

# Exploring the Sustainability of Credit-incentivized Peer-to-Peer Content Distribution

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**Abstract**—Credit-based incentives were proposed to incite peer contributions in P2P content distribution systems. Their effectiveness was extensively analyzed from a game theory perspective. Little attention however has been paid to a potential threat to such systems—the possible condensation of credits in a small number of peers over time. Credits condensation puts system sustainability on the line: many peers gradually run out of credits and cannot keep up a decent download rate. We study the sustainability of credit-based P2P systems running for a long period of time. We first introduce a new queueing network based model for credit circulation in a P2P content trading market. This model enables the study of credit system sustainability via examining the stability of stochastic traffic flows in a network of queues. We show that a stable job circulation, *i.e.*, an equilibrium market state, always exists. A sufficient and necessary condition for asymptotic condensation at equilibrium is proved. We analyze the degree of condensation in finite networks using the Gini index, and relate condensation to P2P network protocols and parameters. Our theoretical results are verified and supported by extensive simulations under realistic settings. We propose counter-actions for preventing and mitigating credit condensation.

**Index Terms**—Peer-to-peer content distribution, Queueing Network, Credit-based system

## I. INTRODUCTION

Incentive engineering has sparked considerable interest since the beginning of research on peer-to-peer (P2P) content distribution. The goal is to incentivize peers to contribute resources such as upload bandwidth and storage capacity [1], [2], [3]. Barter-like schemes (*e.g.*, *tit-for-tat*), while successful in P2P file sharing (*e.g.*, BitTorrent) [4], do not suit streaming applications well in terms of network capacity utilization and download timeliness [5]. Two categories of more general schemes were proposed: *reputation-based* incentives and *credit-based* incentives. The former maintains a reputation score (*e.g.*, upload/download ratio) for a peer, which summarizes the peer’s past contributions [6]; a high score may be rewarded with preferred-upload privilege and a low score may lead to terminated system access [7]. The latter implements credits (or micropayment, virtual currency) among peers, for exchange of desired data chunks [8], [9]. Compared to reputations, credits enable more direct, flexible, and fine-grained incentive solutions that are well-suited for both high-usage and low-usage peers [10].

Credit-based incentives have been proven to be effective in motivating resource contribution from a game theory per-

spective [8], [9]. The security and scalability issues of credit-based P2P systems were also extensively studied [11], [9], [12]. However, the sustainability of such systems, which could run for a long time, has been largely neglected: would credits stay evenly distributed over time, or eventually converge to a small number of peers, *e.g.*, the ones affluent in connections or upload bandwidth? If indeed the credits would converge to a few peers, many peers will be shut out of the P2P service, for lack of purchase power due to bankruptcy. This is the “wealth condensation” [13] phenomenon in a credit-based P2P network.

Such a practical concern can be traced to historical lessons. The “Capitol Hill Baby Sitting Co-op” [14] was established for parents from the same community to exchange baby-sitting services, using coupons. The system enjoyed an initial success only to see its own collapse due to the eventual imbalance of coupon distribution: a small group of parents aggregated the majority of the coupons, leading to a paradox of thrift for the other parents who could no longer afford the service as normal [15]. Similarly, a P2P system experiencing wealth condensation may see degradation in content distribution efficiency. The poor peers with few credits are unable to buy content they need, have little content to sell for revenue, and provide little revenue opportunities for neighbors who are probably also poor. Eventually, the P2P credit flow (and hence the data flow) ceases healthy circulation. Periodically injecting new credits into the system may provide a temporary remedy, which however may lead to another classic economic problem in the long run: the inflation of currency [16].

All these concerns raise intriguing questions on the long-term sustainability of a credit-based P2P economy. As a first effort for the literature, we explore the fundamental causes to wealth condensation in a P2P system. In particular, under what scenarios are the credits prone to aggregation by a small number of peers? What are the important factors influencing credit distribution? What handles do we have for condensation avoidance and mitigation? Is it possible for a credit-driven P2P market to stay in healthy operation over a long period of time?

To answer these questions, we first propose a novel model for the credit transition among peers, using a Jackson queueing network. Each peer is mapped to a queue, each unit credit is mapped to a job, and credit circulation in a P2P market is transformed to job circulation in a queueing network. We discuss advantages of the new queueing network model, and

study the credit distribution among peers by analyzing the queue size distribution in the Jackson network.

To avoid credit condensation, a healthy, stable credit circulation is required. But first of all, will a stable circulation (equilibrium state of queueing network) always exist, given different P2P topologies and trading preferences among peers? We give an affirmative answer which comes from analyzing the existence of positive eigenvectors of the credit transfer probability matrix. While in the equilibrium state, we further prove a sufficient and necessary condition for condensation to happen asymptotically. The condition is based on a carefully designed threshold criterion that connects the average amount of credits per peer in the network to the normalized utilization of the credit queues in the queueing network model. This result sheds light on what parameters in the P2P market are critical to the occurrence of condensation. In particular, if initial wealth at the peers (*i.e.*, the average amount of credits per peer in a close network) exceeds a certain threshold, there will exist at least one peer on which credits are aggregated; otherwise, wealth condensation will not occur, and the credit system can sustain for long.

For wealth condensation in finite networks, we study the degree of condensation, based on the joint distribution of queue sizes computed using the property of Jackson queueing networks. We make approximate simplifications on the joint probability mass function of queue sizes, and apply the Gini index [17] from economics to evaluate the relative skewness of queue lengths. The skewer the distribution of queue sizes is, the more condensed the wealth distribution is. We expound how average peer wealth may affect the skewness: the average wealth at the peers should not be too large, as otherwise condensation will more likely happen; it cannot be too small either, limiting the content download rate. Moreover, pricing on the data chunk transfers influences wealth distribution.

