How Local Information Improves Rendezvous in Cognitive Radio Networks

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***Abstract*—Cognitive Radio Network (CRN) is a promising technique for solving the wireless spectrum scarcity problem. Rendezvous is the fundamental process of CRNs. We aim at designing faster rendezvous algorithms for CRNs. We find that local information such as user's ID and the label of an available channel is very useful for designing faster rendezvous algorithms. First, we propose the Sequence-Rotating Rendezvous (SRR) algorithm. The SRR algorithm can guarantee rendezvous for any two users i and j in $(2P^2 + 2P)$ timeslots, where P is the least prime not less than the total number of channels in the network. Second, we utilize the user's identifier (ID) to design an ID-based Rendezvous (IDR) algorithm. The IDR algorithm can guarantee rendezvous for any two users i and j in $(l + 1)(P_i + 2)(P_j + 2)$ timeslots, where P_i and P_j are the smallest primes which are not less than the numbers of available channels of users i and j respectively. Third, we propose a Channel-Label-based Rendezvous (CLR) algorithm which can guarantee rendezvous for any two users in $((P_i+2)(P_j+2)+P_N)(\lceil log_2 N \rceil+1)$ timeslots, where N is the total number of channels in the network and P_N is the least prime which is not less than N. The theoretical Maximum Time To Rendezvous (MTTRs) of the three algorithms we propose are less than those of the state-of-the-art algorithms in the corresponding categories respectively in certain scenarios. All of our algorithms can be used in multi-user scenarios. We conduct a number of experiments to compare our algorithms with state-of-the-art rendezvous algorithms in different scenarios, the results of which confirm our theoretical analysis.**

***Index Terms*—Cognitive Radio Network, Local Information, Rendezvous Algorithms, Channel Label, identifier**

I. Introduction

The wireless spectrum is an invaluable resource for transmission between wireless devices. Generally, it is divided into the licensed portion for paying users (also called primary users or PUs) and the unlicensed portion for the unlicensed users (also called secondary users or SUs). With the development of wireless communication technology, the unlicensed spectrum is becoming more and more crowded due to rapid growth of wireless devices [12] and wireless services. On the other hand, however, the licensed spectrum remains to be underutilized most of the time [16], [9]. Cognitive radio networks (CRNs) were thus proposed in order to alleviate the spectrum scarcity problem of the unlicensed spectrum, whose idea is to allow SUs to sense (via cognitive radios which have been equipped) and utilize vacant parts of the licensed spectrum opportunistically.[1]

In CRNs, many important processes are constantly being executed, such as broadcasting, routing, data gathering, etc. *Rendezvous* is the fundamental process of CRNs, which aims at finding an available licensed channel for neighboring users to communicate. If rendezvous is not first accomplished, the other tasks of CRNs simply cannot commence. Since PUs have priority over SUs, when a PU is occupying a channel, no SU can access the channel. To construct a communication link on a common channel, some works assume there exists a central controller which has full knowledge of all the users' available channel sets, then the users can be assigned with a common available channel for communication [10], [11], [15], [18]. Alternatively, a central control channel (CCC) can be used, which can emulate the function of a central controller. However, establishing a central controller is costly, which could also turn into a bottleneck and be vulnerable to attacks. Therefore, many distributed rendezvous algorithms have been proposed which do not require any central controller or CCC and the users can run their algorithms independently and locally [13], [7], [19], [20], [21]. Algorithms of this kind are called "blind" rendezvous algorithms and are more practical than centralized ones. Channel Hopping (CH) [1], [2] is a widely adopted technique in blind rendezvous algorithms, by which a user tries to select a channel for rendezvous according to a pre-generated channel hopping sequence. Most blind rendezvous algorithms utilize the labels of channels to construct CH sequences. Some blind rendezvous algorithms utilize users' IDs in their construction of CH sequences.

Existing blind rendezvous algorithms can be categorized into different categories, as follows.

(i) Synchronous/asynchronous algorithms. Synchronous algorithms [24], [22], [17] assume there exists a global clock and every user in the network starts the rendezvous process at the same time. Asynchronous algorithms [6], [7], [3] do not require a global clock and every user can start the rendezvous process at any time.

[1]Unless specifically stated, "users" in the rest of the paper refers to SUs.

TABLE I
Comparisons between rendezvous algorithms

Algorithm	MTTR	Time	Anonymous or Non-anonymous	Oblivious or Non-oblivious	Sequence
JS [14]	$3NP(P-G)+3P$	Asynchronous	Anonymous	Non-oblivious	Global
EJS [13]	$4P(P+1-G)$	Asynchronous	Anonymous	Non-oblivious	Global
DRDS [7]	$3P^2+2P$	Asynchronous	Anonymous	Non-oblivious	Global
CRSEQ [19]	$P(3P-1)$	Asynchronous	Anonymous	Non-oblivious	Global
DSCR [23]	$(2P+\lfloor P/2 \rfloor)(P-G+1)$	Asynchronous	Anonymous	Non-oblivious	Global
SRR(this paper)	$2P^2+2P$	Asynchronous	Anonymous	Non-oblivious	Global
CBH [6]	$2l_p \max\{P_i,P_j\}^2$	Asynchronous	Non-anonymous	Oblivious	Local
A-HCH-Optimal [4]*	$\approx (l+log_2 l)P_iP_j$	Asynchronous	Non-anonymous	Oblivious	Local
A-HCH-η_1 [4]	$(2l+3)P_iP_j$	Asynchronous	Non-anonymous	Oblivious	Local
A-HCH-η_2 [4]	$(l+\left\lceil \sqrt{l} \right\rceil (2+\lceil log_2 l \rceil)+3)P_iP_j$	Asynchronous	Non-anonymous	Oblivious	Local
IDR(this paper)	$(l+1)(P_i+2)(P_j+2)$	Asynchronous	Non-anonymous	Oblivious	Local
MTP [8]	$64max\{\lvert V_i\rvert,\lvert V_j\rvert\}(\lceil loglog\ N \rceil+1)$	Asynchronous	Anonymous	Non-oblivious	Local
CLR(this paper)	$((P_i+2)(P_j+2)+P_N)(\lceil log_2 N \rceil+1)$	Asynchronous	Anonymous	Non-oblivious	Local

