analysis and empirical studies, we discover that the constraint of social awareness can significantly increase the average delay of unicast message deliveries, which increases with the skewness of the social tie distribution among users. On the other hand, the average unicast delay changes little with the increase of network size, which is quite different from the case without social awareness. Third, we observe that with a moderate value of $T$, message delivery can achieve a successful ratio of almost 100% with an expected delay very close to the case of unlimited validity.

The remainder of the paper is organized as follows. We present system overview in Sec. II, and model social-aware epidemic forwarding under unlimited and limited message validity in Sec. III and Sec. IV, respectively. Sec. V presents simulation results and Sec. VI concludes the paper.
II. SYSTEM MODEL AND ASSUMPTIONS

We consider a mobile social network with users \( n_1, n_2, ..., n_N \) moving around all the time in a region \( A \). Message delivery occurs from one node to another when they encounter and are socially connected with each other. The target is to derive the expected delay of unicast message delivery \( E[T_d] \), for one message to be passed from a randomly chosen source node \( n_s \) to a randomly chosen destination \( n_d \) via other nodes as relays.

A. Social Graph

A social graph \( F_N(V, E) \) is defined to represent social ties among the users, with \( V = \{ n_1, n_2, ..., n_N \} \), and a link \((n_i, n_j) \in E \) exists if user \( n_i \) is socially connected with user \( n_j \), e.g., friends, relatives, etc. The two nodes are referred to as social friends or friends for simplicity hereinafter. Existing studies have shown that the social graph among people has a scale-free structure [6]. We therefore use a generic power-law model to describe the degree distribution of \( F_N \):

\[
P(k) = \begin{cases} 0, & k < m, \\ C(m, \gamma) k^{-\gamma}, & m \leq k < N. \end{cases}
\]

Here \( P(k) \) is the probability that a node has degree \( k \). \( m \) is the smallest degree of all nodes in the network, i.e., the smallest possible number of social ties for a user. If \( m > 1 \), \( F_N \) is guaranteed to be connected with probability 1 [7]. \( \gamma \) represents the skewness of the degree distribution: the larger \( \gamma \) is, the more skewed the distribution is. \( C(m, \gamma) \) is the normalization constant. We denote the expectation of the degree distribution as \( \langle k \rangle = \sum_{k=m}^{N-1} k P(k) \).

Mobile social networks in different scenes can be described by different values of \( \gamma \) and \( m \). For instance, among people on campuses or in office buildings, social ties are stronger, and the social graph has a large \( m \) and a small \( \gamma \). On the other hand, in some other public places like cinemas or cafés, usually one is acquainted with few people around, and the social graph resumes a small \( m \) but a large \( \gamma \).

Similar to most existing analytical studies on scale-free topologies [7], [5], we ignore potential correlation of degrees among users in our models. This renders significant simplicity in analytical modeling.

B. Mobility Model

Human mobility patterns are very sophisticated and cannot be captured well by simple mobility models like Random Walk. These simple models typically correspond to an exponentially distributed inter-contact time [2]. However, existing empirical studies report that the inter-contact time of human beings generally follows some power law [8]. But using power law distribution makes the computation of delay extremely difficult. Besides, [9] demonstrates with real traces that exponential decay is still evident [9]. To preserve mathematical tractability and simplify the problem, we assume that the inter-contact time of any pair of nodes is a random variable following exponential distribution with rate \( \lambda \).

C. End-to-End Message Delivery Delay

Nodes \( n_i \) and \( n_j \) are connected at time \( t \) if they are socially connected and within the transmission range of the respective mobile devices at that moment. Denote the set of connections at time \( t \) by \( V(t) \), \( V(t) \subseteq V \). Now and then users with social ties meet and the message is gradually propagated from the source to the destination. We next formally define the message delivery process and the unicast delivery delay.

**Definition 2.1**: An end-to-end delivery is a spatial-temporal path of the message \( \{ n_{t_0}, n_{t_1}, ..., n_{t_f} \} \) where \( n_{t_0} = n_s, n_{t_f} = n_d, (n_{t_i}, n_{t_{i+1}}) \in V(t_{i+1}) \). The end-to-end message delivery delay is \( T_d = t_f - t_0 \).