The skewness of wealth distribution is also studied through extensive simulations which further explicate and verify the effects of different network parameters on the degree of condensation in dynamic P2P systems. We discuss a number of counter-actions for preventing credit condensation, such as the taxation strategy. Our results reveal that, bringing taxes (with a suitable taxing threshold) into credit-based P2P systems can effectively inhibit the skewness of credits distribution when the taxing threshold is well-designed and not too low.

In the rest of the paper, we discuss related work in Sec. II, present the modeling details in Sec. III. Sec. IV proves the existence of the equilibrium of credit distribution. Sec. V analyzes conditions that promote wealth condensation and the impact of different network parameters. Sec. VI presents simulation studies and Sec. VII concludes the paper.

## II. RELATED WORK

Credit (or micropayment, virtual currency) was considered in a number of P2P system designs, for motivating peer cooperation and contribution [8], [9], [12]. The majority of them focus on system protocol design or the implementation of the credits. For example, Lightweight Currency [18] is

an application-layer protocol implementing such a virtual currency. KARMA [9] keeps track of resource contribution and consumption of its peers, using a virtual currency maintained by a set of *bank-set* nodes. PPay [12] discusses a secure payment scheme for P2P file sharing. The BitCoin project [19], which has attracted hundreds of thousands of users by 2011, implements a virtual currency system where users manage the currencies collectively in the peer-to-peer fashion, without involvement of any central authority. All these work focus on implementation of an online currency and are orthogonal to our work, which will assume such an implementation is feasible.

Game theory was applied to the analysis of credit-based P2P systems, on the effectiveness of using credits to motivate peers' maximum contributions at Nash equilibrium [8]. A focus here is the design and analysis of *pricing* schemes, under which peers pay calculated amounts of credits to obtain desired content [20]. Pricing schemes examined include: a single price per peer [20], linear pricing [3], and auction based pricing [1]. The impact of pricing on credit condensation in a P2P market is discussed in Sec. V in this paper.

Few existing work investigates the sustainability of credit-based P2P systems or considers long-term evolution of the credit distribution. The most relevant work is probably by Friedman *et al.* [8], which studies system performance as a function of the total amount of internal currency available. It concludes that too large an amount of total internal currency causes the system to collapse. Via simulations, Hales *et al.* [21] show that insufficient initial credits can lead to a system state where many peers lack credits. Dandekar *et al.* [22] constructs a general credit model with a complete graph, and show through simulations that the system robustness (probability of peer bankruptcy) is related to credit capacity and network density. Different from these two studies, our work will investigate analytically the impact of factors on credit distribution in a P2P system.

In the realm of economics, simple models were proposed to capture the distribution of money [17]. In the field of physics, there also exist models for studying the condensation of materials [13]. Nevertheless, these models are not directly applicable in the scenario of a P2P network. Instead, we propose a new, queueing network based model for analyzing credit distribution within a P2P system.

Queueing models have recently been applied in P2P networks to capture user channel-switching and peer churn behaviors [23], to derive the number of servers in a P2P online storage system [24], as well as to characterize content availability in a bundled P2P swarm system [25] and content retrieval strategies [26]. However, we are not aware of previous research that applies queueing networks to model the circulation of credits.

## III. MOTIVATION AND SYSTEM MODEL

We introduce the credit-based P2P content distribution system and an example to motivate our study in Sec. III-A, and

describe how the P2P system is modeled using a queueing network in Sec. III-B.

### A. The Threat of Wealth Condensation

We consider a mesh P2P content distribution system (*e.g.*, P2P file sharing or streaming), where users exchange data chunks among each other by acting as both a client (downloading from others) and a server (uploading to others). While users are naturally motivated to download data chunks they desire, motivating them to upload has been an important subject of study known as P2P incentive engineering. A credit-based solution employs a virtual currency within the P2P swarm, to incentive peers' mutual upload.

In credit-based P2P content distribution [9], [1], [20], [12], [18], when a peer  $i$  helps upload a data chunk to another peer  $j$ ,  $j$  transfers credits (virtual money) to  $i$  for remuneration. The amount of credits transferred is determined in various ways, *e.g.*, decided by the seller (uploading peer) using a flat or linear rate [20], [3], or settled through an auction [1]. Credit flows therefore accompany data flows within the P2P system, along reverse directions. Through buying from and selling to neighbors, a user should strike to maintain its credit pool at a healthy level, for enjoying a steady downloading rate.

In this paper, we explore long-term evolution of such a credit-based P2P system. The questions at hand to investigate are: Do credits in the system tend to aggregate onto a small number of peers over time? If so, does such wealth condensation really constitute a threat to download performance? The condensation of the majority of wealth to a small number of individuals in a community has been previously observed in the economics literature [27], [17]. We also perform simulation studies based on realistic settings of a P2P live streaming protocol, where results suggest that credit-based P2P systems, without careful design, are also prone to such a phenomenon, which leads to low streaming performance at the peers.

In particular, we simulate a state-of-the-art mesh-based P2P live streaming system with 500 peers in a scale-free overlay topology, running a protocol similar to that of UUSee [28]. Each peer is assigned a certain amount of initial credits; different data chunks transferred between peers may request different amounts of credits to be paid, according to the pricing scheme employed. Chunk transfers (with credit transfers in the inverse direction) among peers are based on chunk availability at the peers.