Remark: N is the total number of channels in the network; P is the smallest prime which is not less than N; G is the number of common available channels; l is the length of user ID; * denotes that this algorithm is a centralized algorithm; l_p is determined by the two users' IDs and the detailed information can be find in [6]; $|V_i|$ and $|V_j|$ are the sizes of the available channel sets of user i and j respectively; P_i and P_j are the smallest primes which are not less than $|V_i|$ and $|V_j|$ respectively; P_N is the smallest prime which is not less than N.

(ii) Anonoymous/non-anonymous algorithms. Anonymous algorithms [7], [13] do not require every user to have a unique ID. Non-anonymous algorithms [6], [4] assume there exists a unique identifer (ID) for every user and make use of a user's ID to generate the channel hopping sequence.

(iii) Oblivious/non-oblivious algorithms. Oblivious algorithms [6], [4] do not require any global labeling of all the channels. Non-oblivious algorithms [6], [7], [13] require global labeling of channels and make use of the labels to guarantee rendezvous.

(iv) Global-sequence-based/semilocal-sequence-based algorithms/local-sequence-based algorithms. Global-sequence-based algorithms [7], [23] construct channel hopping sequences containing all the channels. If a channel is not available to a user, it is replaced by an available channel. Local-sequence-based algorithms [21], [20], [6] generate channel hopping sequences based on only local available channels as if a user is not aware of the existence of other channels. There exist other algorithms like HH [21] and ICH [20], whose hopping sequences are based on the channel labels between the smallest and the largest channel label. We call these semilocal-sequence-based algorithms.

Rendezvous algorithms can be judged according to some common metrics, such as *Maximum Time To Rendezvous (MTTR)* and *Expected Time To Rendezvous (ETTR)*. *Time To Rendezvous (TTR)* is the number of timeslots consumed from the last user starting rendezvous until the completion of rendezvous. ETTR is the expected TTR of an algorithm. MTTR is the maximum TTR needed to achieve rendezvous. In other words, MTTR is the TTR in the worst case.

Although the state-of-the-art rendezvous algorithms are quite well designed and some of their MTTRs can even match or approach the theoretical lower bounds, there still exists room for further improvements. We compare our algorithms with the state-of-the-art rendezvous algorithms in Table I.

In this paper, we introduce some simple but time-efficient rendezvous algorithms which make use of local information. The following are the main contributions of our paper:

1) We make use of the label of an arbitrary available channel of a user and propose the Sequence-Rotating Rendezvous (SRR) algorithm whose MTTR is (P^2+2P) timeslots.
2) Based on the user IDs, we propose the ID-based Rendezvous (IDR) algorithm whose MTTR is $(l+1)(P_i+2)(P_j+2)$ timeslots.
3) Based on the label of an arbitrary available channel of a user, we propose the Channel-Label-based Rendezvous (CLR) algorithm whose MTTR is $((P_i+2)(P_j+2)+P_N)(\lceil log_2 N \rceil+1)$ timeslots.
4) The theoretical MTTRs of the proposed algorithms SRR, IDR, CLR are less than those of the state-of-the-art algorithms of the corresponding categories respectively in certain scenarios.
5) We conduct a number of experiments to compare our algorithms with state-of-the-art algorithms. The results are consistent with our theoretical analysis.

The rest of this paper is organized as follows. Section II introduces the background and some related works. Section III introduces the fundamental model and provides the problem formulation. Section IV presents the details of the SRR algorithm and analyzes its performance. Section V presents the details of the IDR algorithm and analyzes its performance. Section VI presents the details of the CLR algorithm and analyzes its performance. Simulation results are discussed in Section VII. We conclude this paper in Section VIII.

II. Background and Related Works

A. Prime number and Co-prime Numbers

Prime number is a very important concept in number theory, which provides an important foundation for our algorithms. We first introduce the definition of prime number:

Definition 2.1: A prime number is a natural number which is larger than 1 and can be divided with no remainder only by 1 and itself.

The smallest prime number is 2. There is an important theorem about prime number:

Theorem 1: If n is a positive integer and $n \geq 2$, there must exists at least one prime number between n and 2n.

This theorem is called Bertrand-Chebyshev Theorem and the proof of it can be found in [5].

Then we introduce the definition of composite number:

Definition 2.2: A composite number is a natural number which is larger than 1 and has factors besides 1 and itself.

The smallest composite number is 4. There is a property about prime and composite numbers:

Lemma 2.1: Any positive integer larger than 1 is either a prime number or a composite number.

Then we introduce the definition of co-prime numbers:

Definition 2.3: Two nonzero natural integers a and b are said to be co-prime if the only common factor which could divide them with no remainder is 1.

Co-prime numbers have many important properties:

Lemma 2.2: If a and b are co-prime, the least common multiple of them is their product: $a \times b$.

Lemma 2.3: If a and b are two consecutive positive integers, a and b are co-prime.

Lemma 2.4: If a and b are two consecutive positive odd numbers, a and b are co-prime.

