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Review of Barr M. Fixed Points in Cartesian Closed Categories. *Selected Papers of the 4th Workshop on Mathematical Foundations of Programming Semantics. Theoretical Computer Science* *70*, 1 (1990): 65–72.

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Barr proposes to study domain theory using a semantic model general enough to include situations that cannot be covered by standard ZF set theory. The framework is a Cartesian closed category that has finite limits and a natural number object. Let ω be a natural number object with a partial order. We define D as an ω-complete partially ordered object if and only if an arrow $V\colon [\omega \to D] \to D$ exists such that, for any arrow $f\colon A \to [\omega \to D]$ with transpose $\tilde{f}\colon A \times \omega \to D$, we have $f(a,n) \leq V \circ f(a)$ for all $(a,n) \in A \times \omega$, and if $g\colon A \to D$ is any arrow such that $\tilde{f}(a,n) \leq g(a)$ for all $(a,n) \in A \times \omega$, then $V \circ f \leq g$. In such a case, there is an operator fix: $[D \to D] \to D$ satisfying the usual properties of a least fixed point, namely, that $f(\text{fix}(f)) = \text{fix}(f)$ and that if $f(d) = d$, then $\text{fix}(f) \leq d$.

On the whole, the approach is interesting and mathematically elegant. It would be useful, however, if the author elaborated on the advantages of this approach over other models. Inconsistencies in notation are a minor problem: for example, the author uses both π_1 and p_1 for projections, Inc and p for inclusions, id and A for identity functions, and $\langle f, g\rangle$ and (f, g) for maps from X to $Y \times Y$. The paper also contains several typographical errors, such as confusions between f and $\tilde{f}$, between f_0 and $\tilde{f}_0$, between $f \circ n \circ s$ and $f \circ s \circ n \circ g$, between g and $g \circ p_1$, and between $s0$ and $\text{id} \times s0$.