With different settings of parameters (such as the average amount of credits per peer, and specific pricing scheme over data chunks), we have observed that distributions of credits vary when the system has evolved for a long period of time. We evaluate the degree of condensation using the *Gini index*, a measure of inequality of income or wealth [17]. A Gini index ranges between 0 and 1, with 0 representing perfect equality and 1 representing extreme inequality. We will formally define the Gini index in Sec. V-B2.

Fig. 1 presents the distribution of credit spending rates at peers after the system has evolved for a long time, in two cases: (1) We set the initial credits at the peers to be

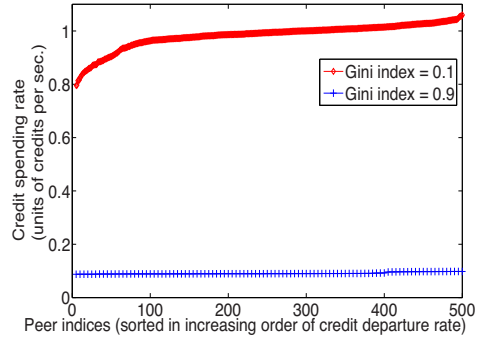


Fig. 1. Distribution of credit spending rates, with and without wealth condensation.

200, and peers charge different credits for selling different chunks, which follow a poisson distribution with an average of 1 credit per chunk; the resulting Gini index=0.9 shows that wealth condensation has occurred. (2) We set the initial credits at each peer as 12, and peers charge 1 credit per chunk uploaded uniformly; the Gini index=0.1 corresponds to relatively balanced credit distribution in the system. The rate of credit spending at a peer reflects the chunk download speeds of the peer. Therefore, the much lower credit spending rates in the first case with condensed credit distribution exhibit the much lower download speeds (and thus much worse streaming performance) in the system in that case. This observation confirms that wealth condensation may lead to reduced levels of content exchange in a P2P system, and seriously deteriorate the overall content distribution performance. In the following sections, we seek to investigate critical network parameters that affect the distribution of wealth, in order to explore counter-actions to avoid the condensation.

### B. From a P2P Market to a Queueing Network

1) *Network of Queues and the Jackson Network*: The classic field of queueing theory studies the stochastic arrival, buffering, and servicing of “jobs” at “servers”. The network of queues branch further considers a network of interconnected queues, where a job departing from a queue  $i$  travels to another neighboring queue  $j$  with probability  $p_{ij}$ . Such probabilities aggregately form the *transfer probability matrix*  $\mathbf{P}$ .

A *Jackson Network* [29] is a network of queues where the arrival process at each queue, as well as the job service times, are *memoryless*. In other words, the arrivals at each queue form a Poisson process, and the job service times are randomly distributed. The average service time at queue  $i$  is  $\frac{1}{\mu_i}$ , taking exponential form as the general setting of queueing networks. It is worth noting that the *all-Poisson arrivals* assumption is possible, since: (i) Stochastically splitting a Poisson flow results in sub-flows that are still Poisson, and (ii) the aggregation of two Poisson flows is still a Poisson flow. A *closed Jackson Network* is one in which jobs circulate within the network, without arrivals or departures. In this case, the transfer probabilities satisfy  $\sum_j p_{i,j} = 1, \forall i$ , by the rule of

TABLE I  
MAPPING BETWEEN A CREDIT-BASED P2P SYSTEM AND A CLOSED  
QUEUEING NETWORK

A P2P Overlay	A Queueing Network
No. of peers, $N$	No. of queues, $N$
A peer $i$	A queue $i$
A unit credit	A job
Total credits of peer $i$ , $B_i$	No. of jobs at queue $i$ , $B_i$
Total credits $M$ in the overlay	Total no. of jobs $M$ in the network
Fraction of purchase made by peer $i$ from peer $j$ , $p_{ij}$	Routing probability, $p_{ij}$
Peer $i$ 's average credit spending rate $\mu_i$	Queue $i$ 's service rate $\mu_i$
Peer $i$ 's average income earning rate $\lambda_i$	Queue $i$ 's arrival rate $\lambda_i$

total probability.

2) *The Mapping and Assumptions:* As a novel contribution, we model a credit-based P2P system using a Jackson queueing Network, by mapping the circulation of credits in a P2P system to the circulation of jobs within a queueing network. We map each P2P user  $i$  into a queue  $i$ . Two queues are neighbors in the queueing network if the corresponding users are neighbors in the P2P system. A unit credit corresponds to a job. The credit pool at a user  $i$  maps to the buffer at queue  $i$ . A user  $i$ 's income earning rate  $\lambda_i$  equals the job arrival rate at queue  $i$ . Spending a credit in the P2P system maps to finishing a job in the queueing network.

For an average credit earning rate  $\lambda$  over time, we approximate the arrival process using a Poisson process. The transfer duration of a unit credit at peer  $i$  is modeled with an exponentially distributed variable with expectation  $\frac{1}{\mu_i}$ . The service rate  $\mu_i$  relates to the *credit spending rate* at peer  $i$  when it has credits. The stochastic nature of the queueing network model naturally reflects the inherent randomness of chunk transactions in a P2P market, due to heterogeneity in peer bandwidth, connectivity, wealth, and operation timing.

We use  $N$  to denote the total number of users/queues, and  $M$  the total number of credits/jobs. The fraction of purchase a user  $i$  made from a neighbor  $j$ , *i.e.*, what fraction of credits paid by  $i$  goes to  $j$ , becomes the job transfer probability  $p_{ij}$ . Reserving a fraction of credits from trading can be modeled by a  $p_{ii} > 0$ .

We first focus on a static P2P network without peer joins and departures, leading to a closed Jackson Network. Each user  $i$  in the network is assigned with an initial credit pool  $c$ . We extend the discussion to dynamic P2P environments in Sec. VI, where an open Jackson network models peer joins and departures.