The expected delay \( E[T_d] \) that we investigate measures message dissemination efficiency in a given mobile social network. In the message propagation process, nodes holding the message are referred to as spreaders, and the others without the message are ignorants. Table I summarizes important notations to appear in our models, for ease of reference.

| \( N \) | the number of users in the mobile social network |
| \( \lambda \) | inter-contact rate between user pairs |
| \( P(k) \) | power-law degree distribution function of social graph |
| \( \gamma \) | parameter denoting skewness of the degree distribution |
| \( m \) | the smallest possible number of social ties per user |
| \( C(m, \gamma) \) | normalization constant in the degree distribution |
| \( (k) \) | expected node degree in social graph \( F_N(V, E) \) |
| \( i(k, t) \) | percentage of ignorants of social degree \( k \) at time \( t \), \( i(t) = \sum_{k=m}^{N-1} i(k, t) \) |
| \( s(k, t) \) | percentage of spreaders of social degree \( k \) at time \( t \), \( s(t) = \sum_{k=m}^{N-1} s(k, t) \) |
| \( \theta(t) \) (or \( \theta(k, t) \)) | probability that a friend of a given ignorant is a spreader |
| \( \alpha(t) \) | auxiliary function, \( \alpha(t) = f'_V \theta(x)dx \) |
| \( e(k, t) \) | percentage of timed-out nodes of social degree \( k \) at time \( t \), \( e(t) = \sum_{k=m}^{N-1} e(k, t) \) |
| \( E[T_d] \) | the expected end-to-end unicast delay |
| \( T \) | the timeout value |
| \( E[T_{d1}] \) | expected end-to-end delay of successful deliveries |

III. MODEL SOCIAL-AWARE EPIDEMIC FORWARDING

We next develop mean-field equations [5] to model epidemic message forwarding in a mobile social network. We assume that the users will always retain the received message without a message expiration deadline here, and will analyze the case with message expiration in the next section.

The key idea is to classify nodes according to their degrees and compute the expected proportional decrement of ignorants of degree \( k \) (in the social graph) in a short interval \( [t, t + \Delta t] \). The analysis is in line with that in [5].

Apparently this decrement is proportional to the current population of degree-\( k \) ignorants, as well as the probability that a degree-\( k \) ignorant contacts a spreader during \( [t, t + \Delta t] \), denoted as \( P(A_k) \):

\[
i(k, t + \Delta t) - i(k, t) = -i(k, t) \cdot P(A_k).
\]

To derive \( P(A_k) \), we consider that the number of spreaders \( g \) among all the \( k \) neighbors of the node in question is a binomial random variable \( g \sim b(k, \theta(k, t)) \), where \( \theta(k, t) \) is
the probability that a friend of an ignorant with $k$ social ties is a spreader (referred to as event $\Theta$). For each spreader friend, denote the event that the ignorant node does not meet it in $[t, t + \Delta t]$ as event $B$. The probability it occurs is simply $P(B) = e^{-\lambda \Delta t}$. Therefore, we have

$$
P(A_k) = 1 - P(A_k^c)
= 1 - \sum_{g=0}^{k} \binom{k}{g} \theta(k, t)^g (1 - \theta(k, t))^{k-g} P(B)^g
= 1 - (1 - (1 - P(B)) \theta(k, t))^k.
$$

Based on the above equation, in the limit $\Delta t \to 0$, we obtain

$$
di(k, t) = -i(k, t) \lim_{\Delta t \to 0} \frac{P(A_k)}{\Delta t}
= -i(k, t) \lim_{\Delta t \to 0} \frac{1 - (1 - (1 - P(B)) \theta(k, t))^k}{\Delta t}
= -i(k, t) \lim_{\Delta t \to 0} \frac{1 - e^{-\lambda \Delta t}}{\Delta t}
= -\lambda ki(k, t) \theta(k, t).
$$

The total population of $k$-degree nodes is a constant, i.e., $i(k, t) + s(k, t) = P(k)$. Because the source of the message delivery is chosen uniformly at random in the network, the initial conditions of the equation set above are

$$i(k, 0) = \frac{N - 1}{N} P(k), \quad k = m, m + 1, \ldots, N - 1.$$

We refine the description of event $\Theta$ by defining a series of independent events $\Theta_{k, k'}, m \leq k' < N$, which means that a given ignorant of degree $k$ connects to a spreader of degree $k'$. $\Theta_{k, k'}$ can be further decomposed into two events: (1) a randomly chosen $k'$-degree node $n_j$ is a spreader, denoted as $\Theta_{1, k'}$; (2) a given $k$-degree ignorant node $n_i$ connects to that $n_j$, denoted as $\Theta_{2, k'}$. Thus we have

$$\theta(k, t) = \sum_{k' = m}^{N-1} P(\Theta_{k, k'}) = \sum_{k' = m}^{N-1} P(\Theta_{1, k'}) P(\Theta_{2, k'}).$$