Lemma 2.5: Suppose a is a prime number and b is a composite number. If a is larger than b, they are co-prime. If b is larger than a, but b is not a multiple of a, they are co-prime.

We then introduce an important theorem for rendezvous:

Theorem 2: If m and n are co-prime numbers, then for any integer a, the integers $a, a+n, a+2n, \cdots, a+(m-1)n$ are m distinct numbers under modulo-m arithmetic.

This theorem is an extension of a lemma in [21].

Proof: Choose any two integers from $a, a+n, a+2n, \cdots, a+(m-1)n$, the absolute value of their difference is kn, where $0 < k < m$. If kn mod m equal 0, then $kn = lm$, where l is a positive integer. Then kn or lm is a common multiple of m and n. From Lemma 2.2 we know mn is the least common multiple of m and n, and because $0 < k < m$, kn is a common multiple of m and n that is less than mn, which is a contradiction. Hence, Theorem 2 holds. ■

Fig. 1 illustrates an example of Theorem 2. In Fig. 1, the first row is a string of numbers starting from 1. The second row is the results of these numbers under modulo-4 arithmetic. The third row is the results of these numbers under modulo-5 arithmetic. We can clearly see that 1, 5, 9, 13, 17 all correspond to 1 under modulo-4 arithmetic, but correspond to 1, 0, 4, 3, 2 respectively under modulo-5 arithmetic, which are all the possible results under modulo-5 arithmetic. Theorem 2 plays an important role in rendezvous algorithms and is the main tool we use to design the following algorithms.

Number	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
mod 4	1	2	3	0	1	2	3	0	1	2	3	0	1	2	3	0	1	2	3	0
mod 5	1	2	3	4	0	1	2	3	4	0	1	2	3	4	0	1	2	3	4	0

Fig. 1. An example of Theorem 2.

B. Related Works

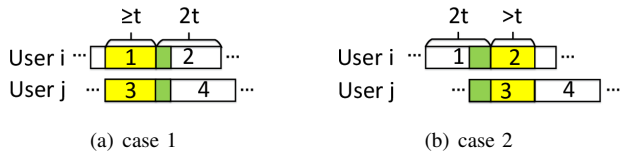
As mentioned in Section I, rendezvous algorithms can be classified into global-sequence-based, semilocal-sequence-based and local-sequence-based algorithms.

(i) Global-sequence-based rendezvous algorithms.

Global-sequence-based rendezvous algorithms design channel hopping sequences based on all the channels in the network. When a channel is not available to a user, the user replaces it with an arbitrary available channel. The JS algorithm [14] uses a CH sequence which consists of two patterns: jump-pattern and stay-pattern. In jump-pattern, the user hops to channels using a pre-selected step length parameter r. In stay-pattern, the user stays on a single channel. The EJS algorithm [13] is an enhanced version of JS and decreases the MTTR from $O(N^3)$ level to $O(N^2)$ level. The DRDS algorithm [7] generalizes CH sequences by constructing a disjoint relaxed difference set. It consists of two stages: listening stage and accessing stage. The CRSEQ [19] algorithm utilizes the property of the triangular numbers generalize the CH sequences for users. It uses the mind of Chinese Remainder Theorem to guarantee rendezvous between users. The DSCR algorithm [23] generalize a CH sequence by constructing a disjoint set cover (DSC). DSC is a famous NP-hard problem and the construction it used is an approximation algorithm.

(ii) Semilocal-sequence-based algorithms. Semilocal-sequence-based algorithms don't use all the channels in the network to generalize CH sequence, but their CH sequences could still include unavailable channels. The HH algorithm [21] supposes that every user can observe a set of successive channels, and builds CH sequences including only these channels. Its CH sequence is composed of three subsequences, which are called fixed, rotating and parity sequences. The ICH algorithm [20] is an improved version of HH, which takes the congestion problem into consideration.

(iii) Local-sequence-based algorithms. Local-sequence-based algorithms generate channel hopping sequences based only on the set of available channels of a user. The CBH algorithm [6] utilizes the difference between the IDs of different users to design CH sequences. The A-HCH-Optimal algorithm [4], A-HCH-η_1 algorithm [4] and A-HCH-η_2 algorithm [4] employ fast/slow sequences to design CH sequences and they also utilize user ID to construct symmetrization classes in the construction of CH sequences. The A-HCH-Optimal algorithm is a centralized algorithm and the A-HCH-η_1 and A-HCH-η_2



(a) case 1 (b) case 2

Fig. 2. An example of different cases of timeslot overlapping.

algorithms can be applied distributively. The MTP algorithm [8] constructs two pointers and let them move at different speeds to guarantee rendezvous.

III. Model and Problem Formulations

In this section, we will introduce the foundation model and some other models which are based on the foundation model. Models are of great importance in the design of rendezvous algorithms, because models constrain the resources that we can use. We design different algorithms under different models to achieve high performance for rendezvous process. We also give the formulation of the rendezvous problem in this section.

A. Foundation Model

Suppose the licensed spectrum is divided into n channels which are non-overlapping. Denote the set of channels as $U = \{1, 2, \cdots, N\}$, where N is the total number of channels in U. We assume that every user in the network is equipped with a cognitive radio. User i has a set of available channels as $V_i = \{v_{i1}, v_{i2}, \cdots, v_{im_i}\}$, where m_i is the number of channels in set V_i. For simplicity, we suppose that V_i will not change during the rendezvous process. Let the length of every timeslot be 2t, where $t = 10ms$ according to IEEE 802.22.