For ease of reference, we summarize the mapping from a P2P network to a Jackson network in Table I.

#### IV. EQUILIBRIUM OF CREDIT DISTRIBUTION: EXISTENCE

To avoid wealth condensation, we need equilibrium states of a P2P market with a healthy and stable credit circulation. Does an equilibrium state exist at all, given the P2P network topology and content trading preferences among peers? - that is our subject of study in this section.

The existence of equilibrium for a queue  $i$  depends on  $\lambda_i$  and  $\mu_i$ . Since the purpose of earning credit is to redeem them for data chunks, it is reasonable to assume that over the long term, the average data earning rate  $\lambda_i$  is upper-bounded by the maximum spending rate  $\mu_i$ , *i.e.*,  $\mu_i \geq \lambda_i, \forall i = 1, \dots, N$ .

Recall  $\mathbf{P}$  is the transfer probability matrix, derived from the network topology and peer trading preferences. Given  $\lambda_i \leq \mu_i, \forall i$ ,  $\vec{\lambda}\mathbf{P}$  provides the new arrival rate vector in the next time instance. The system is at a steady state if  $\vec{\lambda}$  becomes stable:

$$\vec{\lambda}\mathbf{P} = \vec{\lambda}. \quad (1)$$

When arrival rates in  $\vec{\lambda}$  are stable, so are credit departure rates in  $\vec{\lambda}\mathbf{P}$ , and we obtain a stable credit circulation within the network. We show that such an equilibrium of the credit system exists.

**Lemma 1.** *Given the transfer probability matrix  $\mathbf{P}$  that satisfies  $p_{ij} \geq 0, \forall i, \forall j, \sum_{i=1}^N \sum_{j=1}^N p_{ij} > 0$ , and  $\sum_{j=1}^N p_{ij} = 1, \forall i$ ,*

*there always exists a positive  $\vec{\lambda}$  ( $\lambda_i > 0, \forall i$ ) satisfying  $\vec{\lambda}\mathbf{P} = \vec{\lambda}$ .*

Lemma 1 can be proved by applying the *Perron-Frobenius Theorem* [30] on eigenvalues of nonnegative matrices. We prove Lemma 1 in our technical report [31]. A trivial solution to Eq. (1) always exists, by letting  $\vec{\lambda} = 0$ . Lemma 1 shows that a nontrivial solution  $\vec{\lambda} > 0$  can always be found. Can  $\vec{\lambda}$  still be uninteresting by being very small? This is not a worry, since Eq. (1) remains valid under scaling of  $\vec{\lambda}$ .

Lemma 1 shows that a steady state of a credit-based P2P system always exists. We next investigate the detailed credit distribution at the equilibrium state. We first define a *normalized utilization vector*  $\vec{u} = \{u_1, \dots, u_N\}$ :

$$u_i = \frac{\lambda_i/\mu_i}{\max_j(\lambda_j/\mu_j)}, \quad (2)$$

where each *normalized utilization* of peer  $i$ ,  $u_i$ , is between 0 and 1.

By properties of a Jackson Network, the joint equilibrium distribution of credits equals the product of individual peer credit distribution [29]. Recall that  $B_i$  is the amount of credits at peer  $i$  and  $M$  is the total amount of credits. The joint equilibrium distribution can be represented by the following joint probability mass function  $\mathbf{Q}$ .

$$\mathbf{Q}\{B_1 = b_1, \dots, B_N = b_N\} = \frac{1}{Z_M} \prod_{i=1}^N u_i^{b_i}, \quad (3)$$

where  $Z_M = \sum_{\sum_i^N b_i = M} \prod_{i=1}^N u_i^{b_i}$ .

What are the properties of the joint probability  $\mathbf{Q}$ , and under what condition can the budgets be evenly distributed, need further investigation. We analyze these questions and the key factors affecting the phenomenon of wealth condensation in the next section.

## V. EQUILIBRIUM OF CREDIT DISTRIBUTION: PROPERTIES

We analyze wealth condensation in an asymptotic fashion in Sec. V-A, and for finite networks in Sec. V-B. Sec. V-C discusses the role of pricing schemes.

### A. Asymptotic Condensation With Growing Network Sizes

Consider a P2P network with its size growing unboundedly. The total credit  $M$  approaches infinity, while the average peer wealth  $c = \frac{M}{N}$  remains constant (also equal to the amount of initial credits at each peer). In such a growing network, we declare that wealth condensation occurs if the amount of credits at at least one peer grows *unboundedly*. We prove in Theorems 2 and 3 that condensation will not occur if average peer wealth is smaller than a threshold, and vice versa. Let's first define a threshold constant  $T$ :

$$T = \lim_{z \rightarrow 1^-} \int_0^1 \frac{w}{1-zw} f(w) dw, \quad (4)$$

where  $f(w)$  is a continuous, first-order differentiable function over  $[0, 1]$ , satisfying  $u_i = f(u_i), \forall i$ .

**Theorem 2.** *If average peer wealth  $c$  satisfies  $c \leq T$ , then as  $N \rightarrow +\infty$ , there exists a finite constant  $Y$  that upper-bounds the expected wealth  $\bar{B}_i$  at each peer  $i$  at equilibrium, i.e.,  $\bar{B}_i < Y, \forall i$ .*

Proof can be found in our technical report [31]. Theorem 2 shows that when average peer wealth is below a threshold decided by the normalized utilization vector, the expected peer wealth at equilibrium is always bounded, and thus no wealth condensation occurs.