Under the homogenous mixing hypothesis of mean-field theory, we approximate the probability that event $\Theta_{1, k'}$ occurs by the relative density of degree-$k'$ spreaders, i.e., $P(\Theta_{1, k'}) \approx s(k'/k)^k / \sum_{k' = m}^{N-1} s(k'/k)^k$. This constitutes an overestimation because of the locality of epidemic spreading in the social graph $F_N$: Spreaders are always clustered and this cluster continues expanding over time. The majority of degree-$k$ ignorants reside apart from the cluster in the social graph, and therefore the chance for one to get connected to a degree-$k'$ spreader is less than the density.

As degree correlations are neglected, the probability of $\Theta_{2, k'}$ is independent of the degree of emanating node $n_i$. The probability that a random edge in the social graph has an endpoint of degree $k'$ is $\frac{k'(k')/k}{\sum_{k' = m}^{N-1} (k'/k)^k}$, proportional to $k'$. However, we know that during the message forwarding process, there is at least one spreader among the friends of any spreader — the one the latter gets the message from. This edge should be excluded from the calculation of $P(\Theta_{2, k'})$, resulting in an approximation $P(\Theta_{2, k'}) \approx \frac{(k'-1)(k')/k}{\sum_{k' = m}^{N-1} (k'/k)^k}$.

Therefore, we can derive

$$\theta(k, t) \approx \sum_{k' = m}^{N-1} \frac{(k'-1)P(k')}{P(k)} s(k', t) = \sum_{k' = m}^{N-1} \frac{k'-1}{k} s(k', t)$$

$$\sum_{k'=m}^{N-1} \frac{k'-1}{k} \left( P(k') - i(k', t) \right).$$

(3)

Observe that $\theta(k, t)$ does not depend on any particular $k$, but reflects the density of spreaders in the whole system at time $t$. We hereinafter simplify it to $\theta(t)$. It presents an overestimation of $P(\Theta)$, which is also verified in our simulations.

Solution complexity of Eqn. set (1) is high. However, through $\theta(t)$, Eqns. (1) can be collated into only one equation, which offers an insightful picture of system dynamics and leads to a much easier numerical solution.

Integrating Eqn. (1) gives:

$$i(k, t) = i(k, 0)e^{-\lambda \kappa_0 t} = \frac{N - 1}{N} P(k)e^{-\lambda \kappa_0 t},$$

where $\kappa(t) = \int_0^t \theta(x) dx$.

According to Eqn. (3), the following holds

$$\frac{di(k, t)}{dt} = \theta(t) = \sum_{k=m}^{N-1} \frac{k-1}{k} P(k) - i(k, t))$$

$$= \sum_{k=m}^{N-1} \frac{(k-1)P(k)}{k} \left( 1 - \frac{N - 1}{N} e^{-\lambda \kappa_0 t} \right).$$

(5)

The initial condition for Eqn. (5) is $\alpha(0) = 0$.

We proceed to derive the cumulative distribution function (CDF) of $T_d$, $D(t)$. Let $S_1$ be the event that the destination successfully meets a spreader in $dt$ time and $S_{2k}$ be the event that the destination of degree $k$ has not successfully received the message from its $k$ neighbors in $dt$. The following holds:

$$D(t) \approx D(t) = D(t) = D(t < T_d < t+dt)$$

$$= (1 - D(t))P(S_1)$$

$$= (1 - D(t)) \sum_{k=m}^{N-1} P(deg(dest.) = k)(1 - P(S_{2k}))$$

$$= (1 - D(t)) \sum_{k=m}^{N-1} P(k)(1 - (1 - (1 - e^{-\lambda \Delta t})\theta(t))^k)$$

$$\approx (1 - D(t)) \sum_{k=m}^{N-1} k P(k) \lambda \theta(t) dt$$

$$= (1 - D(t)) \lambda \kappa_0 \theta(t) dt.$$

Then we can easily get $D(t) = 1 - e^{-\lambda \kappa_0 t}$. So the expected unicast delay is given by

$$E[T_d] = \int_0^\infty (1 - D(t)) dt = \int_0^\infty e^{-\lambda \kappa_0 t} dt. (6)$$