Because the lengths of all users' timeslots are equal, a single timeslot of user j can overlap with at most two consecutive timeslots of user i. Fig. 2 illustrates this situation in two cases. Suppose timeslot 3 is a single timeslot of user j. The left timeslot of user i which overlaps with timeslot 3 is timeslot 1. The right timeslot of user i which overlaps with timeslot 3 is timeslot 4. The yellow part stands for the overlapping part of timeslot 1 and 3, while the green part stands for the overlapping part of timeslot 2 and 3. In Fig. 2(a), the length of the yellow part is greater than or equal to t. In Fig. 2(b), the length of the green part is greater than t. There is a special case that the length of the yellow part or the green part is zero, which means that the timeslots of user i and user j are aligned. Hence, no matter when the users start their rendezvous process, the maximum overlap of their timeslots is at least t, which is enough for two users to find each other and exchange information. The foundation model is the basis for all the three algorithms which we will propose.

B. Problem Formulations

In this paper, we focus on designing rendezvous algorithms for two-user scenario, which is the base for multiple-user rendezvous. Two-user rendezvous algorithms can be expanded into multiple-user rendezvous algorithms easily or with moderate effort. Here we give the formulations of the three problems we will solve:

Problem 1: Utilize an arbitrary available channel of a user to design an asynchronous anonymous non-oblivious global-sequence-based rendezvous algorithm with a low MTTR.

Problem 2: Utilize the ID of a user to design an asynchronous non-anonymous oblivious local-sequence-based rendezvous algorithm with a low MTTR.

Problem 3: Utilize an arbitrary available channel of a user to design an asynchronous anonymous non-oblivious local-sequence-based rendezvous algorithm with a low MTTR.

IV. Sequence-Rotating Rendezvous Algorithm

In this section, we propose a global-sequence-based rendezvous algorithm which is based on the channel label.

We first propose an algorithm to generate the subsequences to be used.

Algorithm 1 Subsequence Generating Algorithm

```
Input: the set of available channels V = {v_1, v_2, ..., v_m}, and length L;
Initialize array S with length L;
Initialize i := 0;
while i < L do
    if i ≤ m then
        S[i] := v_i;
    else
        Randomly choose a channel from V, denote it as channel (c);
        S[i] := c;
    end if
    i := i + 1;
end while
Output: Sequence S.
```

The main idea of Alg.1 is to generate a subsequence S with length L according to V. The process is rather simple. We fill the first m elements in S with the channels in V. For the rest $(L - m)$ elements, we randomly pick channels from V to fill them.

In Alg. 2, we first find the least prime which is not less than the total number of all channels. Then we randomly choose a channel from V, denote it as channel (c), and access this channel for the first $2P$ timeslots. We call this the **first stage** , the following is the **second stage** .

Then, we generate a subsequence S with length P. If channel (a) is in the available channel set V, the a-th channel in S is channel (a). For the channels that are not exist in V, their places will be replaced by channels which are randomly picked from V. And the last $(P - N)$ channels in S are randomly picked from V. The period is $2P$ timeslots. Then we will rotate the sequence by c steps to the right every period. In every period, we will hop to channels according to rotated sequence.

Fig.3 shows an example of the original generated subsequence and the first rotated subsequence. In this example, $V = \{1, 3, 5, 6\}$ and $n = 6$, so $P = 7$. We generate a sequence as Fig.3(a), where the blue rectangles correspond to randomly

Algorithm 2 Sequence-Rotating Rendezvous Algorithm

Input: the set of available channels $V = \{v_1, v_2, \cdots, v_m\}$ and the total number N of channels;
Find the least prime P that ensures $P \geq N$;
Randomly choose a channel from V, denote it as channel (c);
Initialize $t := 0$;
while $0 \leq t < 2P$ **do**
  Access channel (c);
  $t := t + 1$;
end while
Invoke Alg.1 to generate a subsequence S with channel set V and length P;
while Not rendezvous **do**
  if $t \neq 2P$ and t mod $(2P) = 0$ **then**
    Rotate S to the right by c steps;
  end if
  Access channel $S[t \text{ mod } P]$;
  $t := t + 1$
end while

(a) Original subsequence (b) The first rotated subsequence

Fig. 3. An example of generated subsequence when $V = \{1, 3, 5, 6\}$, $N = 6$ and the first rotated subsequence.

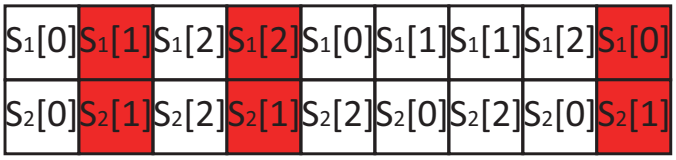


Fig. 4. An example of Theorem 3.

picked channels. A rectangle with number in it, denote it as i, represents the Channel (i). Because the least label in V is 1, the sequence is rotated by 1 step to the right every period. The first rotated sequence is shown in Fig.3(b).

We then introduce a theorem which will be used in the later discuss.

Theorem 3: Suppose there are two aligned subsequences S_1 and S_2 with identical length P and P is a prime. Suppose $0 \leq k_1 \leq P, 0 \leq k_2 \leq P$ and $k_1 \neq k_2$. If in every period we rotate S_1 and S_2 to the right by k_1 and k_2 steps respectively and concatenate them to S_1 and S_2 respectively. Then after P periods, every element of S_1 will meet every element of S_2 and vice versa.