**Theorem 3.** *If the average peer wealth  $c$  is larger than  $T$ , then as  $N \rightarrow +\infty$ , there exists a peer  $i$  whose expected wealth  $\bar{B}_i$  at equilibrium is unbounded, i.e.,  $\bar{B}_i \rightarrow +\infty$ .*

We prove Theorem 3 in our technical report [31]. Theorem 3 states that if average wealth exceeds  $T$ , wealth condenses to at least one of the peers.

The following corollary shows that if we have *symmetric utilization* among the peers, i.e., the ratio of earning rate over spending rate  $\frac{\lambda_i}{\mu_i}$  is equal at any peer  $i$ , wealth condensation is unlikely to happen.

**Corollary.** *Under symmetric utilization at equilibrium, the threshold  $T$  in Theorem 2 and 3 goes to  $+\infty$ , and no wealth condensation will occur as  $N \rightarrow +\infty$ .*

Under symmetric utilization,  $\vec{u} = \{1, \dots, 1\}$ . Furthermore,  $f(w) = 1$  satisfies Eqn. (4). Therefore

$$T = \lim_{z \rightarrow 1^-} \int_0^1 \frac{w}{1-zw} f(w) dw = \lim_{z \rightarrow 1^-} \int_0^1 \frac{1}{1-z} dw \rightarrow +\infty$$

The condition in Theorem 2 becomes  $c < +\infty$ , and is always satisfied. No wealth condensation occurs in this case.

### B. Credit Distribution in Finite Networks

We now turn our attention to the case where network size and total wealth are finite. In such networks, a peer's wealth cannot grow unboundedly and wealth condensation refers to the extremely unbalanced distribution of credits. We study the *skewness* of the distribution, i.e., degree of

condensation, and its dependence on P2P system parameters. We first derive the probability mass function for each queue's length at equilibrium. We then consider the case of symmetric utilization to analyze the impact of  $c$  on the skewness of the wealth distribution and the efficiency of P2P content exchange.

1) *Credit Distribution: Joint Probabilities:* Based on the product form equilibrium distribution in Eqn. (3), we apply the multinomial theorem to derive:

$$\left( \sum_{i=1}^N u_i \right)^M = \sum_{\sum_i b_i = M} \frac{M!}{\prod_i b_i!} \prod_{i=1}^N u_i^{b_i}. \quad (5)$$

We next compute the probability that peer  $i$  has wealth  $b_i$  by giving the marginal probability mass function:

$$\mathbf{Q}\{B_i = b_i\} = \frac{u_i^{b_i} M!}{(M - b_i)! b_i!} \frac{((\sum_{j=1}^N u_j) - u_i)^{M - b_i}}{(\sum_{j=1}^N u_j)^M}. \quad (6)$$

Apply  $u_i = 1, \forall i$ , in cases of symmetric utilization:

$$\mathbf{Q}\{B_i = b_i\} = \frac{M!}{(M - b_i)! b_i!} \left( \frac{N - 1}{N} \right)^M (N - 1)^{-b_i}. \quad (7)$$

Then further plug in  $(N - 1)^{-b_i} = e^{-b_i \ln(N-1)}$ :

$$\mathbf{Q}\{B_i = b_i\} = \left( \frac{N - 1}{N} \right)^M \binom{M}{b_i} e^{-b_i \ln(N-1)}, \quad (8)$$

which is a probability mass function of a non-trivial distribution. Fig. 2 depicts the *Lorenz curve* of the probability distribution with different  $M$  and  $N$ . The Lorenz curve depicts the cumulative distribution [32], and is generated by first sorting the queue lengths, and then plotting the cumulative percentage of bottom peers on the x axis, the percentage of the total credits they have on the y axis.

2) *Average Wealth vs. Skewness of Distribution:* The skewness of credit distribution in Eqn. (8) can be formally measured by the *Gini index* [17]. Computed using the Lorenz curve of the distribution, the Gini index is the ratio of (a) the area between the line of perfect equality and the Lorenz curve, and (b) the total area below the line of perfect equality. The 45° line in Fig. 2 represents perfect equality, for equal wealth at each peer. The more skewed a distribution is, the lower its Lorenz curve is, and the larger the Gini index is.

Fig. 2 shows that the distribution is more skewed with a larger average wealth  $c$ . Our numerical results in Fig. 3 further verify this by plotting the Gini index of the credit distribution vs.  $c$ , in systems of various sizes after they have evolved for a long time (uniform chunk pricing is employed; other settings are similar to those to produce Fig. 1). The curve indicating the Gini index grows rapidly when average wealth increases at first and then slowly towards 1. It shows that in finite networks, the average wealth is still an important factor for credit distribution. Allocating more initial credits to peers increases the risk of wealth condensation.

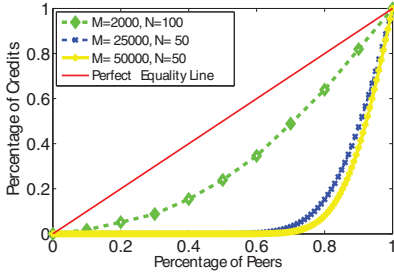


Fig. 2. Lorenz curves of the probability mass function in (8)

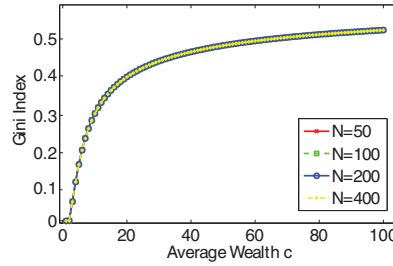


Fig. 3. Gini index vs. average wealth  $c$ .