Based on Eqn. (5) and (6), we have the following theorem:

**Theorem 3.1:** $E[T_d]$ monotonically decreases with $m$ and increases with $\gamma$.

The intuition of the theorem is quite straightforward: more friends and less skewed distribution of the number of friends in the mobile social network lead to more efficient propagation of messages. See a rigorous proof in our technical report [10].
IV. EPIDEMIC FORWARDING WITH MESSAGE EXPIRATION

Since retaining and forwarding messages consume storage and battery power, we next investigate a more practical timeout scenario, where nodes will delete the received message after it has been kept for time $T$. We wish to investigate a proper message timeout threshold, using which a good tradeoff can be achieved between end-to-end message delivery delay and power consumption at relay nodes.

Specifically, a node starts a timer which expires after a threshold $T$ upon the reception of the message. When the timer is up, it deletes the local copy and stores an ‘anti-packet’ to prevent future duplicate receptions. We analyze the end-to-end message delivery in this case in what follows.

Equations (1) to (5) still hold when $t < T$. When $t > T$, we denote $e(k,t)$ as the proportion of nodes of degree $k$, at which the message has expired at time $t$. Denoting $g(k,t) = \theta(t)i(k,t)$, the following mean-field equations model the evolution of the system after $T$:

$$
\frac{di(k,t)}{dt} = -\lambda kg(k,t)
$$

$$
\frac{ds(k,t)}{dt} = \lambda g(k,t) - g(k,t-T)
$$

$$
\frac{de(k,t)}{dt} = \lambda kg(k,t-T).
$$

Eqn. (7) tells that the current net increment of the number of spreaders of degree $k$, i.e., $s(k,t)$, is the difference between the number of new spreaders at time $t$ and the increment occurred at time $t-T$ (since at those nodes the message expires at $T$).

Integrating both sides of Eqn. (7) leads to:

$$
s(k,t) = \lambda k \int_{t-T}^{t} i(k,t)\theta(t)dt + \frac{P(k)}{N}.
$$

From Eqn. (3), (5) and (9), we can easily get (for $t > T$)

$$
\frac{d\alpha(t)}{dt} = \theta(t) = \sum_{k=m}^{N-1} \frac{k-1}{\langle k \rangle} s(k,t)
$$

$$
= \sum_{k=m}^{N-1} \frac{(k-1)P(k)}{\langle k \rangle} \left( \frac{1}{N} + \frac{N-1}{N} (e^{-\lambda k\alpha(t-T)} - e^{-\lambda k\alpha(t)}) \right)
$$

Therefore, a complete description of the system is the following Delay Differential Equation (DDE):

$$
t < 0 : \alpha(t) = 0,
$$

$$
t > T : \frac{d\alpha(t)}{dt} = \sum_{k=m}^{N-1} \frac{(k-1)P(k)}{\langle k \rangle} \left( \frac{1}{N} + \frac{N-1}{N} (e^{-\lambda k\alpha(t-T)} - e^{-\lambda k\alpha(t)}) \right).
$$

The expression of expected delay in Eqn. (6) is still valid. So we can derive the expected delay in this case according to Eqn. (6) and (10). We see that Eqn. (10) degenerates to the simple case of Eqn. (5) as $T \rightarrow \infty$.

We note that in case of message expiration, an end-to-end unicast message delivery may not always be successful, as it is possible that the message times out at all spreaders before they meet any of their friends. Our mean-field equations just describe the expected change in the number of nodes in each category. For instance, Eqn. (7) describes the expected increment of spreaders of degree $k$ at time $t$. Through simulations in Sec. V we find that the expected delay $E[T_d]$ calculated based on Eqn. (6) and (10) can indicate the following value when message deliveries may fail. $E[T_d] = \text{expected end-to-end delay of successful deliveries}$. This metric can be interpreted as an integrated indication of message forwarding hardness: its value is larger if the expected delay among all successful deliveries is larger and/or when the successful delivery ratio (the number of successful unicast message deliveries over the total number of unicast message deliveries) is smaller.