Proof: Because S_1 and S_2 are aligned, $S_1[i]$ will meet $S_2[i]$ in the first period, where $0 \leq i < P$. Without lose of generality, suppose $k_1 > k_2$. Let $k_1 - k_2 = m$, then in the later $(P-1)$ periods, $S_1[i]$ will meet $S_2[(i+m) \text{ mod } P]$, $S_2[(i+2m) \text{ mod } P]$, $\cdots$, $S_2[(i+(P-1)m) \text{ mod } P]$ respectively. According to Theorem 2, $i, i+m, i+2m, \cdots, i+(P-1)m$ are P distinct numbers under modulo-P arithmetic. Hence, $S_1[i]$ meets all the elements of S_2 in P periods and Theorem 3 holds. ■

(a) Situation 1 (b) Situation 2

Fig. 5. Two situations of periods overlapping of user i and j

Fig.4 shows an example of Theorem 3, where the length of S_1 and S_2 is 3. S_1 is rotated to the right by 1 steps every period, and S_2 is rotated to the right by 2 steps every period. We can clearly see that $S_2[1]$ meets $S_1[1], S_1[2], S_1[0]$ in the three consecutive periods.

Theorem 4: Alg.2 can guarantee rendezvous for two asynchronous users in $(2P^2+2P)$ timeslots as long as $V_i \cap V_j \neq \emptyset$.

Proof: We discuss this problem in two cases.

Case 1 : Suppose two users choose the same channel, denote it as channel (c). Without loss of generality, suppose user i starts rendezvous process earlier than user j. For clarity, we divide this case into two subcases.

Subcase 1.1 : Suppose user j starts rendezvous process when user i is still in the first stage, they will both access channel (c) at the same time. Hence, rendezvous is achieved.

Subcase 1.2 : Suppose user j starts rendezvous process when user i is already in the second stage. Then in the following $2P$ timeslots, user i will access each available channel including channel (c) at least once, while user j will stay on channel c in the $2P$ timeslots. Hence, rendezvous can be achieved in the $2P$ timeslots.

Case 2 : Suppose user i and j choose different channels, denote them as channel (c_i) and channel (c_j). When user i and j both get into the second stage, they will both rotate their sequences accordingly. There are two situations of periods overlapping of user i and j, which we will discuss separately.

Subcase 2.1 : Suppose the periods of user i and j are aligned, then the scenario is the same as the setting in Theorem 2 except that we duplicate the current subsequences before rotation and attach them to the corresponding sequences. The difference results in double time delay. According to Theorem 3, rendezvous will be achieved in $(2P^2 + 2P)$ timeslots.

Subcase 2.2 : Suppose the periods of user i and j are not aligned. There are two situations of periods overlapping for user i and j, as illustrated in Fig.5. The yellow rectangle in Fig.5(a) represents the first half of user j's period, while the red rectangle represents the corresponding part in user i's period. The red rectangle includes all the elements of subsequence S_i, because the two halves of user i's period are the same rotated subsequence of S_i. This situation is similar to Subcase 2.1, and according to Theorem 3, rendezvous will be achieved in $(2P^2+2P)$ timeslots. Situation 2 can be analysed in the same way, so we omit it here.

Combine these together, we conclude that Theorem 4 holds. ■

V. ID-BASED RENDEZVOUS ALGORITHM

In this section, we will propose an ID-based asynchronous non-anonymous oblivious rendezvous algorithm. The main idea is to create a channel hopping sequence composed of $(l+1)$ subsequences, where l is the length of ID.

Algorithm 3 ID-based Rendezvous Algorithm

```
Input: the set of available channels V = {v_1, v_2, ..., v_m}, and its binary ID whose length is l bits;
Find the smallest prime P that ensures P ≥ m;
if P = 3 then
    P := 5;
end if
Initialize t := 0, i := 0, j := 0;
Invoke Alg.1 to generate three sequences S_1, S_2, S_3 with channel set V and length P, P+1, P+2 respectively;
Find the channel with the least label in V, denote it as channel (c);
while Not rendezvous do
    i := ⌊t/(l+1)⌋;
    j := t mod (l+1);
    if j < l then
        if ID[j] = 0 then
            Access channel S_2[i mod (P+1)];
        else
            Access channel S_3[i mod (P+2)];
        end if
    else
        Access channel S_1[i mod P];
    end if
    t := t + 1;
end while
```

In Alg.3, we create a channel hopping sequence consisting of $(l+1)$ subsequences. The $(l+1)$ subsequences are generated by Alg.1. There are three kinds of subsequences, which are S_1, S_2 and S_3. The lengths of S_1, S_2 and S_3 are P, $P+1$, $P+2$ respectively.

Fig.6 shows an example of Alg.3. In this example, the length of ID is 5 and user i's ID is 01101. In period 1, the first l elements of period 1 are either $S_2[0]$ or $S_3[0]$. If the corresponding digit of the i-th element in ID is 0, then the i-th element will be $S_2[0]$. If the corresponding digit of the i-th element in ID is 1, then the i-th element will be $S_3[0]$. The last element of period 1 is $S_1[0]$.

Theorem 5: Alg.3 can guarantee rendezvous for two asynchronous users in $(l+1)(P_i+2)(P_j+2)$ timeslots as long as $V_i \cap V_j \neq \emptyset$.

Proof: We discuss the two possible cases:

Case 1 : Suppose $P_i = P_j$. Let $P = P_i = P_j$. We divide this case into two subcases:

Subcase 1.1 : Suppose user i and j's periods are aligned, as illustrated in Fig.7(a). Because user i and user j are two different users, their IDs must be different in at least one digit. Without lose of generality, suppose the k-th digit in ID_i is 0 while the k-th digit in ID_j is 1. Then the k-th element in user i's period corresponds to subsequence S_{2i}, whose length is $P+1$. The k-th element in user j's period corresponds to subsequence S_{3j}, whose length is $P+2$. Then in every period, one element of S_{2i} will meet one element of S_{2j}. According to Lemma 2.3, $P+1$ and $P+2$ are co-prime. Then rendezvous can be achieved in $(l+1)(P+1)(P+2)$ timeslots according to Theorem 2, as long as user i and j have at least one common channel.