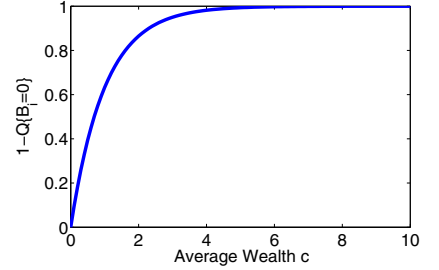


Fig. 4.  $1 - Q\{B_i = 0\}$  vs. average wealth  $c$

### 3) Average Wealth $c$ vs. Content Exchange Efficiency:

Considering the possibility of no credits at a peer  $i$ , the actual average departure rate of credits from  $i$  is  $\mu_i \times (1 - Q\{B_i = 0\})$ . Based on Eqn. (8), we have

$$\begin{aligned} \mu_i (1 - Q\{B_i = 0\}) &= \mu_i \left(1 - \left(\frac{N-1}{N}\right)^M\right) \\ &= \mu_i \left(1 - \left(1 - \frac{1}{N}\right)^{-N}\right)^{-\frac{M}{N}} \\ &\approx \mu_i (1 - e^{-\frac{M}{N}}) = \mu_i (1 - e^{-c}), \end{aligned} \quad (9)$$

when  $N$  is large. Consequently, when the average wealth  $c$  decreases, the actual departure rate decreases too, *i.e.*, the downloading speed drops and the efficiency of content exchange is low. Our numerical results in Fig. 4 validate the above analysis. This observation suggests that the initial credit allocated to peers should not be too small either, to facilitate healthy content exchanges.

### C. Effects of Pricing Schemes

In a credit-based P2P system, the max spending rates in  $\vec{\mu}$  and the transfer probabilities in  $\mathbf{P}$  depend on the content availability and prices at neighbor peers. We next assume a system where all chunks are equivalently useful (*e.g.*, when they are coded with network coding), and focus on the effect of pricing. Recall  $r_{ji}$  is the chunk transfer rate from peer  $j$  to peer  $i$ , and  $s_j$  is the chunk price at peer  $j$ . We have  $\mu_i p_{ij} = r_{ji} s_j$ , and thus  $\mu_i = \sum_{j=1}^N r_{ji} s_j$ , since  $\sum_{j=1}^N p_{ij} = 1, \forall i$ . Consider a simple uniform pricing scheme:  $s_i = \tilde{s}, i = 1, \dots, N$ . In this scenario,  $\mu_i = \tilde{s} \sum_{j=1}^N r_{ji}$ . There are two cases:

1) *Streaming content distribution*, in which  $\sum_{j=1}^N r_{ji}$  equals to the required streaming rate  $\tilde{r}$ . Therefore,  $\mu_i = \tilde{s}\tilde{r}, \forall i$ , *i.e.*, the max spending rates at all peers are equal. Furthermore, since there is no difference among neighbors of peer  $i$ , its credit transfer probabilities are equal, *i.e.*,  $p_{ij} = \frac{1-p_{ii}}{N-1}, \forall j \neq i$ , regardless of the fraction of the credits it reserves (*i.e.*,  $p_{ii}$ ). Combining this property with  $\vec{\lambda}\mathbf{P} = \vec{\lambda}$ , we conclude  $\lambda_i$ 's in the equilibrium  $\vec{\lambda}$  are all equal too.

Therefore, we derive  $\vec{u} = \{1, \dots, 1\}$  in such streaming content distribution. Further by the *corollary* in Sec. V-A, the credits are unlikely to condense to a small number of peers.

2) *Elastic content distribution*, *e.g.*, file sharing, in which the aggregate downloading rate  $\sum_{j=1}^N r_{ji}$  at different peers

may not be the same, and thus the spending rates  $\mu_i$ 's may be different. Though  $\lambda_i$ 's are still equal at all peers,  $\vec{u}$  is unlikely to be an all-1 vector.

If peers charge non-uniform chunk prices, the utilization  $u_i$ 's are different, and wealth condensation could occur, for both streaming and elastic content distribution. Analysis of the credit distribution here is closely dependent on the system-specific protocol implementation.

## VI. SIMULATION STUDIES

We have performed extensive simulations to verify the analytical results, as well as to derive new insights in practical scenarios. We use Java to implement a state-of-the-art mesh-based P2P live streaming system, with a discrete-event P2P simulator that supports various peer dynamics. The streaming protocol is based on a representative P2P streaming systems, UUSEE [28], while credit transfers are enabled together with content downloads. The overlay topologies used are all scale-free. The number of neighbors of a peer follows a Power-law distribution,  $P(D) \sim D^{-k}$ , where the fraction  $P(D)$  of nodes in the system have  $D$  neighbors, and the shape parameter is  $k = 2.5$ . Average number of neighbors is 20. Credit transfer probabilities to neighbors are decided by their data chunks availability during streaming.

We configure the credit earning and spending rates into two cases: (1) symmetric utilization with  $\vec{u} = \{1, 1, \dots, 1\}$ ; (2) asymmetric utilization. In our default setting, initial wealth at each peer is  $c = 100$ . We first fix the network size to 1000, and then investigate networks with dynamic peer joins and departures. Uniform chunk pricing is used by default, at 1 credit per chunk.

### A. Convergence of Credit Distribution

We first investigate whether a stable state of the credit queue length distribution can be reached with  $\lambda_i \leq \mu_i$  at each peer  $i$ , under symmetric utilization.