V. EMPIRICAL STUDIES

In this section, we carry out simulations to verify the epidemic models constructed by our mean-field equations. For each set of parameters, each experiment is run 5000 times and the average results are plotted.

A. DELAY AS A FUNCTION OF NETWORK SIZE

Suppose the unit time of the system to be 0.01 hour. Since existing empirical studies have told that the average inter-contact time between people is around 5 hours [9], we choose an inter-contact rate of $\lambda = 0.002$ for simulations. $k$ is set to be 3, a typical exponent of real life scale-free networks. In Fig. 1, we set $m = 20$ and compare the expected delays derived numerically using our mean-field equations and those produced experimentally through simulations, in networks of different sizes. For comparison purpose, we also plot the delays obtained under epidemic forwarding without social awareness, based on the analytical results in [2]. We observe that the numerical results from our models are typically smaller than the observed delays from simulations, while the gaps are small. This is consistent with our analysis in Sec. III that Eqn. (3) represents an overestimation of the probability that a friend of an ignorant is a spreader. We can also see that the delay under social-aware epidemic forwarding is typically larger than that obtained without social awareness.

Most importantly, we observe that the delay in social-aware epidemic forwarding remains quite stable with the increase of network size. (We also test different configurations of $\gamma$ and $m$ and find identical phenomena.) This counter-intuitive result is in a sharp contrast with the case of no social
leading to a rapid decrement of delivery delay. However, in the absence of social awareness, nodes gain local dynamics which are insensitive to the boost of system scale. Therefore, the diameter only scales as \( \log \log N \) with \( N \), indicating that with \( N \) Cohen et al. [11] demonstrate that with \( \gamma > 2 \), the diameter \( d \) only scales as \( d \sim \log \log N \). Second, average number of friends \( k \) only grows slightly even in case of very large increment of \( N \), as indicated by Fig. 2. Social awareness limits the sights of nodes to their local neighbors in the social graph, essentially transforming message dissemination from a global dynamic into an aggregation of some quite isolated, local dynamics which are insensitive to the boost of system scale. However, in the absence of social awareness, nodes gain much more relay opportunities when the network size grows, leading to a rapid decrement of delivery delay.

### B. Impact of Social Graph Models

We next investigate the expected delay in terms of different social graph models. In the following experiments, we fix \( \lambda = 0.002 \), \( N = 100 \), but vary \( \gamma \) and \( m \) respectively.

In Fig. 3, we fix \( \gamma \) to be 3 and vary \( m \) from 15 to 25. We observe that the delay decreases with the increase of \( m \). On the other hand, in Fig. 4, \( m \) is set to 20 and \( \gamma \) varies over \([2.4, 4.4]\). We can see the delay increases with the increase of \( \gamma \). All these verify our Theorem 3.1 in Sec. III.

### C. Impact of Message Timeout Length

We next study the impact on message delivery when different message timeout intervals are employed. In this set of experiments, we set \( N = 100, m = 20, \gamma = 3 \). We observe a rough match between the numerical results of Eqn. (6) and the metric \( E[T_d] = \) expected end-to-end delay of successful deliveries.

In the former experiments, we derive that in the unlimited validity case, the expected delay \( E[T_d] \) under the identical setting is approximately 66, indicated by the two dashed lines.

From Fig. 5, we observe the weight when the threshold \( T \) is larger than this value of \( E[T_d] \), \( E[T_r] \) becomes very close to this value, which implies a successful delivery ratio of almost 100%. This result tells us that by setting \( T \) to be slightly larger than the expected delay in the unlimited validity case, a good tradeoff can be achieved between message delivery efficiency and energy/storage overhead at the relay nodes in a network.

### VI. Conclusion

Addressing the practical concern that users in mobile social networks may exchange information with their social acquaintances exclusively, we investigate in this paper the delay of unicast message deliveries with social-aware epidemic forwarding. Using mean-field equations, we investigate the average delay under different levels of social-ties among users, considering both cases of limited and unlimited validity of messages. Results demonstrate that the constraint of social awareness significantly increases the delay, which has a positive relationship with the skewness of the social tie distribution. More importantly, we discover that the average delay remains almost constant when the size of the network increases significantly. In the case of limited message validity, we find that with a moderate choice of \( T \), message delivery can achieve a successful ratio of almost 100% with an expected delay very close to the case of unlimited validity. These results provide good implications for the design of message dissemination protocols in mobile social networks.

### REFERENCES