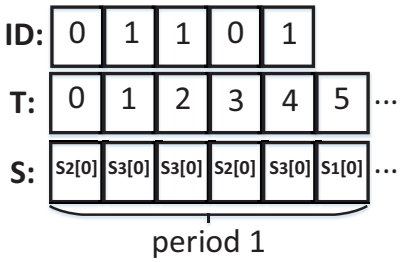
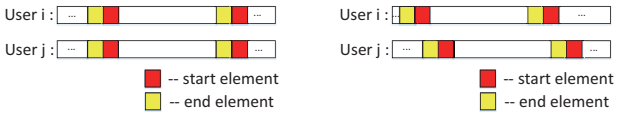


Fig. 6. An example of Alg. 3.



(a) subcase 1 (b) subcase 2

Fig. 7. Two subcases of case 1 and case 2.

Subcase 1.2 : Suppose user i and j's periods are not aligned, as illustrated in Fig.7(b). In this case, the last element in user i's period will meet one of the first $(l+1)$ elements of user j's period. The last element in user i's period corresponds to subsequence S_{1i}. The corresponding element of user j corresponds to subsequence S_{2j} or S_{3j}. From Lemma 2.3 and 2.4, we know that P and $P+1$ are co-prime, P and $P+2$ are also co-prime. Hence, according to Theorem 2, rendezvous can be achieved in $(l+1)P(P+1)$ or $(l+1)P(P+2)$ timeslots respectively.

Case 2 : Suppose $P_i \neq P_j$. We divide this case into two subcases:

Subcase 2.1 : Suppose user i and j's periods are aligned, as illustrated in Fig.7(a). In this case, the last element in user i's period will meet the last element in user j's period. The last element in user i's period corresponds to subsequence S_{i1}. The last element in user j's period corresponds to subsequence S_{j1}. Because $P_i \neq P_j$, rendezvous can be achieved in $(l+1)P_iP_j$ timeslots according to Theorem 2, as long as user i and j have at least one common channel.

Subcase 2.2 : Suppose user i and j's periods are not aligned, as illustrated in Fig.7(b). Without lose of generality, suppose $P_i > P_j$. In this case, the last element in user i's period will meet one of the first $(l+1)$ elements of user j's period. The last element in user j's period will meet one of the first $(l+1)$ elements of user i's period. The last element in user j's period corresponds to subsequence S_{1j}, whose length is P_j. The

corresponding element of user j corresponds to subsequence S_{2i} or S_{3i}, whose lengths are P_i+1 and P_i+2 respectively. Because P_i and P_j are two different primes, $P_i > P_j+2$ or $P_i = P_j+2$. If $P_i > P_j+2$, from Lemma 2.5, we know P_i and P_j+1 are co-prime, P_i and P_j+2 are co-prime. According to Theorem 2, rendezvous can be achieved in $(l+1)P_i(P_j+1)$ or $(l+1)P_i(P_j+2)$ timeslots respectively. If $P_i = P_j+2$, then $P_i+1 = P_2+3$, $P_i+2 = P_j+4$. If $P_j = 2$, $P_i = 2+2 = 4$ is not a prime. In Alg.3, we set P to be 5 if $P = 3$. Because $P_2 \neq 2$ and $P_2 \neq 3$, P_2+3 and P_2+4 can not be divided by P_2 without remainder. From Lemma 2.5, we know that both P_i+1 and P_i+2 are co-prime with P_j. According to Theorem 2, rendezvous can be achieved in $(l+1)(P_i+1)P_j$ or $(l+1)(P_i+2)P_j$ timeslots respectively.

Combine the above together, we conclude that Theorem 5 holds.

■

VI. Channel-Label-based Rendezvous Algorithm

In this section, we will propose a Channel-Label-based Rendezvous (CLR) algorithm which utilizes the label of an arbitrary channel in the available channel set.

Algorithm 4 Channel-Label-based Rendezvous Algorithm

1: Input: the set of available channels $V = \{v_1, v_2, \cdots, v_m\}$ and the number of all channels: N;
2: Find the smallest prime P_N that ensures $P_N \geq N$ and $P_N \geq 5$;
3: Find the smallest prime P that ensures $P \geq m$ and $P \geq 5$;
4: Initialize $l := \lceil log_2 N \rceil$;
5: Initialize $t := 0$;
6: Randomly choose a channel (c) from V;
7: Convert c into a binary string CH whose length is l;
8: **while** Not rendezvous and $t < (l+1)P_N$ **do**
9: Access channel (c);
10: **end while**
11: Invoke Alg.3 to achieve rendezvous with channel set V, binary string CH and length l;

The main idea of Alg.4 is to randomly choose an available channel c from V and use the binary representation of c as the user's ID. It is obvious that the available channel chosen by two users can be identical, but our algorithm can solve this problem, because the label of an available channel is very useful. Alg.4 is composed of two phases, the first phase is the first $(l+2)P_N$ timeslots, the following is the second phase. In the **first phase** , the user stays on the chosen channel (c). In the **second phase** , the user invokes Alg.3 to achieve rendezvous with the binary representation of c as its ID.