Fig. 5 and 6 plot the distribution of credits at the early stage (0 – 20000 seconds) and the later stage (20000 – 40000 seconds) of the experiment, respectively. Each curve in each figure plots the credit distribution at one selected time. In Fig. 5, the flatter curves correspond to earlier times, while the more steeper ones represent later times. Compared to Fig. 5, the curves in Fig. 6 largely overlap, revealing the convergence



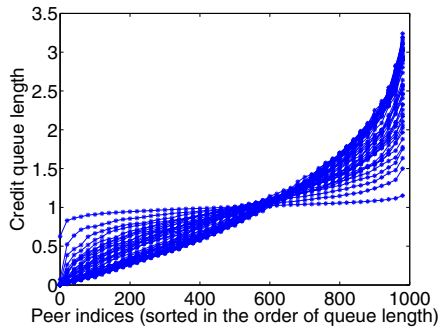


Fig. 5. Credit distribution in the earlier stage.

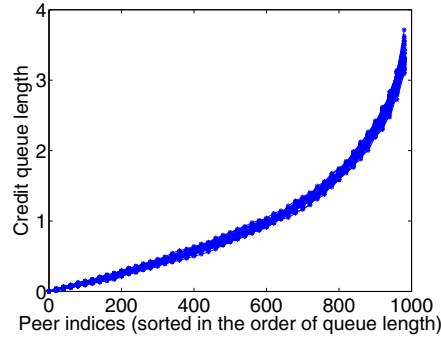


Fig. 6. Credit distribution in the later stage.

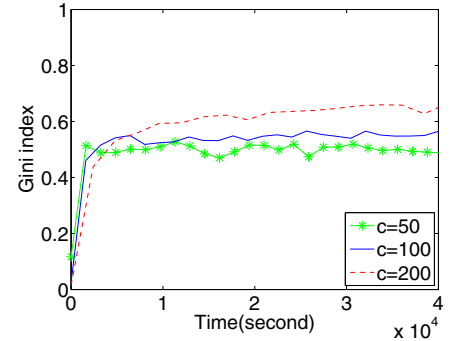


Fig. 7. Evolution of Gini index at  $\vec{u} = \{1, 1, \dots, 1\}$ .

of the credit distribution over time. This validates our analysis in Sec. IV that the distribution of queue lengths stabilizes with the evolution of the system.

To show the existence of stable state from another perspective, we plot in Fig. 7 the Gini index of the credit distribution over time, with different settings of average peer wealth (equal to the amount of initial credits per peer). We observe that the Gini index always converges, regardless of the initial credit amount.

#### B. Impact of Average Wealth

Under the same basic setting from Sec. VI-A, we investigate the impact of  $c$  on the skewness of the credit distribution.

First, with symmetric utilization with  $\vec{u} = \{1, 1, \dots, 1\}$ , Fig. 7 depicts the evolution of the Gini index over time, for different average wealth  $c$ . Besides the convergence of the Gini index over time, it also shows that the larger the average wealth, the larger the Gini index, consistent with our observation in Sec. V-B3.

With an asymmetric utilization, Fig. 8 depicts the evolution of the Gini index over time, at different values of  $c$ . In all cases, the stable state is reachable, and the larger  $c$  is, the larger the stabilized Gini indices are.

#### C. The Role of Taxation

Taxation is a common measure in the field of economics, for balancing resource allocation [33]. We next explore its impact on the credit distribution. For a peer with a wealth above a given *tax threshold*, the system collects a fixed proportion of its income. Whenever the system has collected  $N$  units of credits, it returns a unit to each peer. We adapt the configuration of asymmetric utilization where the risk of condensation is high. Average wealth is set to  $c = 100$ . We explore whether and how the tax rate and tax threshold have impact on credit condensation.

Fig. 9 plots the Gini index of credit queue length, from which we note three observations. First, introducing tax can prevent the system from evolving towards serious skewness of credit distribution. Second, increasing the tax threshold can reduce the Gini index of credit distribution. Third, when the threshold is far smaller than the average amount of credits (50 vs. 100), the curves for tax rate of 0.1 and 0.2 are

almost overlapping. This suggests that increasing tax rate can not effectively help the poor when the threshold is too low. When the tax threshold is close to average wealth instead (80 vs. 100), increasing tax rate can effectively redistribute wealth.

#### D. Dynamic Peer Spending Rates

We now consider a practical case not covered in our analysis: we allow a peer  $i$  to dynamically adjust its maximum spending rate  $\mu_i$  according to its instantaneous credit amount. When the amount is above a threshold  $m$ , it acts more aggressively by increasing its spending speed; otherwise, it sticks to an initial value  $\mu_i^s$ . In particular,  $\mu_i$  is adjusted according to:

$$\mu_i = \begin{cases} \frac{\mu_i^s B_i}{m} & \text{if } B_i > m \\ \mu_i^s & \text{if } B_i \leq m \end{cases}$$

Fig. 10 shows that the stabilized Gini index of credit distribution, under dynamic spending rates, is smaller than that with fixed spending rates. This suggests that allowing peers to dynamically adjust their spending speed is generally beneficial for mitigating credit condensation.

#### E. Impact of Peer Dynamics

We next examine the impact of peer dynamics on the equilibrium and skewness of credit distribution. Upon joining the overlay, a peer is assigned with  $c$  credits; upon departure, it takes away its credits in possess. Therefore, the system studied here corresponds to an open Jackson network, and provides complementary results to the results obtained on closed queueing networks we have obtained so far.

Different from the static network case, the peer queues do not have a stable state in dynamic networks. Nonetheless, the figures in Fig. 11 reveal that the Gini indices of credit distribution still converge into a small range over time, *i.e.*, the credit distribution is pursuing the “equilibrium” state.

Fig. 11 plots the evolution of the Gini index of the credit distribution under different dynamics settings. In Fig. 11 (1), the size of the overlay (1000) is maintained overtime with peer dynamics, by keeping peer arrival rate  $\times$  peer lifespan = expected size of the overlay. We observe that the Gini indices in dynamic overlays are typically smaller than those in a static

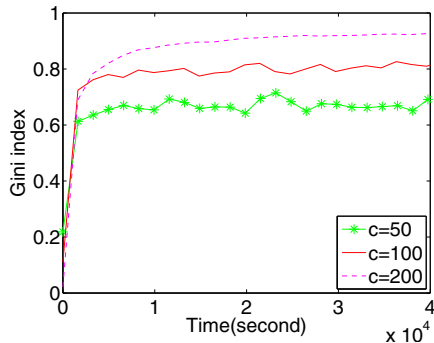


Fig. 8. Evolution of Gini index at  $\bar{u} \neq \{1, 1, \dots, 1\}$ .

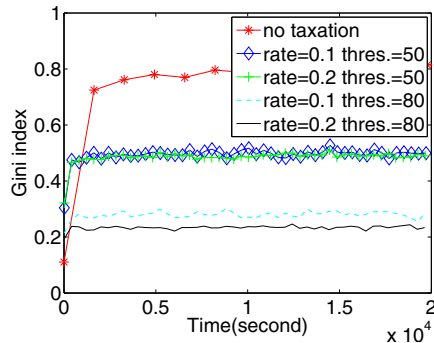


Fig. 9. Skewness of credit distribution at different tax rates and thresholds.

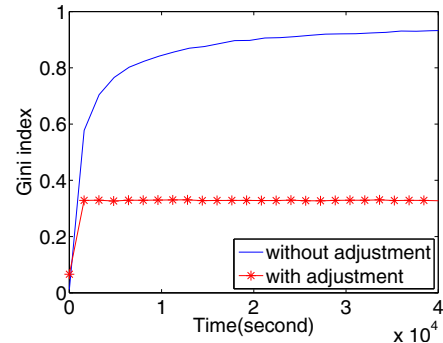


Fig. 10. Static vs. dynamic spending rate: a comparison.

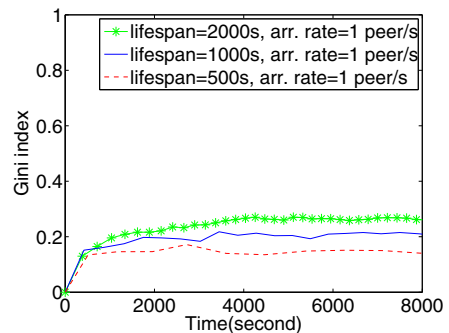
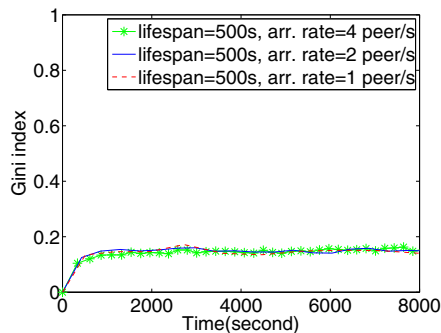
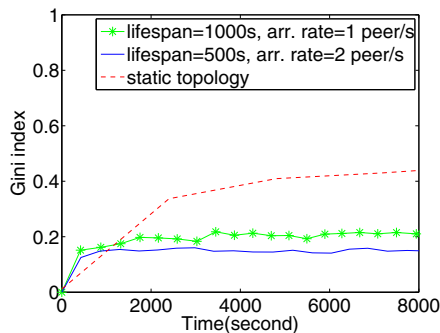


Fig. 11. Impact of dynamics on the skewness of credit distribution: (1) with fixed overlay size; (2) with fixed mean peer lifespan; (3) with fixed peer arrival rate.

overlay, as peers may have departed from the system before having a chance to accumulate an excessive amount of credits.

In Fig. 11 (2) and (3), we fix the average peer lifespan and arrival rate, respectively. From (2), we observe that the skewness of the credit distribution does not change much with different peer arrival rates. From (3), we observe that the skewness increases when peer lifespan is longer. This shows that when the arrival rate is moderate (in contrast to bulk arrival), increasing arrival rate has little impact on credit condensation. However, increasing peer lifespan allows a rich peer to become richer progressively. Overall, we conclude that the phenomenon of wealth condensation follows similar rules in a dynamic network (open queueing network) as in a static network (closed queueing network).

## VII. CONCLUDING REMARKS

This paper focuses on the wealth condensation phenomenon in credit-based P2P content distribution systems. We study whether and how a P2P system with virtual currencies can sustain healthy wealth and data circulation. Our contributions are three fold. First, we propose a novel, queueing network based model to characterize credit circulation in P2P markets. Second, we provide theoretical analysis on the occurrence of condensation and its relation to key P2P network parameters. Third, we perform extensive simulation studies on such relations and derive further insights. We summarize our findings to conclude the paper:

*Average Wealth.* The average peer wealth level turns out to directly affect the occurrence of condensation and the speed of condensation. In a practical system, injecting too many credits into the network may increase the skewness of wealth distribution.

*Credit Spending.* The threshold for average wealth that leads to condensation depends on the maximum rate peers are willing to spend credits. Therefore, in practical system design, encouraging aggressive credit expenditure is a possible measure to prevent credit condensation.

*Taxation.* Applying income tax may mitigate condensation, with appropriately set tax threshold and rate. However, in practice one needs to consider the added system complexity and the side effect on the motivation of sharing.

*Pricing Mechanism.* We observe that pricing policies at peers do affect the occurrence of credit condensation. However, a detailed characterization of non-trivial pricing mechanisms, e.g., pricing through auctions, is beyond the scope of this first attempt to explore the sustainability of credit-based P2P system. We plan to study it in future work.

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