Theorem 6: Alg. 4 can guarantee rendezvous for two asynchronous users i and j in $((P_i+2)(P_j+2)+P_N)(\lceil log_2 N \rceil+1)$ timeslots as long as $V_i \cap V_j \neq \emptyset$.

Proof:

We discuss the two possible cases.

Case 1: Suppose the channels (c_i) and (c_j) chosen by user i and j respectively are different, then the corresponding binary representations of c_i and c_j are different in at least one digit. Therefore, they can replace the roles of IDs for user i and j respectively. Hence, by invoking Alg.3, rendezvous can be achieved in $((P_i+2)(P_j+2)+P_N)(\lceil log_2 N \rceil+1)$ timeslots.

Case 2: Suppose the channels (c_i) and (c_j) chosen by user i and j respectively are the same, let $c = c_i = c_j$. Then channel (c) must be a common channel of user i and j. Without lose of generality, suppose user i starts rendezvous process earlier than user j. We then divide this case into two subcases.

Subcase 2.1 Suppose when user j starts rendezvous process, user i is still in the **first phase** . Then user j will hop to channel (c) in its first timeslot and user i is also in channel (c), hence rendezvous is achieve is achieved in 1 timeslot.

Subcase 2.2 Suppose when user j starts rendezvous process, user i is already in the **second phase** . Because user j stays on channel (c) for $P_N(\lceil log_2 N \rceil+1)$ timeslots and $P_N \geq P_i$, channel (c) will meets every elements of user i's subsequence S_1 (defined in Alg.3). Therefore, rendezvous will be achieved in the **first phase** of user j.

Combine these together, we conclude that Theorem 6 holds.

■

VII. Simulation

In this section, we show the simulation results of comparing our algorithms with state-of-the-art algorithms. We implemented the algorithms in C++ language. Every result in the experiments was obtained through running the corresponding algorithm under the corresponding setting 10000 times independently.

We first compare our SRR algorithm with two representative asynchronous non-anonymous non-oblivious global-sequence-based rendezvous algorithm – EJS and DRDS. From Table I, we can see that SRR can have lowest MTTR when the ratio of common available channels to the total number of channels is small. Denote R_i and R_j as the ratios of $|V_i|$ and $|V_j|$ to N respectively. First, we set $V_i = \{1, 2, \cdots, 0.5N\}, V_j = \{0.5N, 0.5N+1, \cdots, N-1\}$ where N is the total number of channels in the network. In this case, $R_i = R_j = 0.5$ and $G = 1$ where G is the number of common channels. We increase N from 10 to 100 by 10 each time. The result is illustrated in Fig.9, from which we can clearly see that the MTTR of SRR grows slower than that of EJS and DRDS. Then we set $V_i = \{0.1N, 0.1N+2, \cdots, 0.6N-1\}$ and $V_j = \{0.5N, 0.5N+1, \cdots, N-1\}$. In this case, $R_i = R_j = 0.5$ and $G = 0.1N$. The result is shown in Fig. 8. We can clearly see that the MTTR of SRR grows slower than that of EJS and DRDS.

Second, we conduct experiments to compare the MTTRs of the IDR algorithm and the state-of-the-art asynchronous non-anonymous oblivious local-sequence-based algorithms – A-HCH-η_1 and A-HCH-η_2. For this kind of rendezvous algorithms, the crux is in the relationship between the increase of MTTR with the increase of the length of user' ID. Hence, we conduct experiments which fixes R_i, R_j and N but increases the length of ID l step by step. We first fix $R_i = R_j = 0.5$ by setting $V_i = \{1, 2, \cdots, 0.5N\}$ and

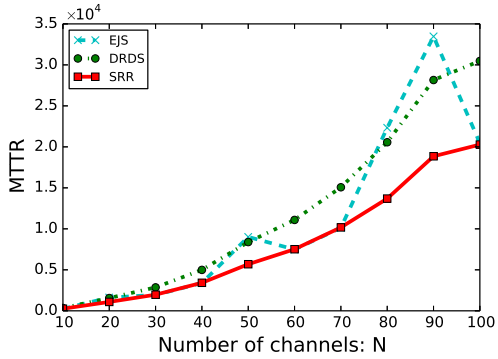


Fig. 8. Comparison between the EJS algorithm, the DRDS algorithm and the SRR algorithm when N increases from 10 to 100, $R_i = R_j = 0.5$ and $G = 1$.

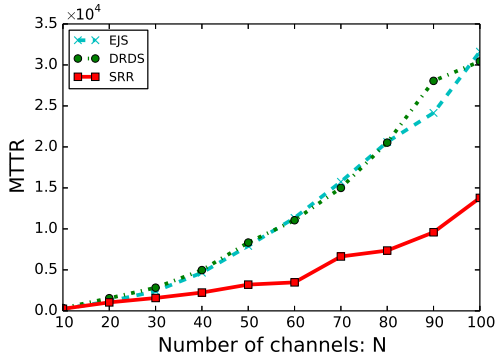


Fig. 9. Comparison between the EJS algorithm, the DRDS algorithm and the SRR algorithm when N increases from 10 to 100, $R_i = R_j = 0.5$ and $G = 0.1N$.

$V_j = \{0.5N, 0.5N + 1, \cdots, N - 1\}$. We fix $N = 50$ and increase l from 10 to 100 by 10 each time. The result is illustrated in Fig.10. We can clearly see that the MTTR of IDR is less than that of A-HCH-η_1 or A-HCH-η_2 in general. Then we repeats this experiment except that we increase N to 100. The result is shown in Fig.11. We can clearly see that the MTTR of IDR is less than that of A-HCH-η_1 or A-HCH-η_2 in general.

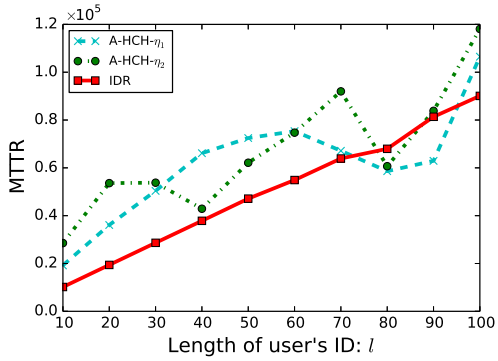


Fig. 10. Comparison between the A-HCH-η_1/η_2 algorithms and the IDR algorithm when l increases from 10 to 100, $R_i = R_j = 0.5$ and $N = 50$.

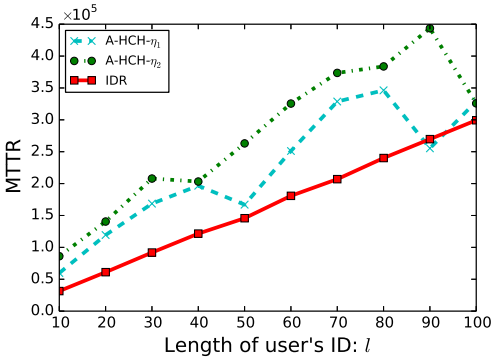


Fig. 11. Comparison between the A-HCH-η_1/η_2 algorithms and the IDR algorithm when l increases from 10 to 100, $R_i = R_j = 0.5$ and $N = 100$.

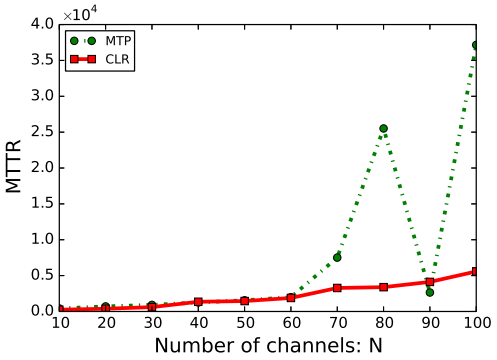


Fig. 12. Comparison between the MTP algorithm and the CLR algorithm when N increases from 10 to 100, $R_i = R_j = 0.2$ and $G = 1$.

Third, we implement experiments to compare our CLR algorithm with the state-of-the-art asynchronous anonymous non-oblivious local-sequence-based algorithm – MTP. For this kind of rendezvous algorithms, the proportion of the available channels to the total number of channels has a dominating influence. We first set $R_i = R_j = 0.2$ by setting $V_i = \{0.3N, 0.3N + 1, \cdots, 0.5N\}, V_j = \{0.5N, 0.5N + 1, \cdots, 0.7N - 1\}$. We increase N from 10 to 100 by 10

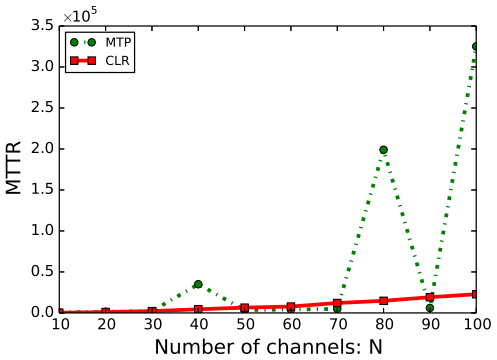


Fig. 13. Comparison between the MTP algorithm and the CLR algorithm when N increases from 10 to 100, $R_i = R_j = 0.5$ and $G = 1$.

each time. The result is illustrated in Fig.12. We can clearly see that the MTTR of CLR is less than that of MTP in general and the MTTR of MTP on some experiment points is very large. This is because the MTP algorithm is designed to be relatively complicated in order to achieve an MTTR of $O(|V_i||V_j|loglog\ N)$ order. However, it results in a large constant coefficient. Hence, the MTTR of the MTP algorithm can be very high, which results in losing superiority. We implement another experiment by increasing R_i and R_j to 0.5 by setting $V_i = \{1, 2, \cdots, 0.5N\}, V_j = \{0.5N, 0.5N + 1, \cdots, N - 1\}$. The result is illustrated in Fig.13. We can see that the situation is similar to that in Fig.12.

VIII. Conclusion

In this paper, we first propose a Sequence-Rotating Rendezvous (SRR) algorithm, which utilizes the label of an arbitrary available channel of a user as local information in generating channel hopping sequence. The SRR algorithm uses some results in number theory and has an MTTR of $(2P^2 + 2P)$ timeslots, where P is the smallest prime which is not less than the total number of channels. Second, we utilize the user's identifier (ID) to design an ID-based Rendezvous (IDR) algorithm. IDR algorithm also uses the thought of number theory and has an MTTR of $(l + 1)(P_i + 2)(P_j + 2)$ timeslots, where l is the length of ID and P_i and P_j are the smallest primes which are not less than the size of available channel sets of user i and j respectively. Third, we propose a Channel-Label-based Rendezvous (CLR) algorithm, which utilizes the binary representation of an arbitrary available channel of a user as the user's ID. CLR algorithm has an MTTR of $((P_i + 2)(P_j + 2) + P_N)(\lceil log_2 N \rceil + 1)$ timeslots, where N is the overall amount of channels in the network and P_N is the least prime which is not less than N. We conducted a number of experiments comparing our algorithms with state-of-the-art rendezvous algorithms and the results show that our algorithms can achieve better performance.

IX. Acknowledgement

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